BC Calculus 9.7-9.10 Review WS #1

Basic Maclaurin Power Series for known elementary functions

1. Write the first 4 terms of the series and the general term .

a) $\sin x =$

b) $\cos x =$

c) $e^x =$

d) $\frac{1}{1+x} =$

e) $\frac{1}{x} =$

f) $\ln x =$

2. Find a Maclaurin polynomial of degree n for the following :

a) $f(x) = xe^{2x}$, n = 4

b) $f(x) = \frac{x}{x+1}$, n = 5

3) Find taylor polynomial of degree n centered at x = c for the following:

$f(x) = \frac{1}{x}$, n = 5, c = 1

Multiple Choice

20. If $f(0)=0$, $f'(0)=1$, $f''(0)=0$, and $f'''(0)=2$, then which of the following is the third-order Taylor polynomial generated by $f(x)$ at $x=0$?

(A) $2x^3+x$ (B) $\frac{1}{3}x^3+\frac{1}{2}x$ (C) $\frac{2}{3}x^3+x$ (D) $2x^3-x$ (E) $\frac{1}{3}x^3+x$

21. Which of the following is the coefficient of x^4 in the Maclaurin polynomial generated by $\cos(3x)$?

(A) $\frac{27}{8}$ (B) 9 (C) $\frac{1}{24}$ (D) 0 (E) $-\frac{27}{8}$

22. Which of the following is the Taylor polynomial generated by $f(x)=\cos x$ at $x=\frac{\pi}{2}$?

(A) $\left(x-\frac{\pi}{2}\right)-\frac{\left(x-\frac{\pi}{2}\right)^3}{3!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$ (B) $1+\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$ (C) $1-\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$

(D) $1-\left(x-\frac{\pi}{2}\right)^2+\left(x-\frac{\pi}{2}\right)^4$ (E) $-\left(x-\frac{\pi}{2}\right)+\frac{\left(x-\frac{\pi}{2}\right)^3}{6}$

23. (Calculator Permitted) Which of the following gives the Maclaurin polynomial of order 5 approximation to $\sin(1.5)$?

(A) 0.965 (B) 0.985 (C) 0.997 (D) 1.001 (E) 1.005

24. Which of the following is the quadratic approximation for $f(x)=e^{-x}$ at $x=0$?

(A) $1-x+\frac{1}{2}x^2$ (B) $1-x-\frac{1}{2}x^2$ (C) $1+x+\frac{1}{2}x^2$ (D) $1+x$ (E) $1-x$

BC Calculus **9.7-9.10** **Review WS #1**

Key

Basic Maclaurin Power Series for known elementary functions

1. Write the first 4 terms of the series and the general term .

a) sin x = $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \frac{(-1)^n x^{(2n+1)}}{(2n+1)!}$

b) cos x = $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \frac{(-1)^n x^{2n}}{(2n)!}$

c) e^x = $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \frac{x^n}{n!}$

d) $\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + \ldots (-1)^n x^n$

e) $\frac{1}{x} = \frac{1}{1-[-(x-1)]} = 1-(x-1)+(x-1)^2-(x-1)^3+\ldots$ $(-1)^n(x-1)^n$

f) ln x =

$\int \frac{1}{x} = \ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \ldots \frac{(-1)^{n+1}(x-1)^n}{n}$ or $\frac{(-1)^{n-1}(x-1)^n}{n}$

2. Find a Maclaurin polynomial of degree n for the following :

a) f(x) = xe^{2x} , n = 4

$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \ldots$

$xe^{2x} = x + 2x^2 + \frac{4x^3}{2!} + \frac{8x^4}{3!} + \ldots$

$\frac{2^n x^{n+1}}{n!}$

b) f(x) = $\frac{x}{x+1}$, n = 5

$\frac{1}{1+x} = 1 - x + x^2 - x^3$

$\frac{x}{1+x} = x - x^2 + x^3 - x^4 + \ldots (-1)^{n+1} x^n$

3) Find taylor polynomial of degree n centered at x = c for the following:

