Name_____ Date_____ Period_____

Worksheet 11.4—Power Series II
Show all work. No calculator except unless specifically stated.

On problems 1-5, find a power series for the given function, centered at the given value of a. Give the first four nonzero terms and the general term.

1. $f(x)=\dfrac{1}{1+x},\ a=0$

2. $f(x)=\dfrac{1}{1+x^2},\ a=0$

3. $f(x)=\dfrac{3}{x+2},\ a=0$

4. $f(x)=\dfrac{x}{1-2x},\ a=0$

5. $f(x)=\dfrac{1}{4-x},\ a=1$

6. Let f be the function given by $f(t) = \dfrac{4}{1+t^2}$ and G be the function given by $G(x) = \int_0^x f(t)dt$.

(a) Find the first four nonzero terms and the general term for the power series expansion of $f(t)$ about $t = 0$.

(b) Find the first four nonzero terms and the general term of the power series expansion of $G(x)$ about $x = 0$.

(c) Find the interval of convergence of the power series in part (b). Justify your answer.

7. Let f be the function given by $f(x) = e^{-2x^2}$

(a) Find the first four nonzero terms and the general term of the power series for $f(x)$ about $x = 0$.

(b) Find the interval of convergence of the power series for $f(x)$ about $x = 0$. Show the analysis that leads to your conclusion.

(c) (Calculator Permitted) Let g be the function given by the sum of the first four nonzero terms of the power series for $f(x)$ about $x = 0$. Show that $|f(x) - g(x)| < 0.02$ for $-0.6 \le x \le 0.6$.

8. The Maclaurin series for $f(x)$ is given by $1+\frac{x}{2!}+\frac{x^2}{3!}+\frac{x^3}{4!}+\text{L}\ +\frac{x^n}{(n+1)!}+\text{L}$

(a) Find $f'(0)$ and $f^{(17)}(0)$.

(b) For what values of x does the given series converge? Show your reasoning.

(c) Let $g(x)=xf(x)$. Write the Maclaurin series for $g(x)$ in terms of a familiar function without using series. Then, write $f(x)$ in terms of the same familiar function.

By recognizing each series in problems 9-12 as a Taylor series evaluated at a particular value of x, find the sum of each of the following convergent series.

9. $1+\frac{2}{1!}+\frac{4}{2!}+\frac{8}{3!}+\cdots+\frac{2^n}{n!}+\cdots$

10. $1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\cdots+\frac{(-1)^n}{(2n+1)!}+\cdots$

11. $1+\frac{1}{4}+\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^3+\cdots+\left(\frac{1}{4}\right)^n+\cdots$

12. $1-\frac{100}{2!}+\frac{10{,}000}{4!}+\cdots+\frac{(-1)^n\cdot 10^{2n}}{(2n)!}+\cdots$