Key

9.5 AP Practice Problems (p.683) – Derivatives, Arc Length of Vector Functions

1. The domain of the vector function $\mathbf{r}(t) = \langle 3t^2 + t, -\ln(t+2) \rangle$ is

(A) the set of all real numbers (B) $\{t \mid t > 0\}$

(C) $\{t \mid t > -2\}$ (D) $\{t \mid t > -2, t \neq -1\}$

Domain of $\ln(t+2)$ is $t > -2$

2. $\lim_{t \to 2} \left\langle t, \frac{t^2 - 2t}{t - 2} \right\rangle =$

(A) $\langle 2, -2 \rangle$ (B) $\langle 2, 0 \rangle$

(C) $\langle 2, 2 \rangle$ (D) The limit does not exist.

$\lim_{t \to 2} \frac{t(t-2)}{t-2} = 2$

$\boxed{\langle 2, 2 \rangle}$

3. The derivative of the vector function $\mathbf{r}(t) = \langle \sin(3t), -\cos(3t) \rangle$ is

(A) $\mathbf{r}'(t) = \langle \cos(3t), \sin(3t) \rangle$

(B) $\mathbf{r}'(t) = \left\langle \frac{1}{3}\cos(3t), \frac{1}{3}\sin(3t) \right\rangle$

(C) $\mathbf{r}'(t) = \langle 3\cos(3t), 3\sin(3t) \rangle$

(D) $\mathbf{r}'(t) = \langle 3\cos(3t), -3\sin(3t) \rangle$

4. The arc length of the curve traced out by $\mathbf{r}(t) = \langle t^3 + 2t, \ln t \rangle$ from $t = 1$ to $t = 5$ is given by

$L = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$

(A) $\int_1^5 \sqrt{(t^3 + 2t)^2 + (\ln t)^2}\, dt$

(B) $\int_1^5 \sqrt{(3t^2 + 2)^2 + \frac{1}{t^2}}\, dt$

(C) $\int_1^5 \sqrt{\frac{9t^4 + 12t^2 + 5}{t^2}}\, dt$

(D) $\int_1^5 \sqrt{(3t^2 + 2)^2 - \frac{1}{t^2}}\, dt$

5. **Which of the following is the second derivative of the vector function $\mathbf{r}(t) = \left\langle 4t, \frac{1}{2}t^2 + e^t \right\rangle$?**

(A) $\langle 4, t + e^t \rangle$ $\boxed{\text{(B)}}$ $\langle 0, 1 + e^t \rangle$

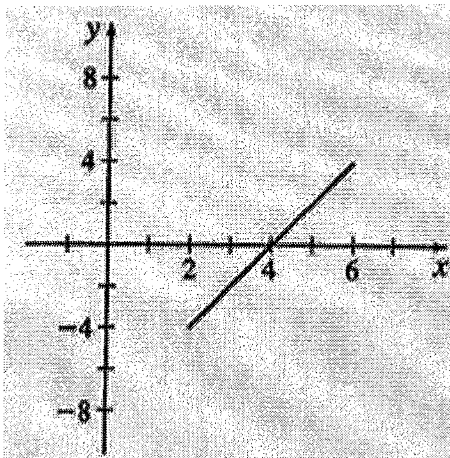
(C) $\langle 1, e^t \rangle$ (D) $\langle 0, e^t \rangle$

6. **Which curve is traced out by the vector function $r(t) = \langle t, 2t - 4 \rangle, 0 \le t \le 4$?**

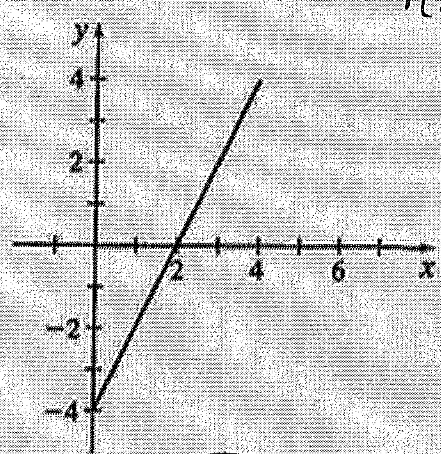
$r(0) = \langle 0, -4 \rangle$

$r(1) = \langle 1, -2 \rangle$

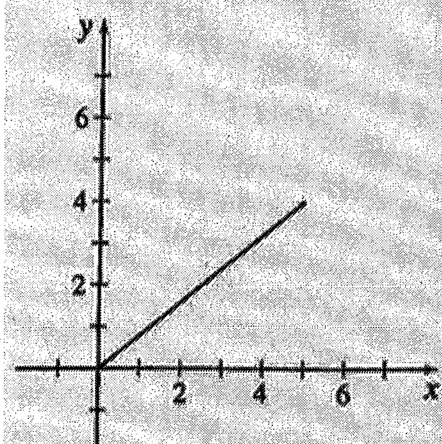
$r(2) = \langle 2, 0 \rangle$



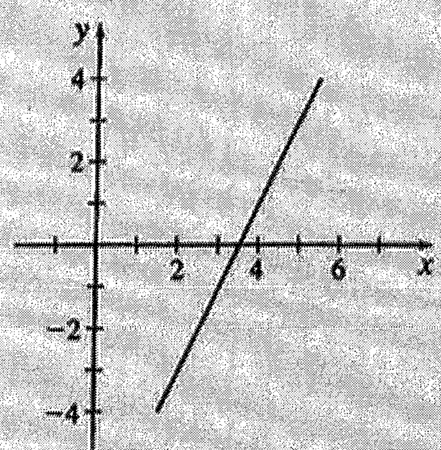
(A)



$\boxed{\text{(B)}}$



(C)



(D)

$r'(t) = \langle 2t, -3 \rangle$

$\boxed{r'(2) = \langle 4, -3 \rangle}$

7. **The tangent vector to the curve traced out by the vector function $\mathbf{r}(t) = \left\langle t^2 + 5, 8 - 3t \right\rangle$ at $t = 2$ is**

$\boxed{\text{(A)}}$ $\langle 4, -3 \rangle$ (B) $\langle 4, 3 \rangle$ (C) $\langle 4, -6 \rangle$ (D) $\langle 9, 2 \rangle$