9.9 Geometric Power Series p.674 #1-36

$$\boxed{* S_n = \frac{a_1}{1-r}}$$

2) Find geometric series centered at 0.

$$f(x) = \frac{4}{5-x} = \frac{4/5}{5/5 - x/5} = \frac{a_1}{1-r}$$

$$\sum_{n=0}^{\infty} \frac{4}{5}\left(\frac{x}{5}\right)^n$$

converges when $\left|\frac{x}{5}\right| < 1$

$$-1 < \frac{x}{5} < 1 \rightarrow -5 < x < 5$$

$$\boxed{I.O.C. \rightarrow (-5, 5)}$$

b)

$$\begin{array}{r} \frac{4}{5} + \frac{4}{25}x + \frac{4}{125}x^2 + \frac{4x^3}{625} \\ 5-x \overline{)4} \\ 4 - \frac{4}{5}x \\ \hline \frac{4}{5}x \\ \frac{4}{5}x - \frac{4}{25}x^2 \\ \hline \frac{4}{25}x^2 \ldots \end{array}$$

4) a)

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \frac{a_1}{1-r}$$

$$|-x| < 1 \quad -1 < x < 1$$

$$\sum_{n=0}^{\infty} (-x)^n$$

$$\boxed{I.O.C \rightarrow (-1, 1)}$$

$$\begin{array}{r} 1 - x + x^2 - x^3 + \ldots \\ 1+x \overline{)1} \\ 1+x \\ \hline -x \\ -x - x^2 \\ \hline x^2 \\ x^2 + x^3 \\ \hline -x^3 \\ -x^3 - x^4 \end{array}$$

Find power series, centered at c, determine interval of convergence

6) $f(x) = \frac{4}{5-x}$, $c=-2$ $\quad \frac{4}{5+2-(x+2)} = \frac{4}{7-(x+2)} = \frac{4/7}{7/7 - \frac{(x+2)}{7}}$

$a_1 = 4/7$ $\quad \sum_{n=0}^{\infty} \frac{4}{7}\left(\frac{1}{7}(x+2)\right)^n$ $\quad \frac{1}{7}|x+2| < 1$

$r = \frac{x+2}{7}$

$-7 < x+2 < 7$

$\boxed{\text{I.O.C. } -9 < x < 5}$

8) $f(x) = \frac{3}{2x-1}$, $c=2$ $\Rightarrow \frac{3}{-1+2x} = \frac{3}{-1+4-(-2(x-2))} = \frac{3}{3-(-2(x-2))}$

$= \frac{\frac{3}{3}}{\frac{3}{3} - \left(-\frac{2}{3}(x-2)\right)}$ $\quad a_1 = 2/3$ $\quad r = \frac{-2}{3}(x-2)$ $\quad \sum_{n=0}^{\infty}\left[\frac{-2}{3}(x-2)\right]^n$ $\quad \frac{2}{3}|x-2| < 1$

$|x-2| < 3/2$ $\quad \frac{-3}{2} < |x-2| < 3/2$ $\quad \boxed{\text{I.O.C. } \frac{1}{2} < x < 7/2}$

$+4/2 \quad +4/2$

10) $h(x) = \frac{1}{2x-5}$ $\quad c=0$ $\quad \frac{1}{-5+2x} = \frac{1}{-5-(-2x)}$ $\quad \frac{1/5}{-5/5 - \left(\frac{-2}{-5}x\right)}$ $\quad a_1 = -1/5$ $\quad r = 2/5x$

$\sum_{n=0}^{\infty} \frac{-1}{5}\left[\frac{2}{5}x\right]^n$ $\quad \left|\frac{2}{5}x\right| < 1$ $\quad \boxed{\frac{-5}{2} < x < 5/2}$

12) $f(x) = \frac{4}{3x+2}$, $c=2$ $\quad \frac{4}{2-(-3x)}$ $\quad \frac{4}{2+6-(-3(x-2))} = \frac{4}{8-(-3(x-2))}$

$\frac{4/8}{8/8 - \left[-3/8(x-2)\right]}$ $\quad a_1 = 1/2$ $\quad r = \frac{-3}{8}(x-2)$ $\quad \sum_{n=0}^{\infty} \frac{1}{2}\left[\frac{-3}{8}(x-2)\right]^n$ $\quad \frac{3}{8}|x-2| < 1$

$-\frac{8}{3} < |x-2| < 8/3$ $\quad \boxed{\text{IOC: } \frac{-2}{3} < x < 14/3}$

