Calculus AB   Chapter 2.2-2.3 Morning Quiz Review

1.

A particle moves along the x-axis so that at time $t \geq 0$, its position is given by $x(t) = t^3 - 3t^2 - 9t + 2$.

(a) At $t = 0$, is the particle moving to the right or to the left? Justify.

(b) At what time(s) does the particle change directions. Justify.

c) Find all values of t for which the particle is moving to the left. Justify

d) Find all values of t for which the particle is moving to the right. Justify

2. ⓐ Determine the x-coordinates at which the graph of the function has a horizontal tangent line. ⓑ Find tangent line equation(s)

$f(x) = \frac{x^2}{x-1}$.

3) If $y = \frac{2-x}{3x+1}$, then $\frac{dy}{dx} =$

(A) $-\frac{7}{(3x+1)^2}$ (B) $\frac{6x-5}{(3x+1)^2}$ (C) $-\frac{9}{(3x+1)^2}$ (D) $\frac{7}{(3x+1)^2}$ (E) $\frac{7-6x}{(3x+1)^2}$

4. Find the equation of the tangent line to $y = \frac{2-x}{5+x}$ at $x = 1$.

5. Given f(x) $= \frac{4x^3}{\sqrt{x}} - 3x\left(\sqrt[5]{x}\right) - 6x^3\left(2\sqrt{x} - 4x\right) - 4\pi x + 3.99$ Find f '(x)

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Key

$v(t) = 3t^2 - 6t - 9$

$a(t) = 6t - 6$

1)

A particle moves along the x-axis so that at time $t \geq 0$, its position is given by $x(t) = t^3 - 3t^2 - 9t + 2$.

(a) At $t = 0$, is the particle moving to the right or to the left? Justify.

$v(0) = -9$ Since $v(0) < 0$, particle moves to the left

(b) At what time(s) does the particle change directions. Justify.

*set $v(t) = 0$

$0 = 3t^2 - 6t - 9$

$0 = 3(t^2 - 2t - 3)$

$= 3(t-3)(t+1)$

$t = 1, 3$ secs.

$\cancel{t = -1}$ $t = 3$

$0 \quad - \quad 3 \quad +$

change directions once when $t = 3$ secs.

c) Find all values of t for which the particle is moving to the left. Justify

$v(t) < 0$ in the interval $[0, 3)$

d) Find all values of t for which the particle is moving to the right. Justify

$v(t) > 0$ in interval $(3, \infty)$

2. Determine the x-coordinates at which the graph of the function has a horizontal tangent line.

set numerator of $f'(x) = 0$

$f(x) = \dfrac{x^2}{x-1}$

$f'(x) = \dfrac{(2x)(x-1) - x^2(1)}{(x-1)^2} = \dfrac{2x^2 - 2x - x^2}{(x-1)^2}$

Use Quotient Rule $\dfrac{f'g - fg'}{g^2}$

$f'(x) = \dfrac{x^2 - 2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$

$0 = x(x-2)$

$\boxed{x = 0, \ x = 2}$

$f(0) = \frac{0}{-1} = 0$

$f(2) = \frac{4}{1} = 4$

Tangent line equations: $y = 0$ and $y = 4$

3) If $y = \dfrac{2-x}{3x+1}$, then $\dfrac{dy}{dx} =$

(A) $-\dfrac{7}{(3x+1)^2}$ (B) $\dfrac{6x-5}{(3x+1)^2}$ (C) $-\dfrac{9}{(3x+1)^2}$ (D) $\dfrac{7}{(3x+1)^2}$ (E) $\dfrac{7-6x}{(3x+1)^2}$

$y' = \dfrac{(-1)(3x+1) - (2-x)(3)}{(3x+1)^2} = \dfrac{-3x - 1 - 6 + 3x}{(3x+1)^2} = \boxed{\dfrac{-7}{(3x+1)^2}}$

4. Find the equation of the tangent line to $y = \dfrac{2-x}{5+x}$ at $x = 1$.

$y' = \dfrac{-1(5+x) - (2-x)(1)}{(5+x)^2} = \dfrac{-5 - x - 2 + x}{(5+x)^2}$

$y' = \dfrac{-7}{(5+x)^2}$

$y(1) = \frac{2-1}{5+1} = \frac{1}{6}$

$y'(1) = \frac{-7}{6^2} = \frac{-7}{36}$

point: $(1, 1/6)$

slope: $m = -7/36$

$\boxed{y - 1/6 = \frac{-7}{36}(x - 1)}$

$4x^{5/2}$ $3x^{6/5}$

5. Given f(x) = $\dfrac{4x^3}{\sqrt{x}} - 3x\sqrt[5]{x} - 6x^3(2\sqrt{x} - 4x) - 4\pi x + 3.99$ Find f '(x)

$f(x) = 4x^{5/2} - 3x^{6/5} - 12x^{7/2} + 24x^4 - 4\pi x + 3.99$

$f'(x) = \frac{5}{2} \cdot 4x^{3/2} - \frac{6}{5} \cdot 3x^{1/5} - \frac{7}{2} \cdot 12x^{5/2} + 4 \cdot 24x^3 - 4\pi + 0$

$f'(x) = 10x^{3/2} - \frac{18}{5}x^{1/5} - 42x^{5/2} + 96x^3 - 4\pi$