9.5 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Numerical and Graphical Analysis In Exercises 1–4, explore the Alternating Series Remainder.

(a) Use a graphing utility to find the indicated partial sum S_n and complete the table.

n	1	2	3	4	5	6	7	8	9	10
S_n										

(b) Use a graphing utility to graph the first 10 terms of the sequence of partial sums and a horizontal line representing the sum.

(c) What pattern exists between the plot of the successive points in part (b) relative to the horizontal line representing the sum of the series? Do the distances between the successive points and the horizontal line increase or decrease?

(d) Discuss the relationship between the answers in part (c) and the Alternating Series Remainder as given in Theorem 9.15.

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} = \frac{\pi}{4}$

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n-1)!} = \frac{1}{e}$

3. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$

4. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!} = \sin 1$

Determining Convergence or Divergence In Exercises 5–26, determine the convergence or divergence of the series.

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

6. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3n+2}$

7. $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$

8. $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$

9. $\sum_{n=1}^{\infty} \frac{(-1)^n(5n-1)}{4n+1}$

10. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+5}$

11. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(n+1)}$

12. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

13. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

14. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{n^2+4}$

15. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{\ln(n+1)}$

16. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ln(n+1)}{n+1}$

17. $\sum_{n=1}^{\infty} \sin \frac{(2n-1)\pi}{2}$

18. $\sum_{n=1}^{\infty} \frac{1}{n} \cos n\pi$

19. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

20. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$

21. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+2}$

22. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{\sqrt[3]{n}}$

23. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$

24. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{1 \cdot 4 \cdot 7 \cdots (3n-2)}$

25. $\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{e^n - e^{-n}} = \sum_{n=1}^{\infty} (-1)^{n+1} \operatorname{csch} n$

26. $\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{e^n + e^{-n}} = \sum_{n=1}^{\infty} (-1)^{n+1} \operatorname{sech} n$

Approximating the Sum of an Alternating Series In Exercises 27–30, approximate the sum of the series by using the first six terms. (See Example 4.)

27. $\sum_{n=0}^{\infty} \frac{(-1)^n 5}{n!}$

28. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4}{\ln(n+1)}$

29. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2}{n^3}$

30. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3^n}$

Finding the Number of Terms In Exercises 31–36, use Theorem 9.15 to determine the number of terms required to approximate the sum of the series with an error of less than 0.001.

31. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$

32. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

33. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3-1}$

34. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$

35. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

36. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$

Determining Absolute and Conditional Convergence In Exercises 37–54, determine whether the series converges absolutely or conditionally, or diverges.

37. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

38. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

39. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

40. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+3}$

41. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

42. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

43. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{(n+1)^2}$

44. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+3)}{n+10}$

45. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

46. $\sum_{n=0}^{\infty} (-1)^n e^{-n^2}$

47. $\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^3-5}$

48. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{4/3}}$

49. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$

50. $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$

51. $\sum_{n=0}^{\infty} \frac{\cos n\pi}{n+1}$

52. $\sum_{n=1}^{\infty} (-1)^{n+1} \arctan n$

53. $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2}$

54. $\sum_{n=1}^{\infty} \frac{\sin[(2n-1)\pi/2]}{n}$

WRITING ABOUT CONCEPTS

55. Alternating Series Define an alternating series.

56. Alternating Series Test State the Alternating Series Test.

57. Alternating Series Remainder Give the remainder after N terms of a convergent alternating series.

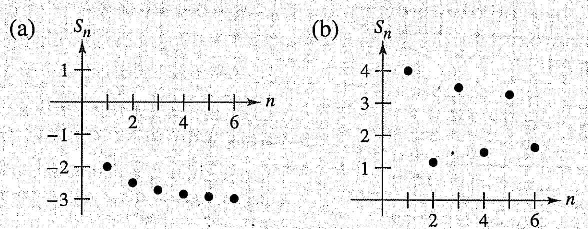
58. Absolute and Conditional Convergence In your own words, state the difference between absolute and conditional convergence of an alternating series.

