AP Calculus BC **9.6 Review Worksheet** **Finding the Right Test**

-Use general guidelines to determine which test would work best for a given series

Finding the best test to use to determine convergence or divergence can save a lot of time and stress. The following guidelines can be used to help quickly find a good test to use.

Guidelines for Choosing the Right Test

1) Does the n^{th} term test for divergence approach something other than 0? If so, it diverges.

2) Is the series one of the special types—geometric, p-series, or telescoping?

3) Is the series alternating, $(-1)^n$ or $(-1)^{n+1}$? Use the alternating series test.

4) Is the series closely related to one of the special series in step 2 or 3? Use Direct or Limit Comparison Test.

5) Are the terms of the series raised to an n^{th} power? Use the Root Test.

6) Can the series be easily integrated? Use the Integral Test.

7) If nothing else works, use the Ratio Test.

Putting it all together.

Example 1: Determine if the following series converge or diverge. Name the test used and the criteria of each test used.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$

(b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^3}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{5^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5}+5}$

(f) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2-6n+3}{n^3-7n+8}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+4}{2^n}$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{8n^3-6n^5}{12n^4-9n^5}$

(k) $\sum_{n=1}^{\infty}\sqrt{\frac{3n+1}{n^5+2}}$

(l) $\sum_{n=1}^{\infty}\frac{3^{n-1}}{n2^n}$

(m) $\sum_{n=1}^{\infty}\left(\frac{2n}{5n-1}\right)^n$

(n) $\sum_{n=2}^{\infty}\frac{1}{(\ln n)^n}$

(o) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$

(p) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\ln n}$

Example 2: Determine if the following series converge or diverge. Name the test used and the criteria of each test used.

1) $\sum_{n=1}^{\infty}\frac{n+1}{3n+1}$

2) $\sum_{n=1}^{\infty}\left(\frac{\pi}{6}\right)^n$

3) $\sum_{n=1}^{\infty} ne^{-n^2}$

4) $\sum_{n=1}^{\infty}\frac{1}{3n+1}$

5) $\sum_{n=1}^{\infty}(-1)^n\frac{3}{4n+1}$

6) $\sum_{n=1}^{\infty}\frac{n!}{10^n}$

7) $\sum_{n=1}^{\infty}\left(\frac{n+1}{2n+1}\right)^n$

8) $\sum_{n=1}^{\infty}\frac{1}{n^2}$

9) $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

10) $\sum_{n=1}^{\infty}\frac{2x}{x^2}$

11) $\sum_{n=1}^{\infty}\frac{1}{(n+1)^2}$

12) $\sum_{n=1}^{\infty}\frac{3n^2}{n^2+n}$

AP Calculus BC **9.6 Review Worksheet** **Finding the Right Test** Key

-Use general guidelines to determine which test would work best for a given series

Finding the best test to use to determine convergence or divergence can save a lot of time and stress. The following guidelines can be used to help quickly find a good test to use.

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2) Is the series one of the special types—geometric, p-series, or telescoping?

3) Is the series alternating, $(-1)^n$ or $(-1)^{n+1}$? Use the alternating series test.

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5) Are the terms of the series raised to an nth power? Use the Root Test.

6) Can the series be easily integrated? Use the Integral Test.

7) If nothing else works, use the Ratio Test.

Putting it all together.

Example 1: Determine if the following series converge or diverge. Name the test used and the criteria of each test used.

(a) $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$

By nth term test, since $\lim_{n\to\infty} f(x) = \frac{1}{4} \neq 0$, series diverges.

(b) $\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)^n$

Geometric series, $r=\frac{2}{7}<1$, converges by GST.

(c) $\sum_{n=1}^{\infty} \frac{4}{n^3}$

p-series test
Convergent series

(d) $\sum_{n=1}^{\infty} \frac{n^2}{5^n}$

By Ratio Test, series converges

$\lim_{n\to\infty} \frac{\frac{(n+1)^2}{5^{n+1}}}{\frac{n^2}{5^n}} = \lim_{n\to\infty} \frac{(n+1)^2}{5^{n+1}} \cdot \frac{5^n}{n^2} = \lim_{n\to\infty} \frac{n^2+2n+1}{5(n^2)} = \frac{1}{5} < 1$

Converges by Ratio Test.

(e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5}+5}$

Limit Comparison Test

$\lim_{n\to\infty} \frac{\frac{1}{\sqrt[3]{n^5+5}}}{\frac{1}{\sqrt[3]{n^5}}} = \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^5+5}} \cdot \frac{\sqrt[3]{n^5}}{1} = 1$

since this is finite and positive and $\frac{1}{\sqrt[3]{n^5}}$ converges by p-series, this series converge by LCT.

(f) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$

$u = \ln n$ $n\,du = dn$

$\frac{du}{dn} = \frac{1}{n}dn$

$\int \frac{1}{n \cdot u^4} n\,du = \int u^{-4}\,du$

$\frac{u^{-3}}{-3} = \left.\frac{-1}{3\ln n}\right]_2^{\infty} = 0 - \frac{-1}{3\ln 2}$

finite, positive convergent

(g) $\sum_{n=1}^{\infty} \frac{5n^2-6n+3}{n^3-7n+8}$

Limit Comparison with $\frac{1}{n}$ (diverges)

$\lim_{n\to\infty} \frac{\frac{5n^2-6n+3}{n^3-7n+8}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{5n^3-6n^2+3n}{n^3-7n+8} = 5$

(finite, positive)

$\boxed{\text{divergent}}$

(h) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

$\frac{(-1)^n}{\sqrt{n}}$

since $\lim_{n\to\infty} \frac{1}{n^{1/2}} = 0$, converges by AST.

