A.P. Calculus AB 4.3 – 4.5 Review

NO CALCULATORS!

1

1. Given $f(x) = x^2 - 2x + 3$, find a) average value in the interval [0, 3] b) find the value of c guaranteed by the theorem

2. Given $f(x) = \sec^2 x$, find the average value in the interval $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

3. If $f(x) = \int_{-2}^{-2x^2} \frac{t}{4-t^3}\,dt$, find $\frac{d}{dx} f(x)$.

4. If $f(x) = \int_{-x}^{3\sqrt{x}} 1-2t\,dt$, find $\frac{d}{dx} f(x)$.

5. Let $\int_{-3}^{6} g(x)\,dx = 10$ and $\int_{3}^{0} g(x)\,dx = -4$

a) If g(x) is even, find $\int_{-6}^{3} g(x)\,dx$

b) If g(x) is odd, find $\int_{0}^{6} g(x)\,dx$

2

6. If $\int_{3} f(x)\,dx = -4$

a) $\int_{7}^{3} 2f(x)\,dx$

b) $\int_{7}^{3} [3f(x) - 2]\,dx$

7. Evaluate $\int \frac{2}{x^2} \sec\left(\frac{3}{x}\right) \tan\left(\frac{3}{x}\right) dx$

8. Evaluate $\int 5x\sqrt{2-x}\,dx$.

9. Evaluate $\int_{4}^{9} \frac{x+1}{\sqrt{x}}\,dx$

10. Evaluate $\int_{0}^{\frac{\pi}{3}} \tan^2 x \sec^2 x \,dx$

A.P. Calculus AB 4.3 – 4.5 Review Solutions 3

NO CALCULATORS!

Avg. value theorem: $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

1. Given $f(x) = x^2 - 2x + 3$, find a) average value in the interval [0, 3] b) find the value of c guaranteed by the theorem

$f(c) = \frac{1}{3-0}\int_0^3 x^2 - 2x + 3\,dx$

$= \frac{1}{3} \cdot \left. \frac{x^3}{3} - \frac{2x^2}{2} + 3x \right]_0^3$

$\frac{1}{3} \cdot \frac{27}{3} - 3^2 + 9$

$= \frac{1}{3}(9 - 9 + 9)$

$= 3$

$f(c) = 3$

Avg. value $= 3$

b) $x^2 - 2x + 3 = 3$

$x^2 - 2x = 0$

$x(x-2) = 0 \quad x = 0, x = 2$

$\boxed{c=2}, \boxed{c=0}$

2. Given $f(x) = \sec^2 x$, find average value in the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$

$f(c) = \frac{1}{\pi/4 - -\pi/4}\int_{-\pi/4}^{\pi/4} \sec^2 x\,dx = \frac{1}{\pi/2} \cdot \left. \tan x \right]_{-\pi/4}^{\pi/4} = \frac{2}{\pi}\left[\tan\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)\right] = \frac{2}{\pi}[1-(-1)]$

$f(c) = \frac{2}{\pi} \cdot 2 = \boxed{\frac{4}{\pi}}$

SFTC: $\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x)) \cdot p'(x)$

$\frac{d}{dx}\left[\int_{g(x)}^{p(x)} f(t)\,dt\right] = f(p(x)) \cdot p'(x) - f(g(x)) \cdot g'(x)$

3. If $f(x) = \int_{-2}^{-2x^2} \frac{t}{4-t^3}\,dt$, find $\frac{d}{dx}f(x)$. use $\boxed{\text{SFTC}}$

$= \frac{-2x^2}{4-(-2x^2)^3} \cdot -4x = \frac{8x^3}{4+8x^6}$

$= \boxed{\frac{2x^3}{1+2x^6}}$

4. If $f(x) = \int_{-x}^{3\sqrt{x}} 1-2t\,dt$, find $\frac{d}{dx}f(x)$.

$\frac{d}{dx}\left[\int_{-x}^{3\sqrt{x}} 1-2t\,dt\right] = [1-2(3\sqrt{x})] \cdot 3 \cdot \frac{1}{2}x^{-1/2} - [1-2(-x)](-1)$

$(1-6\sqrt{x})\frac{3}{2\sqrt{x}} + 1 + 2x$

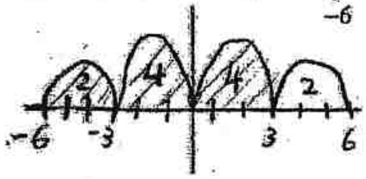
$\frac{3}{2\sqrt{x}} - \frac{18\sqrt{x}}{2\sqrt{x}} + 1 + 2x$

$\frac{3}{2\sqrt{x}} - 9 + 1 + 2x$

$= \boxed{\frac{3}{2\sqrt{x}} - 8 + 2x}$

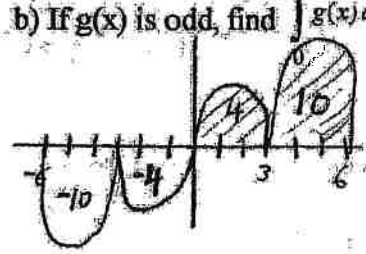
5. Let $\int_{-3}^{6} g(x)\,dx = 10$ and $\int_{3}^{0} g(x)\,dx = -4 \quad = \int_0^3 g(x)\,dx = 4$