59. Think About It Do you agree with the following statements? Why or why not?

(a) If both $\Sigma\, a_n$ and $\Sigma\,(-a_n)$ converge, then $\Sigma\, |a_n|$ converges.

(b) If $\Sigma\, a_n$ diverges, then $\Sigma\, |a_n|$ diverges.

60. HOW DO YOU SEE IT? The graphs of the sequences of partial sums of two series are shown in the figures. Which graph represents the partial sums of an alternating series? Explain.



True or False? **In Exercises 61 and 62, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

61. For the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

the partial sum S_{100} is an overestimate of the sum of the series.

62. If $\Sigma\, a_n$ and $\Sigma\, b_n$ both converge, then $\Sigma\, a_n b_n$ converges.

Finding Values **In Exercises 63 and 64, find the values of p for which the series converges.**

63. $\displaystyle\sum_{n=1}^{\infty} (-1)^n\left(\frac{1}{n^p}\right)$

64. $\displaystyle\sum_{n=1}^{\infty} (-1)^n\left(\frac{1}{n+p}\right)$

65. Proof Prove that if $\Sigma\, |a_n|$ converges, then $\Sigma\, a_n^2$ converges. Is the converse true? If not, give an example that shows it is false.

66. Finding a Series Use the result of Exercise 63 to give an example of an alternating p-series that converges, but whose corresponding p-series diverges.

67. Finding a Series Give an example of a series that demonstrates the statement you proved in Exercise 65.

68. Finding Values Find all values of x for which the series $\Sigma\,(x^n/n)$ (a) converges absolutely and (b) converges conditionally.

Using a Series **In Exercises 69 and 70, use the given series.**

(a) Does the series meet the conditions of Theorem 9.14? Explain why or why not.

(b) Does the series converge? If so, what is the sum?

69. $\displaystyle \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{9} + \frac{1}{8} - \frac{1}{27} + \cdots + \frac{1}{2^n} - \frac{1}{3^n} + \cdots$

70. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} a_n,\ a_n = \begin{cases} \dfrac{1}{\sqrt{n}}, & \text{if } n \text{ is odd} \\ \dfrac{1}{n^3}, & \text{if } n \text{ is even} \end{cases}$

Review **In Exercises 71–80, test for convergence or divergence and identify the test used.**

71. $\displaystyle\sum_{n=1}^{\infty} \frac{10}{n^{3/2}}$

72. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2+5}$

73. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2}$

74. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n+1}$

75. $\displaystyle\sum_{n=0}^{\infty} 5\left(\frac{7}{8}\right)^n$

76. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{2n^2+1}$

77. $\displaystyle\sum_{n=1}^{\infty} 100e^{-n/2}$

78. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n+4}$

79. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}4}{3n^2-1}$

80. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$

81. Describing an Error The following argument, that $0 = 1$, is *incorrect*. Describe the error.

$$\begin{aligned} 0 &= 0 + 0 + 0 + \cdots \\ &= (1-1) + (1-1) + (1-1) + \cdots \\ &= 1 + (-1+1) + (-1+1) + \cdots \\ &= 1 + 0 + 0 + \cdots \\ &= 1 \end{aligned}$$

PUTNAM EXAM CHALLENGE

82. Assume as known the (true) fact that the alternating harmonic series

(1) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \cdots$

is convergent, and denote its sum by s. Rearrange the series (1) as follows:

(2) $1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \cdots$.

Assume as known the (true) fact that the series (2) is also convergent, and denote its sum by S. Denote by s_k, S_k the kth partial sum of the series (1) and (2), respectively. Prove the following statements.

(i) $S_{3n} = s_{4n} + \frac{1}{2}s_{2n}$, (ii) $S \neq s$

This problem was composed by the Committee on the Putnam Prize Competition.