9.7 Maclaurin Series   p.656 #13-30 D2SI

Maclaurin Series: $F(x) = \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} x^n$

Taylor Series: $F(x) = \sum_{n=0}^{\infty} \frac{f^n(c)}{n!} (x-c)^n$

Find the Maclaurin polynomial of degree n for the function

13) $f(x) = e^{-x}$, $n=3$

$f'(x) = -e^{-x}$
$f''(x) = e^{-x}$
$f'''(x) = -e^{-x}$

$f(0) = 1$
$f'(0) = -1$
$f''(0) = 1$
$f'''(0) = -1$

$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

$P_3(x) = 1 + (-1)x + \frac{1}{2}x^2 + \frac{(-1)}{3!}x^3$

$\boxed{P_3(x) = 1 - x + \frac{1}{2}x^2 - \frac{x^3}{6}}$

14) $f(x) = e^{-x}$, $n=5$ $\longrightarrow$ $P_5(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120}$

16) $f(x) = e^{3x}$ $n=4$

$f(x) = e^{3x}$ $f(0) = 1$
$f'(x) = 3e^{3x}$ $f'(0) = 3$
$f''(x) = 9e^{3x}$ $f''(0) = 9$
$f'''(x) = 27e^{3x}$ $f'''(0) = 27$
$f^{(4)}(x) = 81e^{3x}$ $f^{(4)}(0) = 81$

$P_4(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^4(0)x^4}{4!}$

$P_4(x) = 1 + 3x + \frac{9}{2}x^2 + \frac{27}{6}x^3 + \frac{81}{24}x^4$

$\boxed{P_4(x) = 1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \frac{27}{8}x^4}$

17) $f(x) = \sin x$ $n=5$

$f(x) = \sin x$ $f(0) = 0$
$f'(x) = \cos x$ $f'(0) = 1$
$f''(x) = -\sin x$ $f''(0) = 0$
$f'''(x) = -\cos x$ $f'''(0) = -1$
$f^{(4)}(x) = \sin x$ $f^{(4)}(0) = 0$
$f^{(5)}(x) = \cos x$ $f^{(5)}(0) = 1$

$P_5(x) = 0 + 1x + \frac{0x^2}{2!} - \frac{1x^3}{3!} + \frac{0x^4}{4!} + \frac{x^5}{5!}$

$= \boxed{x - \frac{x^3}{6} + \frac{x^5}{120}}$

19) $f(x)=xe^x \quad n=4$

$f(x)=xe^x \qquad f(0)=0$

$f'(x)=xe^x+e^x \qquad f'(0)=1$

$f''(x)=xe^x+2e^x \qquad f''(0)=2$

$f'''(x)=xe^x+3e^x \qquad f'''(0)=3$

$f^{(4)}(x)=xe^x+4e^x \qquad f^{(4)}(0)=4$

$P_4(x)=0+1x+\frac{2x^2}{2!}+\frac{3x^3}{3!}+\frac{4x^4}{4!}$

$\boxed{P_4(x)=x+x^2+\frac{1}{2}x^3+\frac{1}{6}x^4}$

20) $f(x)=x^2e^{-x} \quad n=4$

$f(x)=x^2e^{-x} \qquad f(0)=0$

$f'(x)=2xe^{-x}-x^2e^{-x} \qquad f'(0)=0$

$f''(x)=2e^{-x}-4xe^{-x}+x^2e^{-x} \qquad f''(0)=2$

$f'''(x)=-6e^{-x}+6xe^{-x}-x^2e^{-x} \qquad f'''(0)=-6$

$f^{(4)}(x)=12e^{-x}-8xe^{-x}+x^2e^{-x} \qquad f^{(4)}(0)=12$

$P_4(x)=0+x+\frac{2}{2!}x^2+\frac{3}{3!}x^3+\frac{4}{4!}x^4$

$=\boxed{x+x^2+\frac{1}{2}x^3+\frac{1}{6}x^4}$

22) $f(x)=\frac{x}{x+1}=\frac{x+1-1}{x+1}=\frac{x+1}{x+1}-\frac{1}{x+1}=1-(x+1)^{-1} \quad n=4$