a) If g(x) is even, find $\int_{-6}^{3} g(x)\,dx$



$\int_{-6}^{3} g(x)\,dx = \boxed{10}$

b) If g(x) is odd, find $\int_{0}^{6} g(x)\,dx$



$\int_0^6 g(x)\,dx = \boxed{14}$

4

6. If $\int_3^7 f(x)\,dx = -4$

a) $\int_7^3 2f(x)\,dx = 2\left[-\int_3^7 f(x)\,dx\right]$

$2\cdot(-(-4)) = \boxed{8}$

b) $\int_7^3 [3f(x)-2]\,dx = 3\int_7^3 f(x)\,dx - \int_7^3 2\,dx$

$3\cdot(4) = 12$

$2x\Big]_7^3 = 6-14 = -8$

$= 12-(-8) = \boxed{20}$

7. Evaluate $\int \frac{2}{x^2}\sec\left(\frac{3}{x}\right)\tan\left(\frac{3}{x}\right)dx$

$u = \frac{3}{x} = 3x^{-1}$

$\frac{du}{dx} = -3x^{-2}$

$\frac{du}{dx} = \frac{-3}{x^2}$

$dx = -\frac{x^2}{3}du$

$\int \frac{2}{\cancel{x^2}}\sec(u)\tan(u)\cdot -\frac{\cancel{x^2}}{3}du$

$= -\frac{2}{3}\int \sec u \tan u\,du$

$= -\frac{2}{3}\sec u + C$

$= \boxed{-\frac{2}{3}\sec\left(\frac{3}{x}\right)+C}$

8. Evaluate $\int 5x\sqrt{2-x}\,dx = \int 5x(2-x)^{1/2}dx$

$u = 2-x$

$\frac{du}{dx} = -1$

$dx = -du$

$x = 2-u$

$\int 5x\cdot u^{1/2}(-du)$

$\int 5(2-u)u^{1/2}\cdot(-du)$

$= \int -10u^{1/2} + 5u^{3/2}\,du$

$= \frac{-10u^{3/2}}{3/2} + \frac{5u^{5/2}}{5/2} + C$

$= \frac{2}{3}\left(-10u^{3/2}\right) + \frac{2}{5}\left(5u^{5/2}\right) + C$

$= -\frac{20}{3}u^{3/2} + 2u^{5/2} + C$

$= \boxed{-\frac{20}{3}(2-x)^{3/2} + 2(2-x)^{5/2} + C}$

9. Evaluate $\int_4^9 \frac{x+1}{\sqrt{x}}dx = \int (x+1)x^{-1/2}dx$

$\int_4^9 x^{1/2} + x^{-1/2}\,dx$

$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}$

$\frac{2}{3}x^{3/2} + 2x^{1/2}\Big]_4^9$

$= \frac{2}{3}(9)^{3/2} + 2(9)^{1/2} - \left(\frac{2}{3}(4)^{3/2} + 2(4)^{1/2}\right)$

$\frac{2}{3}(27) + 2(3) - \frac{2}{3}(8) - 2(2)$

$18 + 6 - \frac{16}{3} - 4 = \boxed{\frac{44}{3}}$

10. Evaluate $\int_0^{\pi/3} \tan^2 x\sec^2 x\,dx$

if $x=0$, $u=\tan 0 = 0$

if $x=\pi/3$, $u=\tan(\pi/3)=\sqrt{3}$

$u = \tan x$

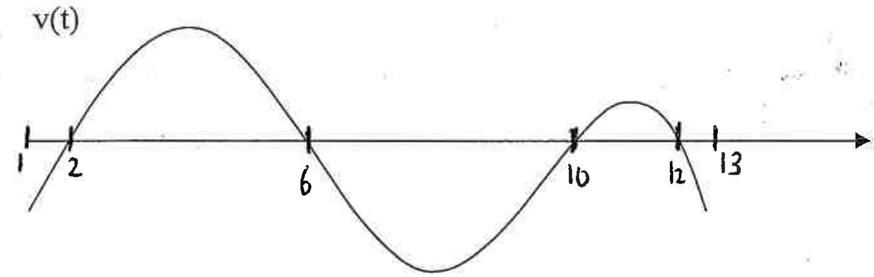
$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int_0^{\pi/3} u^2\cdot\cancel{\sec^2 x}\cdot\frac{du}{\cancel{\sec^2 x}}$

$\frac{u^3}{3}\Big]_0^{\sqrt{3}} = \frac{1}{3}(\sqrt{3})^3 - \frac{1}{3}(0)^3 = \frac{1}{3}(3\sqrt{3}) = \boxed{\sqrt{3}}$

AP Calculus AB (4.3-4.5) Review : PVA Particle Motion Problem #2:



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 13$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 2$, 6, 10 and 12 and the graph has horizontal tangents at $t = 4$, 8, and 11.

The areas of the regions bounded are 1,8, 10, 3, 2 respectively. The position function for the particle is called x and at $t = 1$, $x(1) = -3$

a. Create Sign lines for v(t) and a(t)

b. On what intervals (if any) is the velocity negative? Justify your answer.

c. On what intervals (if any) is the acceleration positive? Justify your answer.

d. On the interval $8 < t < 10$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

e. On the interval $12 < t < 13$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

f. Find the positions of the particle at $t = 2$, $t = 6$ and $t = 10$, and $t = 12$ (use definite integrals.)

g. State the absolute extrema and the t-values where they occur.

h. Find the total distance traveled by the particle from $t = 1$ to $t = 13$. (Use Integral Notation)

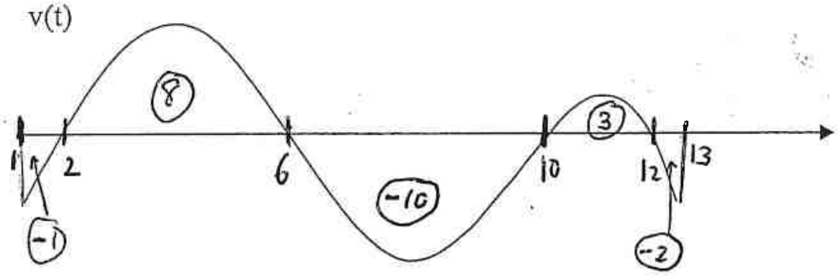
i. Find the total displacement of the particle from $t = 6$ to $t = 13$. (Use Integral Notation)

j. Sketch graph of x(t) below:

6

AP Calculus AB (4.3-4.5) Quiz Review : PVA Particle Motion Problem #2:

Key



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 13$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 2$, 6, 10 and 12 and the graph has horizontal tangents at $t = 4$, 8, and 11.

The areas of the regions bounded are 1,8, 10, 3, 2 respectively. The position function for the particle is called x and at $t = 1$, $x(1) = -3$

a. Create Sign lines for v(t) and a(t)

v(t): − on (1, 2), + on (2, 6), − on (6, 10), + on (10, 12), − on (12, 13)

1 2 6 10 12 13

a(t): + on (1, 4), − on (4, 8), + on (8, 11), − on (11, 13)

1 4 8 11 13

b. On what intervals (if any) is the velocity negative? Justify your answer.

$v(t) < 0$ on $(1,2) \cup (6,10) \cup (12,13)$

c. On what intervals (if any) is the acceleration positive? Justify your answer.

$a(t) > 0$ on $(1,4) \cup (8,11)$

d. On the interval $8 < t < 10$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

speed is decreasing since $v(t) < 0$, $a(t) > 0$ (opposite signs)

e. On the interval $12 < t < 13$ is the speed of the particle increasing or decreasing? Give a reason for your answer.

speed is increasing since $a(t) < 0$, $v(t) < 0$ (same signs)

final position = given position + displacement

$$* \; x(b) = x(a) + \int_a^b v(t)\,dt$$

f. Find the positions of the particle at $t = 2$, $t = 6$ and $t = 10$, and t = 12 (use definite integrals.)

$x(2) = x(1) + \int_1^2 v(t)\,dt$

$x(2) = -3 + (-1) = \boxed{-4}$

$x(6) = x(1) + \int_1^6 v(t)\,dt$

$= -3 + (-1+8) = \boxed{4}$

$x(10) = x(1) + \int_1^{10} v(t)\,dt$

$= -3 + (-1+8-10) =$

$= \boxed{-6}$

$x(12) = x(1) + \int_1^{12} v(t)\,dt$

$= -3 + (-1+8-10+3) = \boxed{-3}$

g. State the absolute extrema and the t-values where they occur.

