9.5c

p.622 61–71 odd
p.623 79–90 D252
p.637 #47–62 D252

*(From #59) $R_N = S - S_N$ bounded by $0 \le R_N \le \int_N^{\infty} f(x)\,dx$

p.622

61) $\sum_{n=1}^{\infty} \frac{1}{n^4}$, 6 terms $\quad S_6 = 1 + \frac{1}{2^4} + \dots + \frac{1}{6^4} \approx 1.0811 \quad R_6 = S - S_6$

$$R_6 \le \int_6^{\infty} \frac{1}{x^4}\,dx = \int x^{-4}\,dx = \left.\frac{x^{-3}}{-3}\right]_6^{\infty} = 0 - \left(\frac{6^{-3}}{-3}\right) = 0.0015 = R_6$$

$$S_6 \le S \le S_6 + R_6 \rightarrow 1.0811 \le \sum_{n=1}^{\infty} \frac{1}{n^4} \le 1.0811 + 0.0015 = \boxed{1.0826}$$

63) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$, 10 terms $\quad S_{10} = 0.9818$

$$R_{10} \le \int_{10}^{\infty} \frac{1}{x^2+1}\,dx \quad \left.\arctan x\right]_{10}^{\infty} = \frac{\pi}{2} - \arctan(10) \approx 0.0997$$

$R_{10} = 0.0997$

$$S_{10} \le S \le S_{10} + R_{10} \quad 0.9818 \le \sum_{n=1}^{\infty} \frac{1}{n^2+1} \le 0.9818 + 0.0997 = \boxed{1.0815}$$

65) $\sum_{n=1}^{\infty} ne^{-n^2}$, 4 terms $\quad S_4 = 0.4049$

$$\int_4^{\infty} xe^{-x^2} \quad u = -x^2 \quad \frac{du}{dx} = -2x \quad dx = \frac{du}{-2x}$$

$$\int xe^{u} \cdot \frac{du}{-2x} \quad \frac{-1}{2}\int e^{u} = \left.\frac{-1}{2}e^{-x^2}\right]_4^{\infty} = \left.\frac{-1}{2e^{x^2}}\right]_4^{\infty} = 0 - \left(-\frac{1}{2e^{16}}\right)$$

$\approx 5.6 \times 10^{-8}$

$$S_4 \le S \le S_4 + R_4 \quad 0.4049 \le \sum_{n=1}^{\infty} ne^{-n^2} \le 0.4049 + 5.6\times10^{-8}$$

67) Find N such that $R_N \le 0.001$ $\quad \sum_{n=1}^{\infty} \frac{1}{n^4}$

$$0 \le R_N \le \int_n^{\infty} \frac{1}{x^4}\,dx \quad \left.-\frac{1}{3x^3}\right]_n^{\infty} = \frac{1}{3N^3} < 0.001$$

$$\frac{1}{3N^3} = \frac{1}{1000}$$

$N^3 = 333.33$

$N > 6.93$

$\boxed{N \ge 7}$

69) $\sum_{n=1}^{\infty} e^{-5n}$ $\quad \int_N^{\infty} e^{-5x}\,dx$ $\quad u=-5x$, $\frac{du}{dx}=-5$, $dx=\frac{du}{-5}$ $\quad \int e^{u}\frac{du}{-5}$ $\quad -\frac{1}{5}e^{-5x}\Big]_n^{\infty} = \frac{-1}{5e^{5x}}\Big]_n^{\infty}$

$= \frac{-1}{\infty} + \frac{1}{5e^{5N}}$

$\frac{1}{5e^{5N}} < 0.001$ $\quad e^{5N} > 200$

$e^{5N} = 200$

$5N = \ln 200$

$N = \frac{1}{5}\ln 200 = 1.06$

$N > 1.06$

$\boxed{N \geq 2}$

71) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ $\quad \int_N^{\infty} \frac{1}{x^2+1}\,dx$ $\quad \tan^{-1}(x)\Big]_N^{\infty} = \frac{\pi}{2} - \tan^{-1}(N) < 0.001$

