9.8b p.666 #3-44 D3S2

14) $\sum_{n=0}^{\infty}(-1)^{n+1}(n+1)x^n$ $\lim_{n\to\infty}\left|\frac{(-1)^{n+2}(n+2)x^{n+1}}{(-1)^{n+1}(n+1)x^n}\right| = |x|$ $-1<x<1$

*test endpts:

$x=-1$ $\sum_{n=0}^{\infty}(-1)^{n+1}(n+1)(-1)^n$ diverges

$x=1$ $\sum(-1)^{n+1}(n+1)(1)^n$ diverges $\boxed{-1<x<1}$

18) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{(n+1)(n+2)}$ $\lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{n+1}}{(n+2)(n+3)}\frac{(n+1)(n+2)}{(-1)^n x^n}\right| = \lim_{n\to\infty}|x| = |x|<1$

Interval: $-1<x<1$

*Test endpts: $x=-1$ $\sum_{n=0}^{\infty}\frac{(-1)^n(-1)^n}{(n+1)(n+2)} =$ converges by Comparison Test to p-series.

*Test $x=1$ $\sum\frac{(-1)^n}{(n+1)(n+2)}$ AST converges

$\boxed{\text{I.O.C. } -1\le x\le 1}$

24) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{n2^n}$ $\lim_{n\to\infty}\left|\frac{(-1)^{n+2}(x-2)^{n+1}}{(n+1)2^{n+1}}\cdot\frac{n2^n}{(-1)^{n+1}\cancel{(x-2)}^n}\right| = \left|\frac{x-2\,.\,n}{2(n+1)}\right| = \frac{1}{2}|x-2|<1$

$-2<x-2<2$

$0<x<4$

test $x=0$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-\cancel{2})^n}{n\cancel{2^n}} = \frac{(-1)}{n}$ diverges

test $x=4$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(2)^n}{n2^n} = \frac{(-1)^{n+1}}{n}$ AST converges $\boxed{\text{I.O.C}: 0<x\le 4}$

34) $\sum_{n=1}^{\infty} \frac{n!(x+1)^n}{1\cdot 3\cdot 5 \cdots (2n-1)}$ $\quad \lim_{n\to\infty} \left| \frac{(x+1)^{\cancel{n+1}}\, (n+1)!\, \cancel{(2n-1)}}{(2n+1)\cancel{(2n-1)}\, \cancel{n!}\, \cancel{(x+1)^n}} \right| = \frac{1}{2}|x+1| < 1$

$-2 < x+1 < 2$

$-3 < x < 1$

*test endpts.

$x=1 \quad a_n = \frac{n!\,2^n}{1\cdot 3\cdot 5 \cdots (2n-1)} = \frac{2\cdot 4\cdot 6 \cdots 2n}{2n-1}$ diverges >1

$1\cdot 2\cdot 3\cdot 4\cdot 5$
$2\cdot 2\cdot 2\cdot 2\cdot 2$

$x=-3 \quad a_n = \frac{n!(-2)^n}{1\cdot 3 \cdots (2n-1)} = (-1)^n \frac{2\cdot 4 \cdots 2n}{1\cdot 3 \cdots (2n-1)}$ diverges AST

$\boxed{\text{I.O.C. } -3<x<1}$

35) $\sum_{n=1}^{\infty} \frac{(x-c)^{n-1}}{c^{n-1}}$ $\quad \lim_{n\to\infty} \left| \frac{(x-c)^n}{c^n} \cdot \frac{c^{n-1}}{(x-c)^{n-1}} \right| = \frac{1}{c}(x-c)$ $\quad$ Radius $= c$

$\frac{1}{c}|x-c| < 1$

$-c < x-c < c$

$0 < x < 2c$

test endpts:

$x=0 \rightarrow \sum_{n=1}^{\infty} \frac{(-c)^{n-1}}{c^{n-1}} = (-1)^{n-1}$ diverges

$x=2c \quad \sum \frac{c^{n-1}}{c^{n-1}} = 1$ diverges

$\boxed{\text{IOC: } 0<x<2c}$

38) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-c)^n}{nc^n}$ $\quad \lim_{n\to\infty} \left| \frac{(-1)^{n+2}(x-c)^{\cancel{n+1}}}{(n+1)c^{n+1}} \cdot \frac{nc^n}{(-1)^{n+1}\cancel{(x-c)^n}} \right| = \frac{1}{c}|x-c| < 1$

$-c < x-c < c$

$0 < x < 2c$

test endpts: $R=c \quad x=c$

$x=0, \ \sum_{n=1}^{\infty} \frac{-1}{n}$ diverges

$x=2c \quad \sum \frac{(-1)^{n+1}(c)^n}{n\,c^n}$ converges by AST harmonic series

$\boxed{\text{I.O.C. } 0<x\leq 2c}$