9.5a Alternating Series Test p. 637 11–32 all

Alternating Series: series whose terms alternate signs.

<u>Alternating Series Test</u>: Let $a_n > 0$

The alternating series: $\sum_{n=1}^{\infty} (-1)^n a_n$ and $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ will <u>converge</u> if:

1) $\lim_{n\to\infty} a_n = 0$ and 2) $a_{n+1} \le a_n$ for all n.

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12) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot n}{2n-1}$ a) $\lim_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2} \neq 0$ diverges by n^{th} term test.

14) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ a) $\lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$ ✓ b) $\frac{1}{\ln(n+2)} < \frac{1}{\ln(n+1)}$ Since $a_{n+1} < a_n$, 2^{nd} condition for convergence passes.

<u>Converges by Alternating Series Test (AST)</u>

16) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot n}{n^2+1}$ a) $\lim_{n\to\infty} \frac{n}{n^2+1} = 0$ ✓ b) $\frac{n}{(n+1)^2+1} < \frac{n}{n^2+1}$ Since $a_{n+1} < a_n$, 2^{nd} condition passes.

<u>Converges by AST.</u>

18) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot n^2}{n^2+5}$ a) $\lim_{n\to\infty} \frac{n^2}{n^2+5} = 1 \neq 0$; <u>Diverges by n^{th} Term Test.</u>

20) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n+1)}{n+1}$ a) $\lim_{n\to\infty} \frac{\ln(n+1)}{n+1} = 0$ ✓ b) $\frac{\ln(n+2)}{n+2} < \frac{\ln(n+1)}{n+1}$ ✓

↳ L'Hopital's $\lim_{n\to\infty} \frac{\frac{1}{n+1}}{1} = 0$ ✓ <u>Converges by AST</u>

22) $\sum_{n=1}^{\infty} \frac{1}{n} \sin\left[\frac{(2n-1)\pi}{2}\right] = \sum \frac{1}{n}(-1)^{n+1}$ a) $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

b) $\frac{1}{n+1} < \frac{1}{n}$ ✓ $a_{n+1} < a_n$,

<u>Converges by AST.</u>

24) $\sum_{n=1}^{\infty} \frac{1}{n} \cos(n\pi) = \sum_{n=1}^{\infty} \frac{1}{n}(-1)^n$

a) $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

b) $\frac{1}{n+1} < \frac{1}{n}$ ✓ $a_{n+1} < a_n$ <u>converges by AST</u>

26) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$

a) $\lim_{n\to\infty} \frac{1}{(2n+1)!} = 0$ ✓

b) $\frac{1}{(2[n+1]+1)!} < \frac{1}{(2n+1)!}$, Since $a_{n+1} < a_n$ converges by AST.

28) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{\sqrt[3]{n}}$

a) $\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt[3]{n}} = n^{1/6} = \infty$ diverges by n^{th} term test.

30) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1\cdot 3\cdot 5\cdot \dots (2n-1)}{1\cdot 4\cdot 7 \dots (3n-2)} \to 3\cdot\left[\frac{5}{4}\cdot\frac{7}{7}\cdot\frac{9}{10}\cdot\frac{11}{13}\cdot\frac{13}{16}\cdot\frac{15}{19}\cdots\frac{(2n-1)}{(3n-5)}\right]\cdot\frac{1}{3n-2}$

$\lim_{n\to\infty} 3\cdot\frac{5}{4}\cdot\frac{2n-1}{3n-5} = \frac{15}{4}\cdot\frac{2}{3}\cdot\frac{1}{3n-2} = \boxed{0}$ ✓

$a_{n+1} < a_n$, Converges by AST.

32) $\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{e^n + e^{-n}}$

$a_n = \frac{2}{e^n + \frac{1}{e^n}} = \frac{2}{\frac{e^{2n}+1}{e^n}} = \frac{2e^n}{e^{2n}+1}$ $\lim_{n\to\infty} \frac{2e^n}{e^{2n}+1} \to L'H \frac{2e^n}{2e^{2x}}$

$f(x) = \frac{2e^x}{e^{2x}+1}$

a) $\lim_{n\to\infty} \frac{\cancel{2}e^n}{\cancel{2}e^{2n}} = \lim_{n\to\infty} \frac{1}{e^n} = 0$ ✓

b) $f'(x) = \frac{-2e^x(e^{2x}+1)}{(e^{2x}-1)^2} < 0$ This implies $a_{n+1} < a_n$ Converges by AST.