9.6 AP Practice Problems (p.689) – Motion along a Curve of Vector Functions

Key

1. If the velocity vector of a particle in motion is $v(t) = \langle t + 3, 6t^2 \rangle$, then the acceleration vector of the particle at $t = 2$ is

(A) $\langle 0, 24 \rangle$ (B) $\langle 1, 24 \rangle$ (C) $\langle 1, 12 \rangle$ (D) $\langle 2, 24 \rangle$

$a(t) = \langle 1, 12t \rangle$

$a(2) = \langle 1, 24 \rangle$

2. A particle moves along the plane curve

$$r(t) = \langle e^{2t}, t^4 + 2t^2 \rangle$$

What is the acceleration vector of the particle at $t = 0$?

(A) $a(0) = \langle 1, 4 \rangle$ (B) $a(0) = \langle 4, 0 \rangle$

(C) $a(0) = \langle 4, 4 \rangle$ (D) $a(0) = \langle 4e^2, 4 \rangle$

$r'(t) = \langle 2e^{2t}, 4t^3 + 4t \rangle$

~~$r'(0) = \langle 2e^0, 0 \rangle$~~

$r''(t) = \langle 4e^{2t}, 12t^2 + 4 \rangle$

$r''(0) = \langle 4e^0, 12(0)^2 + 4 \rangle$

$= \langle 4, 4 \rangle$

3. A particle moves in the xy-plane along the curve

$$r(t) = \langle 2\sqrt{t}, 3t^2 \rangle \quad t > 0$$

What is the velocity of the particle at $t = 9$?

(A) $\left\langle \frac{1}{3}, 54 \right\rangle$ (B) $\left\langle \frac{2}{3}, 54 \right\rangle$ (C) $\langle 3, 27 \rangle$ (D) $\left\langle \frac{1}{3}, 36 \right\rangle$

$v(t) = \langle 2 \cdot \frac{1}{2} t^{-1/2}, 6t \rangle$

$v(9) = \langle \frac{1}{\sqrt{9}}, 6(9) \rangle = \langle 1/3, 54 \rangle$

(OMIT)

4. A particle of mass m is moving along the curve traced out by $\mathbf{r}(t) = (2t^2, e^t)$. Using Newton's Second Law of Motion, $\mathbf{F} = m\mathbf{a}$, the force acting on the particle at time t is

(A) $(4m, e^t)$ **(B)** $(4m, me^t)$

(C) $(4m, e^m)$ (D) $(4, e^m)$

5. A particle moves along the plane curve

$$\mathbf{r}(t) = (1 - 2\cos t, 2\sin t)$$

(a) Find the velocity of the particle.

(b) Find the acceleration of the particle.

a) $v(t) = \langle 2\sin(t), 2\cos(t) \rangle$

b) $a(t) = \langle 2\cos(t), -2\sin(t) \rangle$

6. A particle moves along the plane curve $\mathbf{r}(t) = (2\sin t, 3\cos t)$.

(a) Find the speed of the particle.

(b) What is the speed of the particle at $t = \frac{\pi}{2}$?

a) $||r'(t)|| = \sqrt{(2\cos(t))^2 + (-3\sin(t))^2} = \sqrt{4\cos^2 t + 9\sin^2 t}$

$= \sqrt{4(1-\sin^2 t) + 9\sin^2 t} = \boxed{\sqrt{4 + 5\sin^2 t}}$

b) $\left|\left| r'(\pi/2) \right|\right| = \sqrt{4 + 5\left[\sin(\pi/2)\right]^2} = \sqrt{4+5(1)} = \sqrt{9} = \boxed{3}$