$\frac{f^{(n)}(c)}{n!}(x-c)^n$

c = 1

f(x) = $\frac{1}{x}$ ~~ln x~~, n = 5, c = 1

$f(x) = \frac{1}{x}$ $\quad f(1) = 1$

$f'(x) = \frac{-1}{x^2}$ $\quad f'(1) = -1$

$f''(x) = 2/x^3$ $\quad f''(1) = 2$

$f'''(x) = -6/x^4$ $\quad f'''(1) = -6$

$f^{(4)}(x) = 24/x^5$ $\quad f^{(4)}(1) = 24$

$f^{(5)}(x) = \frac{-120}{x^6}$ $\quad f^{(5)}(1) = -120$

$P_5(x) = 1 - 1(x-1) + \frac{2}{2!}(x-1)^2 - \frac{6}{3!}(x-1)^3 + \frac{24}{4!}(x-1)^4 - \frac{120}{5!}(x-1)^5$

$P_5(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 - (x-1)^5$

Multiple Choice

20. If $f(0)=0$, $f'(0)=1$, $f''(0)=0$, and $f'''(0)=2$, then which of the following is the third-order Taylor polynomial generated by $f(x)$ at $x=0$?

(A) $2x^3+x$ (B) $\frac{1}{3}x^3+\frac{1}{2}x$ (C) $\frac{2}{3}x^3+x$ (D) $2x^3-x$ $\boxed{\text{(E) } \frac{1}{3}x^3+x}$

$$f(x)\approx P_3(x)=0+1(x-0)+\frac{0}{2!}(x-0)^2+\frac{2}{3!}(x-0)^3$$

$$=x+\frac{1}{3}x^3$$

21. Which of the following is the coefficient of x^4 in the Maclaurin polynomial generated by $\cos(3x)$?

(A) $\frac{27}{8}$ (B) 9 (C) $\frac{1}{24}$ (D) 0 (E) $-\frac{27}{8}$

$f=\cos(3x)$
$f'=-3\sin(3x)$
$f''=-9\cos(3x)$
$f'''=27\sin(3x)$
$f^{(4)}=81\cos(3x)$

$$\frac{f^4(0)}{4!}(x-0)^4=\frac{81\cos(0)}{4!}x^4=\frac{81}{4\cdot3\cdot2}x^4=\boxed{\frac{27}{8}}$$

22. Which of the following is the Taylor polynomial generated by $f(x)=\cos x$ at $x=\frac{\pi}{2}$?

(A) $\left(x-\frac{\pi}{2}\right)-\frac{\left(x-\frac{\pi}{2}\right)^3}{3!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$ (B) $1+\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$ (C) $1-\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$

(D) $1-\left(x-\frac{\pi}{2}\right)^2+\left(x-\frac{\pi}{2}\right)^4$ $\boxed{\text{(E) } -\left(x-\frac{\pi}{2}\right)+\frac{\left(x-\frac{\pi}{2}\right)^3}{6}}$

$f=\cos x$ $\quad f(\pi/2)=0$
$f'=-\sin x$ $\quad =-1$
$f''=-\cos x$ $\quad =0$
$f'''=\sin x$ $\quad =1$
$f^{(4)}=\cos x$ $\quad =0$

$$P_4(x)=0-1(x-\pi/2)+\frac{0}{2!}(x-\pi/2)^2+\frac{1}{3!}(x-\pi/2)^3$$

$$=\boxed{-(x-\pi/2)+\frac{(x-\pi/2)^3}{3!}}$$

23. (Calculator Permitted) Which of the following gives the Maclaurin polynomial of order 5 approximation to $\sin(1.5)$?

(A) 0.965 (B) 0.985 (C) 0.997 $\boxed{\text{(D) } 1.001}$ (E) 1.005

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}$$

$$\sin(1.5)\approx(1.5)-\frac{(1.5)^3}{3!}+\frac{(1.5)^5}{5!}=1.00078\approx1.001$$

24. Which of the following is the quadratic approximation for $f(x)=e^{-x}$ at $x=0$? Maclaurin

$\boxed{\text{(A) } 1-x+\frac{1}{2}x^2}$ (B) $1-x-\frac{1}{2}x^2$ (C) $1+x+\frac{1}{2}x^2$ (D) $1+x$ (E) $1-x$

$$e^x\approx1+x+\frac{x^2}{2!} \qquad e^{-x}=1-x+\frac{x^2}{2}$$