Abs max value is 4 at $x=6$

Abs min value is -4 at $x=2$

h. Find the total distance traveled by the particle from $t = 1$ to $t = 13$. (Use Integral Notation)

$$\int_1^{13} |v(t)|\,dt = \boxed{24}$$

i. Find the total displacement of the particle from $t = 6$ to $t = 13$. (Use Integral Notation)

$$\int_6^{13} v(t)\,dt = -10 + 3 - 2 = \boxed{-9}$$

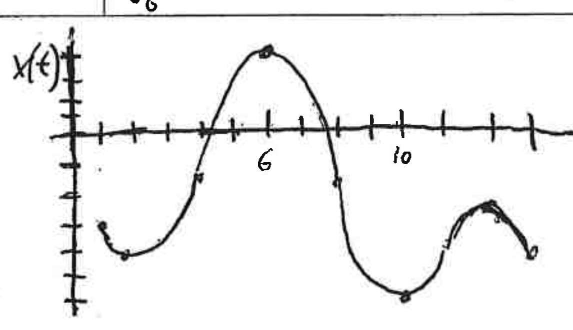
j. Sketch graph of x(t) below:

x(t): ↘ on (1, 2), ↗ on (2, 6), ↘ on (6, 10), ↗ on (10, 12), ↘ on (12, 13)

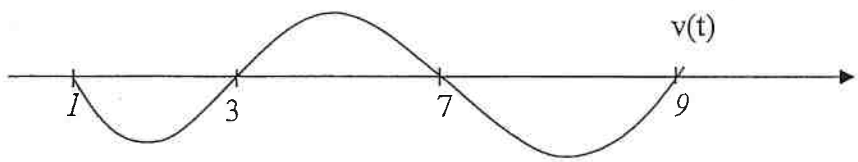
1 2 6 10 12 13

x(t): ∪ on (1, 4), ∩ on (4, 8), ∪ on (8, 11), ∩ on (11, 13)

1 4 8 11 13



7



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 9$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 1$, 3, 7 and 9 and the graph has horizontal tangents at $t = 2$, 5, and 8.

The areas of the regions bounded are 2, 3, and 6 respectively. The position function for the particle is called x and at $t = 1$, $x(1) = 2$.

a. Create Sign lines for v(t) and a(t)

b. On what intervals (if any) is the velocity negative? Justify your answer.

c. On what intervals (if any) is the acceleration positive? Justify your answer.

d. On the interval $5 < t < 7$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

e. On the interval $7 < t < 8$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

f. Find the positions of the particle at $t = 3$, $t = 7$ and $t = 9$. (use definite integrals.)

g. State the absolute extrema and the t-values where they occur.

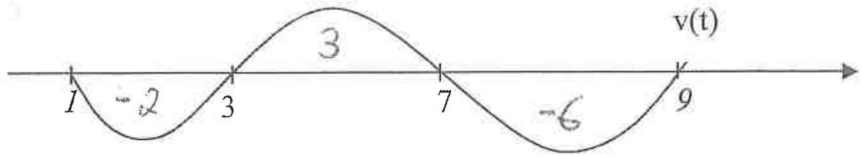
h. Find the total distance traveled by the particle from $t = 1$ to $t = 9$. (Use Integral Notation)

i. Find the total displacement of the particle from $t = 3$ to $t = 9$. (Use Integral Notation)

j. Sketch graph of x(t) below:

8

AP Calculus AB (4.3-4.5) PVA Particle Motion: Velocity Graph Practice Problem



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 9$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 1, 3, 7$ and 9 and the graph has horizontal tangents at $t = 2, 5,$ and 8.

The areas of the regions bounded are 2, 3, and 6 respectively. The position function for the particle is called x and at $t = 1$, $x(1) = 2$.

a. Create Sign lines for v(t) and a(t)

$v(t)$: − on (1, 3), + on (3, 7), − on (7, 9)
1 3 7 9

$a(t)$: − on (1, 2), + on (2, 5), − on (5, 8), + on (8, 9)
1 2 5 8 9

b. On what intervals (if any) is the velocity negative? Justify your answer.

$(1,3) \cup (7,9)$ b/c $v(t) < 0$

c. On what intervals (if any) is the acceleration positive? Justify your answer.

$(2,5) \cup (8,9)$ b/c $v'(t) > 0$

d. On the interval $5 < t < 7$, is the speed of the particle increasing or decreasing? Give a reason for your answer. Decreasing speed b/c $v(t) > 0$ and $a(t) < 0$ (opposite signs)

e. On the interval $7 < t < 8$, is the speed of the particle increasing or decreasing? Give a reason for your answer. Increasing speed b/c $v(t) < 0$ and $a(t) < 0$ (same signs)

f. Find the positions of the particle at $t = 3$, $t = 7$ and $t = 9$. (use definite integrals.)

$x(3) = x(1) + \int_1^3 v(t)\,dt = 2 + (-2) = 0$

$x(7) = x(3) + \int_3^7 v(t)\,dt = 0 + 3 = 3$

$x(9) = x(7) + \int_7^9 v(t)\,dt = 3 + (-6) = -3$

g. State the absolute extrema and the t-values where they occur.

Abs min at -3 where $t = 9$

Abs max at 3 where $t = 7$

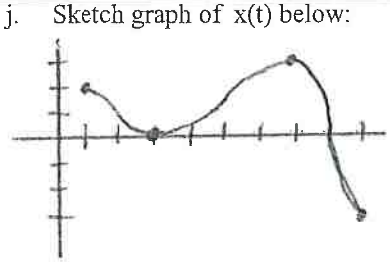
h. Find the total distance traveled by the particle from $t = 1$ to $t = 9$. (Use Integral Notation)

$\int_1^9 |v(t)|\,dt = 2 + 3 + 6 = \boxed{11}$

i. Find the total displacement of the particle from $t = 3$ to $t = 9$. (Use Integral Notation)

$\int_3^9 v(t)\,dt = 3 - 6 = \boxed{-3}$

j. Sketch graph of x(t) below:



$v(t)$: − − + + − −

$a(t)$: − + + − − +

1 2 3 5 7 8 9

A.P. Calculus AB **4.3 – 4.5 (Morning Quiz Review Session)**

NO CALCULATORS!