$f(x)=1-(x+1)^{-1} \qquad f(0)=0$

$f'(x)=+1(x+1)^{-2} \qquad f'(0)=1$

$f''(x)=-2(x+1)^{-3} \qquad f''(0)=-2$

$f'''(x)=6(x+1)^{-4} \qquad f'''(0)=6$

$f^{(4)}(x)=-24(x+1)^{-5} \qquad f^{(4)}(0)=-24$

$P_4(x)=0+1x-\frac{2x^2}{2!}+\frac{6x^3}{3!}-\frac{24x^4}{4!}$

$\boxed{P_4(x)=x-x^2+x^3-x^4}$

23) $f(x)=\sec x \quad n=2$

$f(x)=\sec x \qquad f(0)=1$

$f'(x)=\sec x\tan x \qquad f'(0)=0$

$f''(x)=\sec^3 x+\sec x\tan^2 x \qquad f''(0)=1$

$P_2(x)=1+0x+\frac{1}{2}(1)x^2$

$\boxed{P_2(x)=1+\frac{1}{2}x^2}$

* Find the n^{th} Taylor polynomial centered at c: $F(x)=\sum_{n=0}^{\infty}\frac{f^{n}(c)}{n!}(x-c)^n$

25) $f(x)=\frac{1}{x}$, $n=4$, $c=1$

$f(x)=x^{-1}$ $\quad f(1)=1$

$f'(x)=-x^{-2}$ $\quad f'(1)=-1$

$f''(x)=2x^{-3}$ $\quad f''(1)=2$

$f^{(3)}(x)=-6x^{-4}$ $\quad f'''(1)=-6$

$f^{4}(x)=24x^{-5}$ $\quad f^{(4)}(1)=24$

$P_4(x)=1+-1(x-1)^1+2\frac{(x-1)^2}{2!}-6\frac{(x-1)^3}{3!}+24\frac{(x-1)^4}{4!}$

$\boxed{P_4(x)=1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4}$

26) $f(x)=\frac{2}{x^2}=2x^{-2}$ $\quad n=4$ $\quad c=2$

$f(x)=2x^{-2}$ $\quad f(2)=\frac{1}{2}$

$f'(x)=-4x^{-3}$ $\quad f'(2)=-\frac{1}{2}$

$f''(x)=12x^{-4}$ $\quad f''(2)=3/4$

$f'''(x)=-48x^{-5}$ $\quad f'''(2)=-3/2$

$f^{(4)}(x)=240x^{-6}$ $\quad f^{(4)}(2)=15/4$

$P_4(x)=\frac{1}{2}-\frac{1}{2}(x-2)+\frac{3}{4}\cdot\frac{(x-2)^2}{2!}-\frac{3}{2}\frac{(x-2)^3}{3!}+\frac{15}{4}\cdot\frac{(x-2)^4}{4!}$

$\boxed{P_4(x)=\frac{1}{2}-\frac{1}{2}(x-2)+\frac{3}{8}(x-2)^2-\frac{1}{4}(x-2)^3+\frac{5}{32}(x-2)^4}$

28) $f(x)=x^{1/3}$, $n=3$ $\quad c=8$

$f(x)=x^{1/3}$ $\quad f(8)=2$

$f'(x)=\frac{1}{3}x^{-2/3}$ $\quad f'(8)=\frac{1}{12}$

$f''(x)=\frac{-2}{9}x^{-5/3}$ $\quad f''(8)=-1/144$

$f'''(x)=\frac{10}{27}x^{-8/3}$ $\quad f'''(8)=5/3456$

$P_3(x)=2+\frac{1}{12}(x-8)-\frac{1}{144}\cdot\frac{(x-8)^2}{2!}+\frac{5}{3456}\cdot\frac{(x-8)^3}{3!}$