$-\tan^{-1}(N) < -1.5698$

$N > \tan(+1.5698)$

$N > 1003$

$\boxed{N \geq 1004}$

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p.623 79-90

Determine convergence/divergence of series

79) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ $\quad f(x) = \frac{1}{2x-1}$ positive, continuous, decreasing for $x \geq 1$

$\int_1^{\infty} \frac{1}{2x-1}$ $\quad u=2x-1$, $\frac{du}{dx}=2$, $dx=\frac{du}{2}$ $\quad \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|2x-1|\Big]_1^{\infty} = \frac{1}{2}(\infty) - \frac{1}{2}\ln = \infty$

Diverges by Integral Test.

80) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$ $\quad f(x) = \frac{1}{x\sqrt{x^2-1}}$ continuous, positive, decreasing for $x \geq 2$

$f(x) = \frac{x^{-1}}{(x^2-1)^{1/2}}$ $\quad \frac{-x^{-2}(x^2-1)^{1/2} - x^{-1}(\frac{1}{2})(x^2-1)^{-1/2}(2x)}{(x^2-1)}$

$\int_2^{\infty} \frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x\Big]_2^{\infty} = \boxed{\frac{\pi}{2} - \frac{\pi}{3}}$ converges by Integral Test p.637 47-62 D2S2 Determine conditional/Absolute convergence.

47) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^2}$ $a_n = \frac{1}{(n+1)^2}$ Since $\frac{1}{(n+1)^2} < \frac{1}{n^2}$ and $\frac{1}{n^2}$ converges by p-series test,

$\frac{1}{(n+1)^2}$ converges absolutely

48) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$ a) $\lim_{n\to\infty} \frac{1}{n+1} = 0$

b) $\frac{1}{n+2} \le \frac{1}{n+1}$, series converge by AST

$\int \frac{1}{n+1} dn$ $\int_1^{\infty} \frac{1}{x+1} dx$ $\ln|x+1| \Big]_1^{\infty} = \infty$ diverges so series converge conditionally.

51) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{(n+1)^2}$ $\lim_{n\to\infty} \frac{n^2}{(n+1)^2} = 1 \neq 0$, series diverge by n^{th} term test.

52) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+3)}{n+10}$ $\lim_{n\to\infty} \frac{2n+3}{n+10} = 2 \neq 0$ Diverges by n^{th} term test

55) $\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^3-1}$ $a_n = \frac{n}{n^3-1}$ $b_n = \frac{1}{n^2}$ $\lim_{n\to\infty} \left| \frac{\frac{n}{n^3-1}}{\frac{1}{n^2}} \right| = \lim_{n\to\infty} \left| \frac{n}{n^3-1} \cdot \frac{n^2}{1} \right| = 1$

The series converges Absolutely.

Limit is finite and $\frac{1}{n^2}$ converges by p-series test.

56) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1.5}}$ = $\frac{1}{n^{1.5}}$ p-series convergent series

converges absolutely

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59) $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ a) $\lim_{n\to\infty} \frac{1}{n+1} = 0$ converges by AST

b) $\frac{1}{n+2} < \frac{1}{n+1}$

$a_n = \frac{1}{n+1}$

$b_n = \frac{1}{n}$ $\lim_{n\to\infty} \left| \frac{\frac{1}{n+1}}{\frac{1}{n}} \right| = \lim_{n\to\infty} \left| \frac{1}{n+1} \cdot \frac{n}{1} \right| = 1$ a_n Diverges by Limit comparison Test to harmonic series.

Series converge conditionally.

60) $\sum_{n=1}^{\infty} (-1)^{n+1} \arctan n$

$\lim_{n\to\infty} \arctan n = \pi/2 \neq 0$ Diverges by n^{th} term test.