9.8 Power Series p.666 #1-44 D2S3

* Find radius of convergence using Ratio Test

* Ratio Test converge when $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$

6) $\sum_{n=0}^{\infty}(2x)^n$ $\quad \lim_{n\to\infty}\left|\frac{(2x)^{n+1}}{(2x)^n}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(2x)^n}\cdot(2x)}{\cancel{(2x)^n}}\right| = |2x| < 1$

$|x| < 1/2$ $\quad \boxed{R = \frac{1}{2}}$

7) $\sum_{n=1}^{\infty}\frac{(2x)^n}{n^2}$ $\quad \lim_{n\to\infty}\left|\frac{\frac{(2x)^{n+1}}{(n+1)^2}}{\frac{(2x)^n}{n^2}}\right| \quad \lim_{n\to\infty}\left|\frac{\cancel{(2x)^{n+1}}}{(n+1)^2}\cdot\frac{n^2}{\cancel{(2x)^n}}\right| \quad \lim_{n\to\infty}\left|\frac{(2x)\cdot n^2}{n^2+2n+1}\right| = |2x| < 1$

$|x| < 1/2$

$\boxed{R = 1/2}$

* Find Interval of Convergence (IOC). Be sure to check convergence at endpoints of interval

11) $\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$ $\quad \lim_{n\to\infty}\left|\frac{(x/2)^{n+1}}{(x/2)^n}\right| \quad \lim_{n\to\infty}\left|\frac{\cancel{\left(\frac{x}{2}\right)^n}\left(\frac{x}{2}\right)}{\cancel{(x/2)^n}}\right| = \left|\frac{x}{2}\right| < 1 \quad |x| < 2$

* Geometric series converge only if $\left|\frac{x}{2}\right| < 1$ $\quad R = 2$

$\boxed{\text{IOC: } -2 < x < 2}$

16) $\sum_{n=0}^{\infty}\frac{(3x)^n}{(2n)!}$ $\quad \lim_{n\to\infty}\left|\frac{\frac{(3x)^{n+1}}{(2n+2)!}}{\frac{(3x)^n}{(2n)!}}\right| = \lim_{n\to\infty}\left|\frac{(3x)^n\cdot(3x)}{(2n+2)!}\cdot\frac{\cancel{(2n)!}}{\cancel{(3x)^n}}\right| = \lim_{n\to\infty}\left|3x\cdot\frac{1}{4n^2+6n+2}\right|$

$(2n+2)! \downarrow (2n+2)(2n+1)\cancel{(2n)!}$

$\frac{3x}{\infty} = 0$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0$

I.O.C $\rightarrow$ $\boxed{-\infty < x < \infty}$

17) $\sum_{n=0}^{\infty}(2n)!\left(\frac{x}{2}\right)^n$

$\lim_{n\to\infty}\left|\frac{(2n+2)!\left(\frac{x}{2}\right)^{n+1}}{(2n)!\left(\frac{x}{2}\right)^n}\right| = \lim_{n\to\infty}\left|\frac{(2n+2)(2n+1)}{1}\cdot\left(\frac{x}{2}\right)\right| = \infty$ $\quad \boxed{\text{Series converge only for } x=0}$

21) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$ $\lim_{n\to\infty} \left| \frac{\frac{(x-5)^{n+1}}{(n+1)5^{n+1}}}{\frac{(x-5)^n}{n5^n}} \right| = \lim_{n\to\infty} \left| \frac{(x-5)^{\cancel{n+1}}}{(n+1)5^{n+1}} \cdot \frac{n5^n}{\cancel{(x-5)^n}} \right|$

$= \lim_{n\to\infty} \left| \frac{n(x-5)}{5(n+1)} \right| = \lim_{n\to\infty} \left| \frac{n}{5n+5} \cdot |x-5| \right| = \frac{1}{5}|x-5| < 1$ $R = 5$

center: $x = 5$

$-5 < x-5 < 5 \rightarrow 0 < x < 10$

* Test endpts: when $x=0$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-5)^n}{n5^n} = \frac{(-1)}{n}$ series diverges (harmonic series)

when $x=10$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(+5)^n}{n5^n} = \frac{(-1)^{n+1}}{n}$ Alternating harmonic series converges

$\boxed{\text{I.O.C.} \rightarrow 0 < x \leq 10}$

22) $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)4^{n+1}}$ $\lim_{n\to\infty} \left| \frac{\frac{(x-2)^{n+2}}{(n+2)4^{n+2}}}{\frac{(x-2)^{n+1}}{(n+1)4^{n+1}}} \right| = \lim_{n\to\infty} \left| \frac{(x-2)^{n+2}}{(n+2)4^{n+2}} \cdot \frac{(n+1)4^{n+1}}{(x-2)^{n+1}} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1)}{4(n+2)} \cdot (x-2) \right| = \frac{1}{4}|x-2| < 1$ Radius $= 4$

Center: $x = 2$

$-4 < x-2 < 4 \rightarrow -2 < x < 6$

* Test endpoints: $\sum_{n=0}^{\infty} \frac{(-2-2)^{n+1}}{(n+1)4^{n+1}} = \frac{-4^{n+1}}{(n+1)4^{n+1}} = \frac{(-1)^{n+1}}{n+1}$ converges: AST for harmonic series