9

1. Given $f(x) = \frac{4(x^2+1)}{x^2}$, find a) average value in the interval [1, 3] b) find the value of c guaranteed by the theorem

2. If $f(x) = \int_{3x^4}^{-5} \frac{5}{2\sqrt{t}}\,dt$, find $\frac{d}{dx} f(x)$.

3. If $f(x) = \int_{-2x^2}^{\sqrt{x}} t^2 - 2t\,dt$, find $\frac{d}{dx} f(x)$.

4. Let $\int_{-3}^{6} g(x)\,dx = -4$ and $\int_{0}^{3} g(x)\,dx = 2$

a) If g(x) is even, find $\int_{-6}^{3} g(x)\,dx$

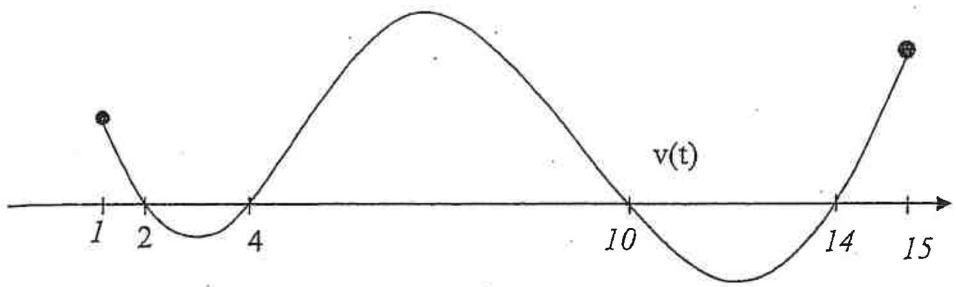
b) If g(x) is odd, find $\int_{0}^{6} g(x)\,dx$

5. Evaluate $\int \sqrt{\tan x}\sec^2 x\,dx$.

6. Evaluate $\int 2x\sqrt{4-x}\,dx$

7. Evaluate $\int_{0}^{1} x^2(x^3+1)^3\,dx$

10



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 15$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 2$, 4, 10 and 14 and the graph has horizontal tangents at $t = 3$, 7, and 12.

The areas of the regions bounded are **1, 2, 14, 6, and 3** respectively. The position function for the particle is called x and at $t = 1$, $x(1) = 3$.

a. Create Sign lines for v(t) and a(t)

b. On what intervals (if any) is the velocity negative? Justify your answer.

c. On what intervals (if any) is the acceleration positive? Justify your answer.

d. On the interval $3 < t < 4$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

e. On the interval $10 < t < 12$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

f. Find the positions of the particle at $t = 2$, $t = 4$ and $t = 10$, $t = 14$. (use definite integrals.)

g. State the absolute extrema and the t-values where they occur.

h. Find the total distance traveled by the particle from $t = 1$ to $t = 15$. (Use Integral Notation)

i. Find the total displacement of the particle from $t = 2$ to $t = 15$. (Use Integral Notation)

A.P. Calculus AB **4.3 – 4.5 (Morning Quiz Review Session)** 11

NO CALCULATORS!

1. Given $f(x) = \frac{4(x^2+1)}{x^2}$, find a) average value in the interval [1, 3] b) find the value of c guaranteed by the theorem $f(x) = \frac{4x^2+4}{x^2} = (4x^2+4)x^{-2} = 4+4x^{-2}$

$\text{Avg. value} = \frac{1}{3-1}\int_1^3 4+4x^{-2}\,dx$

$= \frac{1}{2}\cdot 4x + \frac{4x^{-1}}{-1}\Big]_1^3$

$\frac{1}{2}\cdot\left[12-\frac{4}{3}-(4-4)\right]$

$= \frac{1}{2}\left[\frac{36}{3}-\frac{4}{3}\right] = \frac{1}{2}\left(\frac{32}{3}\right) = \boxed{\frac{16}{3}}$

b) set $f(x) = \frac{16}{3}$

$4 + \frac{4}{x^2} = \frac{16}{3}$

$\frac{4}{x^2} = \frac{16}{3} - \frac{4}{1}$

$\frac{4}{x^2} = \frac{4}{3}$ $x^2 = 3$

$x = \pm\sqrt{3}$

$\boxed{c = \sqrt{3}}$

2. If $f(x) = \int_{3x^4}^{-5} \frac{5}{2\sqrt{t}}\,dt$, find $\frac{d}{dx} f(x)$. $\boxed{\text{SFTC}}$

$f(x) = \int_{-5}^{3x^4} -\frac{5}{2\sqrt{t}} \rightarrow \frac{d}{dx}\int_{-5}^{3x^4} \frac{-5}{2\sqrt{t}} = \frac{-5}{2\sqrt{3x^4}}\cdot 12x^3 = \frac{-30x^3}{\sqrt{3}\,x^2}$

$= \boxed{\frac{-30x}{\sqrt{3}}}$

3. If $f(x) = \int_{-2x^2}^{\sqrt{x}} t^2 - 2t\,dt$, find $\frac{d}{dx} f(x)$. $\boxed{\text{SFTC}}$

$(x - 2\sqrt{x})\cdot\frac{1}{2}x^{-1/2} - \left[4x^4 + 4x^2\right](-4x)$

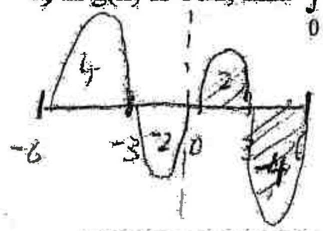
$(x-2\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right) + 16x^5 + 16x^3$

$\boxed{\frac{\sqrt{x}}{2} - 1 + 16x^5 + 16x^3}$

4. Let $\int_{-3}^{6} g(x)\,dx = -4$ and $\int_0^3 g(x)\,dx = 2$

a) If g(x) is even, find $\int_{-6}^{3} g(x)\,dx = \boxed{-4}$

b) If g(x) is odd, find $\int_0^6 g(x)\,dx = \boxed{-2}$



5. Evaluate $\int \sqrt{\tan x}\,\sec^2 x\,dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int u^{1/2}\cdot\cancel{\sec^2 x}\cdot\frac{du}{\cancel{\sec^2 x}} = \frac{u^{3/2}}{3/2} + C = \boxed{\frac{2}{3}(\tan x)^{3/2} + C}$

6. Evaluate $\int 2x\sqrt{4-x}\,dx$ $x = 4-u$

$u = 4-x$

$\frac{du}{dx} = -1$

$dx = -du$

$\int 2x\cdot u^{1/2}\cdot(-du)$

$\int 2(4-u)(u^{1/2})(-1)\,du$

$\int -2u^{1/2}(4-u)\,du$

$\int -8u^{1/2} + 2u^{3/2}\,du$

$= -\frac{8u^{3/2}}{3/2} + 2\frac{u^{5/2}}{5/2} + C$

$= -8\left(\frac{2}{3}\right)u^{3/2} + 2\left(\frac{2}{5}\right)u^{5/2} + C$

$= -\frac{16}{3}(4-x)^{3/2} + \frac{4}{5}(4-x)^{5/2} + C$

7. Evaluate $\int_0^1 x^2(x^3+1)^3\,dx$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$\int x^2\cdot u^3\cdot\frac{du}{3x^2}$