(i) $\sum_{n=1}^{\infty} \frac{3^n+4}{2^n}$

$\boxed{\text{divergent}}$ by nth term test

(j) $\sum_{n=1}^{\infty} \frac{8n^3-6n^5}{12n^4-9n^5}$

$8n^4-6n$

By nth term test the $\lim_{n\to\infty} f(x) = \frac{2}{3} \neq 0$, series converges

(k) $\sum_{n=1}^{\infty}\sqrt{\frac{3n+1}{n^5+2}}$ $\quad \frac{n}{n^5} = \sqrt{\frac{1}{n^4}} = \frac{1}{n^2}$ (convergent)

$\lim_{n\to\infty}\sqrt{\frac{\frac{3n+1}{n^5+2}}{\frac{1}{n^4}}} = \sqrt{\frac{3n^5+n^4}{n^5+2}} = \sqrt{\frac{3}{1}} = \sqrt{3}$

By LCT, series also converges

(l) $\sum_{n=1}^{\infty}\frac{3^{n-1}}{n2^n}$ $\quad$ n^{th} term test inconclusive

$\lim_{n\to\infty}\frac{\frac{3^n}{(n+1)2^{n+1}}}{\frac{3^{n-1}}{n2^n}}$ $\quad$ Ratio Test

$\lim_{n\to\infty}\frac{3^n}{(n+1)2^{n+1}}\cdot\frac{n2^n}{3^{n-1}}$

$\lim_{n\to\infty}\frac{3n\cdot 2^n}{(n+1)\cdot 2} = \frac{3}{2} > 1$

divergent series.

(m) $\sum_{n=1}^{\infty}\left(\frac{2n}{5n-1}\right)^n$ $\quad \lim_{n\to\infty}\sqrt[n]{\left(\frac{2n}{5n-1}\right)^n} = \lim_{n\to\infty}\frac{2n}{5n-1} = \frac{2}{5} < 1$

Root Test.

converges by root test

(n) $\sum_{n=2}^{\infty}\frac{1}{(\ln n)^n}$ $\quad \lim_{n\to\infty}\sqrt[n]{\frac{1}{(\ln n)^n}} = \frac{1}{\ln n} < 1$ $\boxed{\text{converges}}$

Root Test

(o) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$ $\quad$ n^{th} term test $= e \neq 0$

$\boxed{\text{divergent series}}$

(p) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\ln n}$ $\quad$ By AST, since $\lim_{n\to\infty}\frac{1}{\ln n} = 0$

series converges (conditionally)

Example 2: Determine if the following series converge or diverge. Name the test used and the criteria of each test used.

1) $\sum_{n=1}^{\infty}\frac{n+1}{3n+1}$ $\quad$ n^{th} term test

$\lim_{n\to\infty}\frac{n+1}{3n+1} = \frac{1}{3} \neq 0$

series diverge.

2) $\sum_{n=1}^{\infty}\left(\frac{\pi}{6}\right)^n$ $\quad$ geometric

$r = \pi/6$

convergent series

3) $\sum_{n=1}^{\infty} ne^{-n^2}$ $\quad$ Integral Test

$\frac{n}{e^{n^2}}$ convergent series

4) $\sum_{n=1}^{\infty}\frac{1}{3n+1}$ $\quad$ Limit comparison test

compare to $\frac{1}{n}$ (divergent)

$\lim_{n\to\infty}\left(\frac{\frac{1}{n}}{\frac{1}{3n+1}}\right) = \frac{1}{n}\cdot\frac{3n+1}{1} = 3$

Since $\frac{1}{n}$ diverges, then $\frac{1}{3n+1}$ also diverges by LCT.

5) $\sum_{n=1}^{\infty}(-1)^n\frac{3}{4n+1}$ $\quad$ By AST, since $\lim_{n\to\infty}\frac{3}{4n+1} = 0$,

then series converges

6) $\sum_{n=1}^{\infty}\frac{n!}{10^n}$ $\quad$ Ratio Test (or comparative growth rate)

$\lim_{n\to\infty}\frac{\frac{(n+1)!}{10^{n+1}}}{\frac{n!}{10^n}}$ $\quad \lim_{n\to\infty}\frac{(n+1)!}{10^{n+1}}\cdot\frac{10^n}{n!}$

$= \lim_{n\to\infty}\frac{(n+1)}{10} = \infty$

diverges by Ratio Test.

7) $\sum_{n=1}^{\infty}\left(\frac{n+1}{2n+1}\right)^n$ $\quad$ Root test.

$\lim_{n\to\infty}\sqrt[n]{\left(\frac{n+1}{2n+1}\right)^n} = \frac{1}{2} < 1$

$\boxed{\text{Converges by Root test}}$

8) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ $\quad$ Converges by p-series test

9) $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ $\quad$ converges by geometric series test.

10) $\sum_{n=1}^{\infty}\frac{2x}{x^2}$ $\quad \int_0^{\infty}\frac{2}{x}\,dx \quad 2\ln x\Big]_0^{\infty}$

diverges

By Integral Test, series also diverges.

11) $\sum_{n=1}^{\infty}\frac{1}{(n+1)^2}$ $\quad$ By DCT Compare to $\frac{1}{n^2}$

Since $\frac{1}{n^2}$ converge and $\frac{1}{n^2} > \frac{1}{(n+1)^2}$, series also converge

12) $\sum_{n=1}^{\infty}\frac{3n^2}{n^2+n}$ $\quad$ By n^{th} term test

$\lim_{n\to\infty}\frac{3n^2}{n^2+n} = 3 \neq 0$,

Divergent series.