Name_____ Date_____ Period_____

Worksheet 11.2—Taylor Polynomials
Show all work. No calculator except unless specifically stated.

Short Answer/Free Response

On problems 1-5, find a Maclaurin polynomial of degree *n* for each of the following.

1. $f(x) = e^{-x},\ n = 3$

2. $f(x) = e^{2x},\ n = 4$

3. $f(x) = \cos x,\ n = 8$

4. $f(x) = xe^{2x},\ n = 4$

5. $f(x) = \dfrac{1}{x+1},\ n = 5$

On problems 6-8, find a Taylor polynomial of degree *n* centered at $x = c$ for each of the following.

6. $f(x) = \frac{1}{x},\ n = 5,\ c = 1$

7. $f(x) = \ln x,\ n = 5,\ c = 1$

8. $f(x) = \sin x,\ n = 6,\ c = \frac{\pi}{4}$

9. (Calculator Permitted) Use your answer from problem 1 to approximate $f\left(\frac{1}{2}\right)$ to four decimal places.

10. (Calculator Permitted) Use your answer from problem 7 to approximate $f(1.2)$ to four decimal places.

11. Suppose that function $f(x)$ is approximated near $x = 0$ by a sixth-degree Taylor polynomial $P_6(x) = 3x - 4x^3 + 5x^6$. Give the value of each of the following:

(a) $f(0)$ (b) $f'(0)$ (c) $f'''(0)$ (d) $f^{(5)}(0)$ (e) $f^{(6)}(0)$

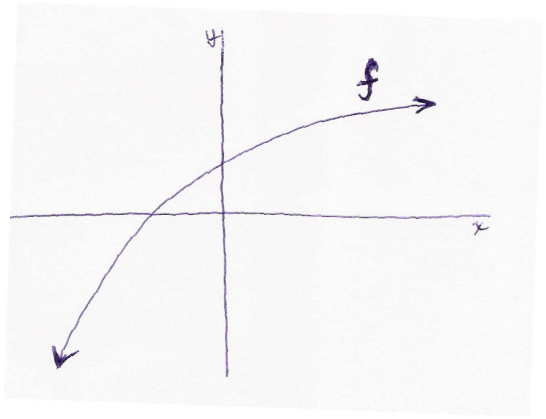
12. (Calculator Permitted) Suppose that g is a function which has continuous derivatives, and that $g(5)=3,\ g'(5)=-2,\ g''(5)=1,\ g'''(5)=-3$

(a) What is the Taylor polynomial of degree 2 for g near 5? What is the Taylor polynomial of degree 3 near 5?

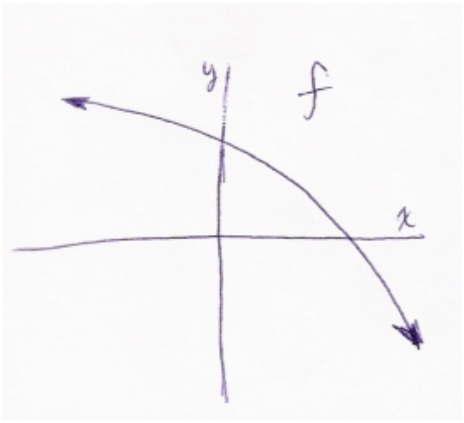
(b) Use the two polynomials that you found in part (a) to approximate $g(4.9)$.

For problems 13-16, suppose that $P_2(x) = a + bx + cx^2$ is the second degree Taylor polynomial for the function f about $x = 0$. What can you say about the signs of a, b, and c, if f has the graphs given below?

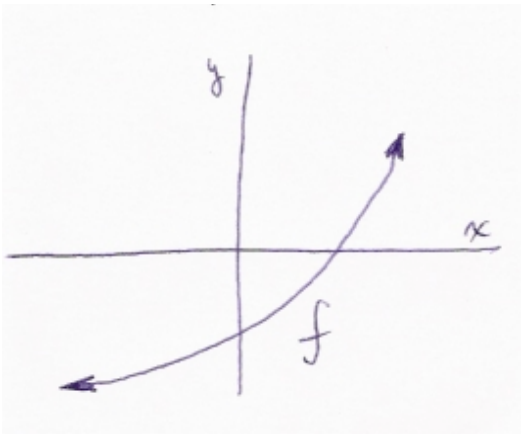
13.



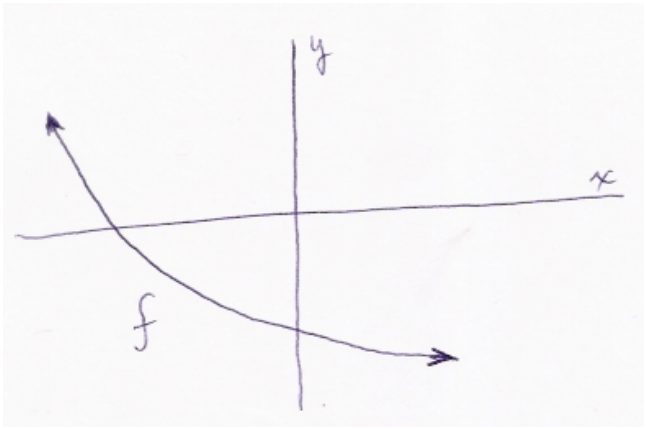
14.



15.



16.



17. Show how you can use the Taylor approximation $\sin x \approx x - \frac{x^3}{3!}$ for x near 0 to find $\lim_{x \to 0} \frac{\sin x}{x}$.

18. Use the fourth-degree Taylor approximation of $\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ for x near 0 to find $\lim_{x \to 0} \frac{1 - \cos x}{x}$.

19. Estimate the integral $\int_0^1 \frac{\sin t}{t} dt$ using a Taylor polynomial for $\sin t$ about $t = 0$ of degree 5.

Multiple Choice

20. If $f(0)=0$, $f'(0)=1$, $f''(0)=0$, and $f'''(0)=2$, then which of the following is the third-order Taylor polynomial generated by $f(x)$ at $x=0$?

(A) $2x^3+x$ (B) $\frac{1}{3}x^3+\frac{1}{2}x$ (C) $\frac{2}{3}x^3+x$ (D) $2x^3-x$ (E) $\frac{1}{3}x^3+x$

21. Which of the following is the coefficient of x^4 in the Maclaurin polynomial generated by $\cos(3x)$?

(A) $\frac{27}{8}$ (B) 9 (C) $\frac{1}{24}$ (D) 0 (E) $-\frac{27}{8}$

22. Which of the following is the Taylor polynomial generated by $f(x)=\cos x$ at $x=\frac{\pi}{2}$?

(A) $\left(x-\frac{\pi}{2}\right)-\frac{\left(x-\frac{\pi}{2}\right)^3}{3!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$ (B) $1+\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$ (C) $1-\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$

(D) $1-\left(x-\frac{\pi}{2}\right)^2+\left(x-\frac{\pi}{2}\right)^4$ (E) $-\left(x-\frac{\pi}{2}\right)+\frac{\left(x-\frac{\pi}{2}\right)^3}{6}$

23. (Calculator Permitted) Which of the following gives the Maclaurin polynomial of order 5 approximation to $\sin(1.5)$?

(A) 0.965 (B) 0.985 (C) 0.997 (D) 1.001 (E) 1.005

24. Which of the following is the quadratic approximation for $f(x) = e^{-x}$ at $x = 0$?

(A) $1 - x + \frac{1}{2}x^2$ (B) $1 - x - \frac{1}{2}x^2$ (C) $1 + x + \frac{1}{2}x^2$ (D) $1 + x$ (E) $1 - x$