$\sum_{n=0}^{\infty} \frac{(6-2)^{n+1}}{(n+1)4^{n+1}}$ $\frac{4^{n+1}}{(n+1)4^{n+1}} = \frac{1}{(n+1)}$. $\sum_{n=1}^{\infty} \frac{1}{n+1}$ diverges using Direct comparison test with harmonic series $\frac{1}{n}$

$\boxed{\text{I.O.C} \rightarrow -2 \leq x < 6}$

26) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ $\lim_{n\to\infty} \left| \frac{x^{2n+3}}{2n+3} \cdot \frac{2n+1}{x^{2n+1}} \right| = \lim_{n\to\infty} \left| x^2 \cdot \frac{2n+1}{2n+3} \right| = |x^2| < 1$

$R = 1$

Interval $\rightarrow$ $-1 < x < 1$

center $= 0$

*Test endpts:

at $x=-1$ $\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^{2n+1}}{2n+1} = \sum \frac{-(-1)^n}{2n+1}$ converges by AST.

at $x=1$ $\sum \frac{(-1)^n(1)^{2n+1}}{2n+1} = \sum \frac{(-1)^n}{2n+1}$ Converges by AST

$\boxed{\text{I.O.C.} \rightarrow -1 \leq x \leq 1}$

27) $\sum_{n=1}^{\infty} \frac{n}{n+1}(-2x)^{n-1}$ $\lim_{n\to\infty} \left| \frac{(n+1)(-2x)^n}{n+2} \cdot \frac{n+1}{n(-2x)^{n-1}} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^2}{n^2+2n} \cdot (-2x)^{-1} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1)^2}{n^2+2n} \cdot (-2x) \right| = |2x| < 1$ $x < 1/2$ center $x=0$

radius $= 1/2$

Interval: $\frac{-1}{2} < x < 1/2$

*Test endpts:

when $x = \frac{-1}{2}$ $\sum_{n=1}^{\infty} \frac{n}{n+1} \cdot (1)^{n-1}$ diverges by n^{th} term test

$\lim_{n\to\infty} \frac{n}{n+1} \neq 0$

when $x = 1/2$ $\sum_{n=1}^{\infty} \frac{n}{n+1}(-1)^{n-1}$ a) $\lim_{n\to\infty} \frac{n}{n+1} \neq 0$ diverges by AST

$\boxed{\text{I.O.C.} \rightarrow \frac{-1}{2} < x < 1/2}$

31) $\sum_{n=1}^{\infty} \frac{2\cdot 3\cdot 4 \cdots (n+1)x^n}{n!} = \sum_{n=1}^{\infty} (n+1)x^n \quad \lim_{n\to\infty}\left|\frac{(n+2)x^{n+1}}{(n+1)x^n}\right| = |x| < 1$

radius = 1

Interval $-1 < x < 1$

center = $x = 0$

*Test endpts $x=-1$ $\sum_{n=1}^{\infty}(n+1)(-1)^n$ diverges by n^{th} term test

$x=1$ $\sum_{n=1}^{\infty}(n+1)(1)^n$ diverges by n^{th} term test

$\boxed{\text{I.O.C. } -1<x<1}$

32) $\sum_{n=1}^{\infty}\left[\frac{2\cdot 4\cdot 6\cdots(2n)}{3\cdot 5\cdot 7\cdots(2n+1)}\right]x^{2n+1} \quad \lim_{n\to\infty}\left|\frac{2n+2}{(2n+3)}\cdot x^{2n+3}\cdot\frac{2n+1}{(2n)x^{2n+1}}\right| = |x^2| < 1$

$R=1$

Interval: $-1 < x < 1$

center: $x=0$

*Test endpoints: $x=-1$ $\sum_{n=1}^{\infty}\frac{2n+2}{2n+3}(-1)^{2n+3}$ Diverges by Direct comparison

$x=1$ $\longrightarrow$ $\frac{1}{2n+1}$ diverges

$\boxed{\text{IOC: } -1<x<1}$

36) $\sum_{n=0}^{\infty}\frac{(n!)^k x^n}{(kn)!} \quad \lim_{n\to\infty}\left|\frac{(n+1)!^k x^{n+1}}{(kn+k)!}\cdot\frac{(kn)!}{n!x^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^k}{(kn+k)!}\cdot x\right| = \frac{|x|}{k^k} < 1$

$k(n+1)!$ $(kn+k)(kn+k-1)(kn+k-2)$

$(k+nk)(k-1+nk)(k-2)+nk)\,(1+nk)(nk)$

converges if $\frac{|x|}{k^k} < 1 \rightarrow \boxed{R = k^k}$

37) $\sum_{n=0}^{\infty}\left(\frac{x}{k}\right)^n$ geometric series $\frac{x}{k} < 1$ $\boxed{-k < x < k}$

41) $\sum_{n=0}^{\infty}\frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \dots = \boxed{\sum_{n=1}^{\infty}\frac{x^{n-1}}{(n-1)!}}$

42) $\sum_{n=0}^{\infty}(-1)^{n+1}(n+1)x^n = \boxed{\sum_{n=1}^{\infty}(-1)^n(n)x^{n-1}}$