$= \frac{1}{3}\int u^3\,du$

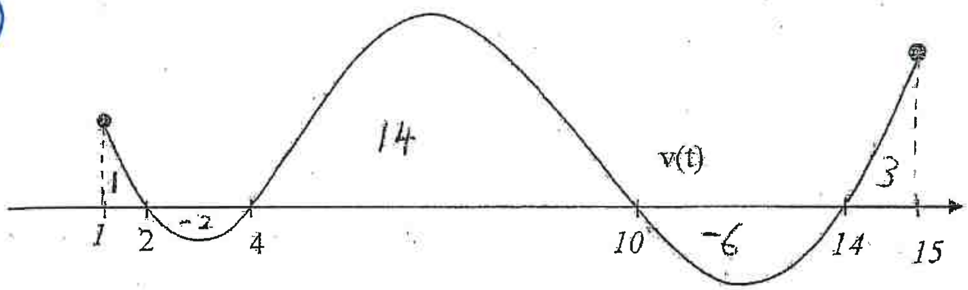
= if $x=0$, $u = 0^3+1 = 1$

if $x=1$, $u = 1^3+1 = 2$

$\frac{1}{3}\cdot\frac{u^4}{4}\Big]_1^2 = \frac{1}{3}\left(\frac{2^4}{4} - \frac{1^4}{4}\right)$

$= \frac{1}{3}\left(\frac{15}{4}\right) = \frac{15}{12} = \boxed{\frac{5}{4}}$

12



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 15$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 2$, 4, 10 and 14 and the graph has horizontal tangents at $t = 3$, 7, and 12.

The areas of the regions bounded are **1, 2, 14, 6, and 3** respectively. The position function for the particle is called x and at $t = 1$, $x(1) = 3$.

a. Create Sign lines for v(t) and a(t)

$v(t)$: + on (1, 2), − on (2, 4), + on (4, 10), − on (10, 14), + on (14, 15)

$a(t)$: − on (1, 3), + on (3, 7), − on (7, 12), + on (12, 15)

b. On what intervals (if any) is the velocity negative? Justify your answer.

$(2,4) \cup (10,14)$ b/c $v(t) < 0$

c. On what intervals (if any) is the acceleration positive? Justify your answer.

$(3,7) \cup (12,15)$ b/c $a(t) > 0$

d. On the interval $3 < t < 4$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

decreasing speed b/c $v(t) < 0$, $a(t) > 0$, (opposite signs)

e. On the interval $10 < t < 12$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

increasing speed b/c $a(t) < 0$, $v(t) < 0$, (same signs)

f. Find the positions of the particle at $t = 2$, $t = 4$ and $t = 10$, $t = 14$. (use definite integrals.)

$x(2) = x(1) + \int_1^2 v(t)\,dt = 3 + 1 = 4$

$x(10) = x(2) + \int_2^{10} v(t)\,dt = 4 + -2 + 14 = 16$

$x(14) = x(10) + \int_{10}^{14} v(t)\,dt = 16 - 6 = 10$

g. State the absolute extrema and the t-values where they occur.

Test other critical pt and endpt.

$x(4) = x(2) + \int_2^4 v(t)\,dt = 4 - 2 = 2$

$x(15) = x(14) + \int_{14}^{15} v(t)\,dt = 10 + 3 = 13$

Abs. min is 2 at $t=4$
Abs. max is 16 at $t=10$

h. Find the total distance traveled by the particle from $t = 1$ to $t = 15$. (Use Integral Notation)

$\int_1^{15} |v(t)|\,dt = 1 + 2 + 14 + 6 + 3 = \boxed{26}$

i. Find the total displacement of the particle from $t = 2$ to $t = 15$. (Use Integral Notation)

$\int_2^{15} v(t)\,dt = -2 + 14 - 6 + 3 = \boxed{9}$

13

AP Calculus Ch. 4 Test Review WS 2 (Non-Calculator)

1. $\int \frac{x^2+x+1}{\sqrt{x}}\,dx$

2. $\int 2x\sqrt{1-3x^2}\,dx$

3. $\int 5\sqrt{x}\,(4-3x^2)\,dx$

4. $\int 5x\sec^2(3x^2)\,dx$

5. $\int x^2\sqrt{7-x}\,dx$

6. $\int_1^2 x(1-2x^2)^3\,dx$

14

7. Find $f'(x)$ if $f(x) = \int_{2x^3}^{\sqrt{x}} \sqrt{1-t^2}\,dt$

8. Find $f'(x)$ if $f(x) = \int_{3x^2}^{\pi} \sqrt{1-t^2}\,dt$

9. $\int_{-5}^{6} |x+2|\,dx$

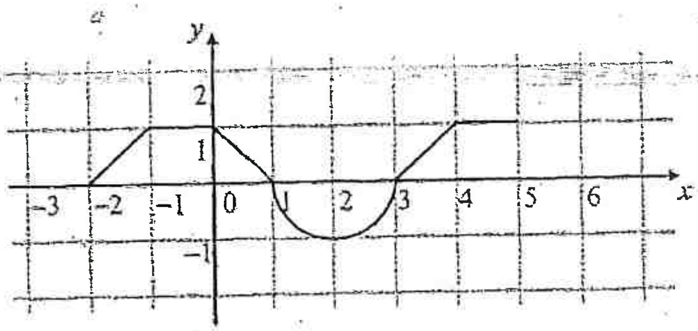
10. $\int_{-2}^{7} |x-4|\,dx$

11. If $a(t) = 12t^2 + 18t - 4$ and $x(1) = 3$ and $v(-1) = 9$, find the below:

a) Find the specific function for $v(t)$

b) Find the specific function for $x(t)$

12. The graph of f below consists of a semicircle, triangles, and squares. Find the average value of f on the interval [-2, 5]



AP Calculus Ch. 4 Test Review WS 2 (Non-Calculator) Solution Key $\frac{1}{2}$ 15

1. $\int \frac{x^2+x+1}{\sqrt{x}}dx$ $\int \frac{x^2}{x^{1/2}}+\frac{x}{x^{1/2}}+\frac{1}{x^{1/2}}dx$

$\int x^{3/2}+x^{1/2}+x^{-1/2}dx$

$=\frac{x^{5/2}}{5/2}+\frac{x^{3/2}}{3/2}+\frac{x^{1/2}}{1/2}+C$

$=\boxed{\frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}+2x^{1/2}+C}$

2. $\int 2x\sqrt{1-3x^2}\,dx$ $=\int 2x(1-3x^2)^{1/2}dx$

$u=1-3x^2$

$\frac{du}{dx}=-6x$

$dx=\frac{du}{-6x}$

$\int 2\cancel{x}\cdot u^{1/2}\cdot\frac{du}{-6\cancel{x}}$

$-\frac{1}{3}\int u^{1/2}du$

$=-\frac{1}{3}\left(\frac{u^{3/2}}{3/2}\right)=-\frac{1}{3}\left(\frac{2}{3}\right)u^{3/2}+C$