$+6/3 \quad +6/3 \quad +6/3$

$c=0$

14) $g(x) = \dfrac{4x-7}{2x^2+3x-2} = \dfrac{A}{2x-1} + \dfrac{B}{x+2} = \dfrac{A(x+2)}{2x-1} + \dfrac{B(2x-1)}{x+2}$

$(2x-1)(x+2)$

$4x-7 = A(x+2)+B(2x-1)$

$= Ax+2A+2Bx-B$

$4x-7 = (A+2B)x + 1(2A-B)$

$4 = A+2B$ | $2A-B = -7$

$A+2B=4$
$2(2A-B=-7)$

$A+2B=4$
$4A-2B=-14$

$5A=-10$
$A=-2$
$B=3$

$g(x) = \dfrac{-2}{2x-1} + \dfrac{3}{x+2}$

$= \dfrac{-2}{-1+2x}$

$\dfrac{-2/-1}{\frac{-1}{-1}-\left(\frac{-2x}{-1}\right)}$

$= \dfrac{2}{1-(+2x)}$

$a_1 = 2$
$r = 2x$

$\sum_{n=0}^{\infty} 2(2x)^n$

$-1 < 2x < 1$

$\frac{-1}{2} < x < \frac{1}{2}$

$\dfrac{3}{2+x} = \dfrac{3/2}{2/2-\left(-\frac{x}{2}\right)}$

$a_1 = 3/2$
$r = -x/2$

$= \sum_{n=0}^{\infty} \frac{3}{2}\left(-x/2\right)^n$

$\left|-\frac{x}{2}\right| < 1$

$-2 < x < 2$

$\boxed{\text{Sum} \to \sum \frac{3}{2}\left(-x/2\right)^n + 2(2x)^n \qquad \text{I.O.C. } \frac{-1}{2} < x < 1/2}$

13) $g(x) = \dfrac{3x}{x^2+x-2}$ $\quad c=0$ $\quad \dfrac{A}{x+2} + \dfrac{B}{(x-1)}$ $\quad \dfrac{A(x-1)}{x+2} + \dfrac{B(x+2)}{x-1}$

$(x+2)(x-1)$

$3x = A(x-1) + B(x+2)$

$3x = Ax - A + Bx + 2B$

$= (A+B)x + 1(-A+2B)$

$A+B=3$ $\quad -A+2B=0$

$$\begin{array}{r} A+B=3 \\ -A+2B=0 \\ \hline 3B=3 \end{array}$$

$B=1$

$A=2$

$g(x) = \dfrac{2}{x+2} + \dfrac{1}{x-1}$

$= \dfrac{\frac{2}{2}}{\frac{2}{2}+\frac{x}{2}}$ $\quad \dfrac{-1}{1-x}$

$= \dfrac{1}{1+\frac{1}{2}(x)}$ $\quad r=x$

$\sum\left(\frac{-1}{2}x\right)^n + \sum(-1)x^n = \sum\left[\frac{1}{(-2)^n} - 1\right]x^n$

$= \left[\dfrac{1-(-2)^n}{(-2)^n}\right]x^n$

$\lim_{n\to\infty}\left[\dfrac{\left(1-(-2)^{n+1}\right)x^{n+1}}{-2^{n+1}} \cdot \dfrac{(-2)^n}{\left[1-(-2)^n\right]x^n}\right]$

$= \lim_{n\to\infty}\left[\dfrac{1-(-2)^{n+1}}{-2-(-2)^{n+1}}x\right] = |x|$

$|x| < 1$ so

$\boxed{\text{IOC } -1<x<1}$

15) $f(x) = \dfrac{2}{1-x^2}$, $c=0$

$= \dfrac{1}{1-x} + \dfrac{1}{1+x} = \sum(x^n) + \sum(-x)^n = \sum_{n=0}^{\infty}\left(1+(-1)^n\right)x^n =$

$2x^0 + 2x^2 + 2x^4 + 2x^6 + \dots$

$\boxed{2x^{2n}}$

$\lim_{n\to\infty}\left|\dfrac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\dfrac{2x^{2(n+1)}}{2x^{2n}}\right|$

$= \lim_{n\to\infty}\left|\dfrac{2x^{2n+2}}{2x^{2n}}\right| = \left|\dfrac{\cancel{2}\cdot\cancel{x^{2n}}\cdot x^2}{\cancel{2}\cancel{x^{2n}}\cdot 1}\right| = |x^2| < 1$ $\quad -1<x<1$

$\boxed{\text{I.O.C } -1<x<1}$