$\boxed{P_3(x)=2+\frac{1}{12}(x-8)-\frac{1}{288}(x-8)^2+\frac{5}{20736}(x-8)^3}$

29) $f(x)=\ln x$ $\quad n=4$ $\quad c=1$

$f(x)=\ln x$ $\quad f(1)=0$

$f'(x)=x^{-1}$ $\quad f'(1)=1$

$f''(x)=-x^{-2}$ $\quad f''(1)=-1$

$f'''(x)=2x^{-3}$ $\quad f'''(1)=2$

$f^{(4)}(x)=-6x^{-4}$ $\quad f^{(4)}(1)=-6$

$P_4(x)=0+1(x-1)-1\cdot\frac{(x-1)^2}{2!}+2\cdot\frac{(x-1)^3}{3!}-6\cdot\frac{(x-1)^4}{4!}$

$\boxed{P_4(x)=(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\frac{1}{4}(x-1)^4}$

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12) $f(x)=x^2e^x$

$f(x)=x^2e^x \qquad f(0)=0$

$f'(x)=x^2e^x+2xe^x \qquad f'(0)=0$

$f''(x)=x^2e^x+4xe^x+2e^x \qquad f''(0)=2$

$f'''(x)=x^2e^x+6xe^x+6e^x \qquad f'''(0)=6$

$f^{(4)}(x)=x^2e^x+8xe^x+12e^x \qquad f^{4}(0)=12$

$P_2(x)=0+0+\frac{2x^2}{2!}$

$P_3(x)=\frac{2x^2}{2!}+\frac{6x^3}{3!}$

$P_4(x)=x^2+x^3+\frac{12x^4}{4!}=x^2+x^3+\frac{x^4}{2}$

c) $P_2''(0)=2$ 2nd derivative of $P(x)=2$ at 0

$P_3'''(0)=6$ 3rd " of $P(x)=6$

$P_4''''(0)=12$ 4th " of $P(x)=12$

d) $f^n(0)=P_n^n(0)$

18) $f(x)=\sin(\pi x) \quad n=3$

$f(x)=\sin(\pi x) \qquad f(0)=0$

$f'(x)=\pi\cos(\pi x) \qquad f'(0)=\pi$

$f''(x)=-\pi^2\sin(\pi x) \qquad f''(0)=0$

$f'''(x)=-\pi^3\cos(\pi x) \qquad f'''(0)=-\pi^3$

$P_3(x)=0+\pi x+\frac{0x^2}{2!}-\frac{\pi^3x^3}{3!}$

$\boxed{P_3(x)=\pi x-\frac{\pi^3}{6}x^3}$

24) $f(x)=\tan x \quad n=3$

$f(x)=\tan x \qquad f(0)=0$

$f'(x)=\sec^2x \qquad f'(0)=1$

$f''(x)=2\sec^2x\tan x \qquad f''(0)=0$

$f'''(x)=4\sec^2x\tan^2x+2\sec^4x \qquad f'''(0)=2$

$P_3(x)=0+1x+0x^2+\frac{2x^3}{3!}$

$\boxed{P_3(x)=1x+\frac{1}{3}x^3}$

30) $f(x)=x^2\cos x \quad n=2 \quad c=\pi$

$f(x)=x^2\cos x \qquad f(\pi)=-\pi^2$

$f'(x)=\cos x-x^2\sin x \qquad f'(\pi)=-2\pi$

$f''(x)=2\cos x-4x\sin x-x^2\cos x \qquad f''(\pi)=-2+\pi^2$

$P_2(x)=-\pi^2-2\pi(x-\pi)+\frac{(-2+\pi^2)(x-\pi)^2}{2!}$

$\boxed{P_2(x)=-\pi^2-2\pi(x-\pi)+\frac{(\pi^2-2)}{2}(x-\pi)^2}$

42) $f(1/5)\approx \boxed{0.0328}$

44) $f\left(\frac{7\pi}{8}\right)\approx -6.7954$