$=\boxed{\frac{-2}{9}(1-3x^2)^{3/2}+C}$

3. $\int 5\sqrt{x}\,(4-3x^2)\,dx$ $=\int 5x^{1/2}(4-3x^2)dx$

$=\int 20x^{1/2}-15x^{5/2}dx$

$=20\left(\frac{x^{3/2}}{3/2}\right)-15\left(\frac{x^{7/2}}{7/2}\right)+C$

$=20\cdot\frac{2}{3}x^{3/2}-15\cdot\frac{2}{7}x^{7/2}+C$

$=\boxed{\frac{40}{3}x^{3/2}-\frac{30}{7}x^{7/2}+C}$

4. $\int 5x\sec^2(3x^2)\,dx$

$u=3x^2$

$\frac{du}{dx}=6x$

$dx=\frac{du}{6x}$

$\int 5\cancel{x}\cdot\sec^2(u)\cdot\frac{du}{6\cancel{x}}$

$\frac{5}{6}\int\sec^2u\,du$

$\frac{5}{6}\tan u+C$

$=\boxed{\frac{5}{6}\tan(3x^2)+C}$

5. $\int x^2\sqrt{7-x}\,dx$ $=\int x^2(7-x)^{1/2}dx$

$u=7-x\Rightarrow x=7-u$

$\frac{du}{dx}=-1$

$dx=-du$

$\int x^2\cdot u^{1/2}(-du)$

$\int(7-u)^2u^{1/2}(-du)$

$\int -u^{1/2}(49-14u+u^2)du$

$\int -49u^{1/2}+14u^{3/2}-u^{5/2}du$

$-49\left(\frac{u^{3/2}}{3/2}\right)+14\left(\frac{u^{5/2}}{5/2}\right)-\frac{u^{7/2}}{7/2}+C$

$-49\cdot\frac{2}{3}u^{3/2}+14\cdot\frac{2}{5}u^{5/2}-\frac{2}{7}u^{7/2}+C$

$\boxed{-\frac{98}{3}(7-x)^{3/2}+\frac{28}{5}(7-x)^{5/2}-\frac{2}{7}(7-x)^{7/2}+C}$

6. $\int_1^2 x(1-2x^2)^3dx$ $\int\cancel{x}\cdot u^3\cdot\frac{du}{-4\cancel{x}}=\frac{-1}{4}\int u^3du$

$u=1-2x^2$

$\frac{du}{dx}=-4x$

$dx=\frac{du}{-4x}$

if $x=1$, $u=1-2(1)^2=-1$

if $x=2$, $u=1-2(2)^2=-7$

$=-\frac{1}{4}\frac{u^4}{4}\Big]_{-1}^{-7}=-\frac{1}{16}(-7)^4-\left(-\frac{1}{16}(1)\right)$

$=\frac{-2401}{16}+\frac{1}{16}$

$=\frac{-2400}{16}=\boxed{-150}$

$\frac{d}{dx}\int_{g(x)}^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x) - f(g(x))\cdot g'(x)$

SFTC

7. Find $f'(x)$ if $f(x) = \int_{2x^3}^{\sqrt{x}} \sqrt{1-t^2}\,dt$ $\quad \sqrt{x} \to x^{1/2}$

$f'(x) = \sqrt{1-(\sqrt{x})^2}\cdot \frac{1}{2}x^{-1/2} - \sqrt{1-(2x^3)^2}\cdot 6x^2$

$= \boxed{\dfrac{\sqrt{1-x}}{2\sqrt{x}} - 6x^2\sqrt{1-4x^6}}$

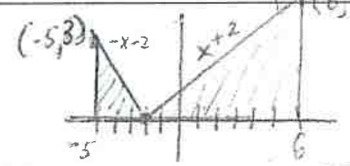
$\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$

SFTC

8. Find $f'(x)$ if $f(x) = \int_{3x^2}^{\pi} \sqrt{1-t^2}\,dt = \int_{\pi}^{3x^2} -\sqrt{1-t^2}\,dt$

$f'(x) = -\sqrt{1-(3x^2)^2}\cdot 6x = \boxed{-6x\sqrt{1-9x^4}}$

9. $\int_{-5}^{6} |x+2|\,dx$



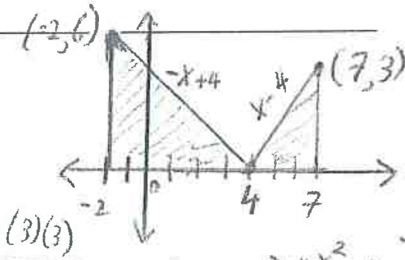
$\frac{1}{2}bh = \frac{1}{2}(3)(3) + \frac{1}{2}(8)(8) = \frac{9}{2} + \frac{64}{2} = \boxed{\frac{73}{2}}$

OR

$\int_{-5}^{-2} -x-2\,dx + \int_{-2}^{6} x+2\,dx = -\frac{x^2}{2} - 2x\Big]_{-5}^{-2} = \frac{9}{2}$

$+ \frac{x^2}{2} + 2x\Big]_{-2}^{6} = 64/2 \to \frac{9}{2} + \frac{64}{2} = \boxed{\frac{73}{2}}$

10. $\int_{-2}^{7} |x-4|\,dx$



$\frac{1}{2}bh = \frac{1}{2}(6)(6) + \frac{1}{2}(3)(3)$

$= \frac{36}{2} + \frac{9}{2} = \boxed{\frac{45}{2}}$

OR

$\int_{-2}^{4} -x+4\,dx + \int_{4}^{7} x-4\,dx$

$-\frac{x^2}{2} + 4x\Big]_{-2}^{4} = 18$

$\frac{x^2}{2} - 4x\Big]_{4}^{7} = 9/2$

$18 + 9/2 = \boxed{\frac{45}{2}}$

11. If $a(t) = 12t^2 + 18t - 4$ and $x(1) = 3$ and $v(-1) = 9$, find the below:

$v(t) = \int a(t)\,dt = \int 12t^2 + 18t - 4\,dt = \frac{12t^3}{3} + \frac{18t^2}{2} - 4t + C$

$v(t) = 4t^3 + 9t^2 - 4t + C$

$9 = 4(-1)^3 + 9(-1)^2 - 4(-1) + C$

$9 = -4 + 9 + 4 + C$

$0 = C$

$v(t) = 4t^3 + 9t^2 - 4t$

$x(t) = \int v(t)\,dt = \int 4t^3 + 9t^2 - 4t\,dt$

$x(t) = \frac{4t^4}{4} + \frac{9t^3}{3} - \frac{4t^2}{2} + K$

$x(t) = t^4 + 3t^3 - 2t^2 + K$

a) Find the specific function for v(t)

$\boxed{v(t) = 4t^3 + 9t^2 - 4t}$

b) Find the specific function for x(t)

$3 = 1^4 + 3(1)^3 - 2(1)^2 + K$

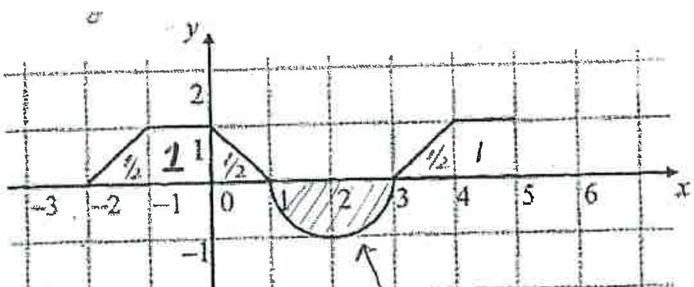
$3 = 1 + 3 - 2 + K$

$3 = 4 - 2 + K$

$1 = K$

$\boxed{x(t) = t^4 + 3t^3 - 2t^2 + 1}$

12. The graph of f below consists of a semicircle, triangles, and squares. Find the average value of f on the interval [-2, 5]



$= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(1) = \boxed{-\pi/2}$

Avg. value $= \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{5-(-2)}\int_{-2}^{5} f(x)\,dx$

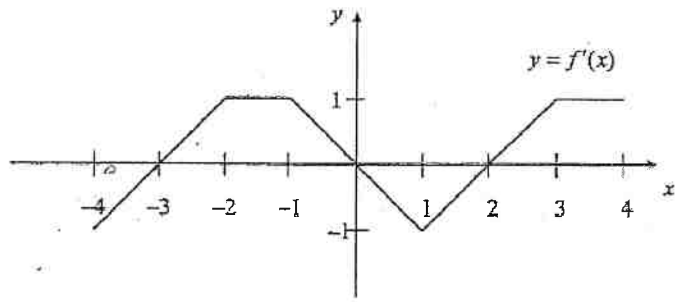
$= \frac{1}{7}\left(\frac{7-\pi}{2}\right)$

$\boxed{\text{Avg. value} = \frac{7-\pi}{14}}$ or $\frac{1}{2} - \frac{\pi}{14}$

$\int_{-2}^{5} f(x)\,dx = \frac{1}{2} + 1 + \frac{1}{2} - \pi/2 + \frac{1}{2} + 1 = \frac{7}{2} - \frac{\pi}{2} = \frac{7-\pi}{2}$

17

1. Find the average value of f '(x) on [-4, 4]



2. a) Find the average value of $f(x) = 4 - x^2$ on [0, 2].

b) Find the c-value guaranteed by the average value theorem.

3. $$\int \frac{x-2}{\sqrt[4]{x^2 - 4x}}\,dx$$

4. $$\int_{-5}^{2} |x+3|\,dx$$

18

5. $\int_{\sqrt{7}}^{0} x\sqrt{16-x^2}\,dx$

6. $\int \sqrt[3]{\cos x}\,\sin x dx$

7. $\int x\sqrt{1-x}dx$

8. Find $\frac{d}{dx}\left[\int_{-4x}^{\sqrt{x}} 1-t^2 dt\right]$

9. Given f''(x) = 1 – 2x and f'(-1) = 6 and f(0) = 14 find the below

a. Find the specific equation for f'(x)

b. Find the specific equation for f(x)

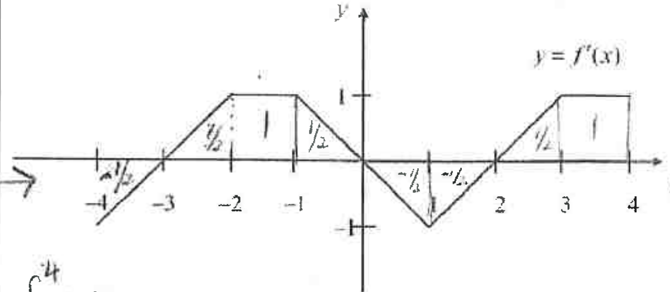
AP Calculus Chapter 4 Morning Review Session (part 1: Non-Calculator)

1. Find the average value of f '(x) on [-4, 4]

$$\text{Avg. value} = \frac{1}{b-a}\int_a^b f'(x)\,dx$$

$$= \frac{1}{4--4}\int_{-4}^{4} f'(x)\,dx \qquad \int_{-4}^{4} f'(x)\,dx = 2$$

$$= \frac{1}{8}(2) = \frac{2}{8} = \boxed{\frac{1}{4}}$$



$$\int_{-4}^{4} f'(x)\,dx = -\frac{1}{2} + \frac{1}{2} + 1 + \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{1}{2} + 1 = 2$$

2. a) Find the average value of f(x) = 4 – x^2 on [0, 2].

$$\text{Avg. value} = \frac{1}{2-0}\int_0^2 4 - x^2\,dx$$

$$\int_0^2 4-x^2\,dx = 4x - \frac{x^3}{3}\Big]_0^2 = 8 - \frac{8}{3} - \left(4(0) - \frac{0^3}{3}\right)$$

$$\text{Avg. value} = \frac{1}{2}\left(8 - \frac{8}{3}\right) = \frac{1}{2}\left(\frac{16}{3}\right) = \boxed{\frac{8}{3}}$$

b) Find the c-value guaranteed by the average value theorem.

set $f(x) = \text{Avg. value}$

$$4 - x^2 = \frac{8}{3}$$
$$-x^2 = \frac{8}{3} - 4$$
$$-x^2 = -\frac{4}{3}$$
$$x^2 = \frac{4}{3}$$
$$x = \pm\sqrt{\frac{4}{3}}$$
$$\boxed{c = \frac{2}{\sqrt{3}}}$$

3. $\displaystyle\int \frac{x-2}{\sqrt[4]{x^2-4x}}\,dx = \int \frac{x-2}{(x^2-4x)^{1/4}}\,dx$

$u = x^2 - 4x$

$\frac{du}{dx} = 2x - 4$

$dx = \frac{du}{2x-4}$

$$\int \frac{x-2}{u^{1/4}} \cdot \frac{du}{2x-4}$$

$$\int \frac{x-2}{u^{1/4}} \cdot \frac{du}{2(x-2)}$$

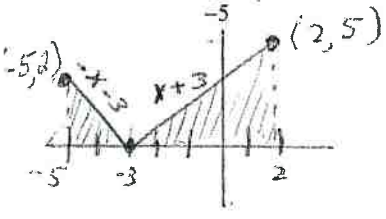
$$\frac{1}{2}\int u^{-1/4}\,du$$

$$= \frac{1}{2}\left(\frac{u^{3/4}}{3/4}\right)$$

$$= \frac{1}{2}\cdot\frac{4}{3}u^{3/4} + C$$

$$= \boxed{\frac{2}{3}(x^2-4x)^{3/4} + C}$$

4. $\displaystyle\int_{-5}^{2} |x+3|\,dx$



$$\frac{1}{2}(2)(2) + \frac{1}{2}(5)(5)$$

$$2 + \frac{25}{2} = \boxed{14.5 \text{ or } \frac{29}{2}}$$

OR

$$\int_{-5}^{-3} -x-3\,dx + \int_{-3}^{2} x+3\,dx$$

$$\frac{-x^2}{2} - 3x\Big]_{-5}^{-3} + \frac{x^2}{2} + 3x\Big]_{-3}^{2}$$

$$\frac{-9}{2} + 9 - \left(\frac{-25}{2} + 15\right) \qquad \frac{4}{2} + 6 - \left(\frac{9}{2} - 9\right)$$

$$2 + 12.5 = \boxed{14.5 \text{ or } \frac{29}{2}}$$

20

5. $\int_{\sqrt{7}}^{0} x\sqrt{16-x^2}\,dx$

if $x = \sqrt{7}$, $u = 16 - \sqrt{7}^2 = 9$
if $x = 0$, $u = 16 - 0 = 16$

$u = 16 - x^2$
$\frac{du}{dx} = -2x$
$dx = \frac{du}{-2x}$

$\int x \cdot u^{1/2} \cdot \frac{du}{-2x}$

$-\frac{1}{2}\int u^{1/2}\,du$

$-\frac{1}{2}\left(\frac{u^{3/2}}{3/2}\right)$

$-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}$

$-\frac{1}{3}u^{3/2}\Big]_9^{16} = -\frac{1}{3}(16)^{3/2} - \left(-\frac{1}{3}(9)^{3/2}\right)$

$= -\frac{1}{3}(64) + \frac{1}{3}(27)$

$= \boxed{\frac{-37}{3}}$

6. $\int \sqrt[3]{\cos x}\,\sin x\,dx$

$u = \cos x$
$\frac{du}{dx} = -\sin x$
$dx = \frac{du}{-\sin x}$

$\int u^{1/3} \cdot \cancel{\sin x} \cdot \frac{du}{-\cancel{\sin x}}$

$-1\int u^{1/3}\,du$

$-\frac{u^{4/3}}{4/3} + C$

$\boxed{-\frac{3}{4}(\cos x)^{4/3} + C}$

7. $\int x\sqrt{1-x}\,dx$

$u = 1 - x \longrightarrow x = 1 - u$
$\frac{du}{dx} = -1$
$dx = -du$

$\int x \cdot u^{1/2}(-du)$

$\int (1-u)\,u^{1/2}(-du)$

$\int -u^{1/2} + u^{3/2}\,du$

$-\frac{u^{3/2}}{3/2} + \frac{u^{5/2}}{5/2} + C$

$-\frac{2}{3}u^{3/2} + \frac{2}{5}u^{5/2} + C$

$\boxed{-\frac{2}{3}(1-x)^{3/2} + \frac{2}{5}(1-x)^{5/2} + C}$

8. Find $\frac{d}{dx}\left[\int_{-4x}^{\sqrt{x}} 1 - t^2\,dt\right]$

$\boxed{\text{use SFTC}}$

$= \left(1 - (\sqrt{x})^2\right)\cdot\frac{1}{2}x^{-1/2} - \left(1 - (-4x)^2\right)(-4)$

$= \boxed{\frac{1-x}{2\sqrt{x}} + 4 - 64x^2}$

9. Given f''(x) = 1 – 2x and f'(-1) = 6 and f(0) = 14 find the below

a. Find the specific equation for f'(x)

$f'(x) = \int f''(x)\,dx = \int 1 - 2x\,dx = x - \frac{2x^2}{2} + C$

$f'(x) = x - x^2 + C$

$6 = (-1) - (-1)^2 + C$

$6 = -1 - 1 + C$

$8 = C$

$\boxed{f'(x) = x - x^2 + 8}$

b. Find the specific equation for f(x)

$f(x) = \int f'(x)\,dx = \int x - x^2 + 8\,dx = \frac{x^2}{2} - \frac{x^3}{3} + 8x + K$

$f(x) = \frac{x^2}{2} - \frac{x^3}{3} + 8x + K$

$14 = 0 - 0 + 0 + K$

$14 = K$

$\boxed{f(x) = \frac{x^2}{2} - \frac{x^3}{3} + 8x + 14}$

*Formulas to Memorize for Test

Ch. 4 Formula Sheet

Summation Formulas:

1) $\sum_{i=1}^{n} 1 = n$

2) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5) $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$

Area using Limit Definition

$$\lim_{n\to\infty}\sum_{i=1}^{n}(width) * f(left\ endpoint + width * i)$$

width $= \frac{b-a}{n}$

Trapezoid Area:

$$Area = \frac{1}{2}w(h_1 + h_2)$$

Derivative Formulas:

Power Rule:

$$\frac{d}{dx}x^n = nx^{n-1}$$

Trig Derivatives:

$\frac{d}{dx}\sin u = \cos u * u'$	$\frac{d}{dx}\cos u = -\sin u * u'$
$\frac{d}{dx}\tan u = \sec^2 u * u'$	$\frac{d}{dx}\cot u = -csc^2 u * u'$
$\frac{d}{dx}\sec u = \sec u \tan u * u'$	$\frac{d}{dx}\csc u = -\csc u \cot u * u'$

Integral Formulas:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

FFTC (First Theorem)

$$\int_a^b \boldsymbol{f(x)dx = F(b) - F(a)}$$

SFTC (Second Theorem)

$$\frac{d}{dx}\int_a^{p(x)} f(t)dt = f(p(x)) * p'(x)$$

*variations of FFTC

$$\int_a^b v(t)dt = x(b) - x(a)$$

$$\boldsymbol{x(b) = x(a) + \int_a^b v(t)dt}$$

final position = initial position + displacement

$$v(b) = v(a) + \int_a^b a(t)dt$$

variations of FFTC (continued)…

$$\int_a^b f'(x)dx = f(b) - f(a)$$

$$\int_a^b f''(x)dx = f'(b) - f'(a)$$

$$\int_a^b a(t)dt = v(b) - v(a)$$

Trig Integrals:

$\int sin\, u\, du = -cos\, u + C$ $\quad$ $\int cos\, u\, du = sin\, u + C$

$\int sec^2 u\, du = tan\, u + C$ $\quad$ $\int sec\, u\, tan\, u\, du = sec\, u + C$

$\int csc^2 u\, du = -cot\, u + C$ $\quad$ $\int csc\, u\, cot\, u\, du = -csc\, u + C$

Average Value Theorem

$$Avg.Value = \frac{1}{b-a}\int_a^b f(x)dx$$