Name:______________________________ Period: ________

AP Calculus AB

Integrals Unit

Notes Packet

Part 2

(Fundamental Theorems of Calculus,

Avg. Value Theorem, U-Substitution,

Even & Odd Functions, Particle Motion)

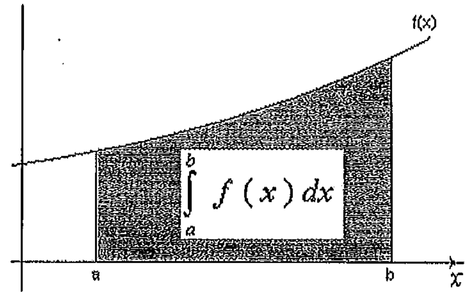
AP Calculus AB January 2024 Class Calendar

Monday	Tuesday	Wednesday	Thursday	Friday
1 Winter Break	2 Teacher Workday (No School)	3 4.2 – Sigma Notation HW: pg 263—264 #1 – 19 odds	4 4-2b—Riemann Sums HW: pg 263—264 #25– 35 odds	5 4-2c and 4-6—Riemann Sums and Trapezoid Rule HW:(4.2) **pg 263-264** #37,39,41, 45, 49 **(4.6) pg. 310** #1, 5, 9 (trapezoid rule only)
8 4-1a—Integrals HW: pg 251—252 #7– 23 odds	9 4-1b—Trig Integrals and baby differential equations HW: pg 251-252 #25 – 31 odds, 35,37	10 Quiz Review 4.1, 4.2, 4.6 HW: pg 251—252 #53,55,57	11 Quiz Review 4.1, 4.2, 4.6	12 **Quiz (4.1, 4.2, 4.6)**
15 **MLK Day No School**	16 Teacher Workday *(No School)*	17 4-3 and 4-4a Definite Integrals HW: (4.3) pg 273—275 #41, 43 (4.4) Pg 288—289 # 11,13,19,21	18 4-4b—Average Value Formula and SFTC (second fundamental theorem of calculus) HW: pg 288—289 #35, 37, 39, 51, 53, 75, 77, 81	19 4-5a—U-substitution HW: pg 301—302 #9-17 odds, 47, 49
22 4-5b—U-substitution with definite integrals HW: pg 301—303 #19-25 odds, #55-61 odds	23 4.5b – U-Substitution Review	24 Test Review Day & Intro to TI-84 Graphing Calculator Problems	25 Ch. 4 Test Review Day	26 **Ch. 4 Test (Non-Calculator Part 1)**
29 Ch. 4 Calculator Review	30 Ch. 4 Calculator Review	31 **Ch. 4 Test (Calculator Portion – Part 2)**	Feb 1 5-2—Natural log Integrals HW: pg 334—336 #9 – 33 odds, 49 – 55 odds	Feb 2 5-4—Integral of e^x HW: pg 354 #91 – 107 odds, 113, 115

Calculus **Chapters 4.3/4.4a** **1st Fundamental Theorem of Calculus**

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is the antiderivative of f.



Recall:

*The general derivative is a **slope-finding function** or formula : (ex. $f'(x) = 2x + 1$)

*The specific derivative is the **actual slope** at a point (ex: $f'(3) = 7$)

Likewise…

The indefinite integral is an **Area-Finding Function** or formula (Ex: $\int 2x\,dx = x^2 + C$)

The definite integral is the **Actual Area** of the region for an interval (Ex: $\int_1^3 2x\,dx = 8$)

*If a function is **continuous** on a closed interval, then the function is able to be integrated on that interval

Class Examples:

1. Evaluate $\int_1^4 (3x^2 + 4x - 1)dx$

**NOTE: For definite integrals, we don't need to worry about the constant of integration " +C ". It will always wash out.

2. Evaluate $\int_{-2}^{1} 2x\,dx$

②

Integral Properties:

1) $\int_a^a f(x)dx = 0$

2) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (given that c is between a and b)

Example 3: If $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, find the below:

a) $\int_0^6 f(x)dx$

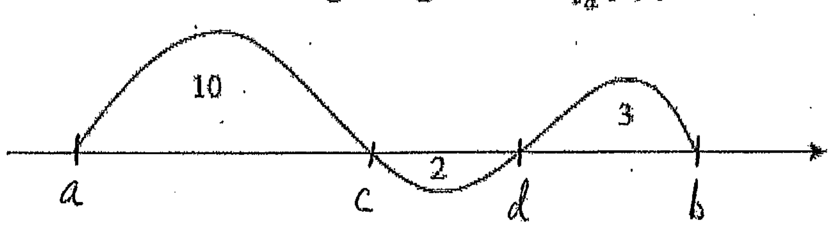
b) $\int_6^3 f(x)dx$

c) $\int_3^3 f(x)dx$

d) $\int_3^6 (-5f(x)+3)dx$

Ex. 4: If $\int_3^8 f'(x)dx = 10$ and $f(8) = 6$, find $f(3)$.
*Reminder that the FFTC can be used as an equation solving tool to find the value of an antiderivative at a specific point. Recall that $\int_a^b f'(x)dx = f(b) - f(a)$

Ex. 5: The area for each region is given. Find $\int_a^b f(x)dx$

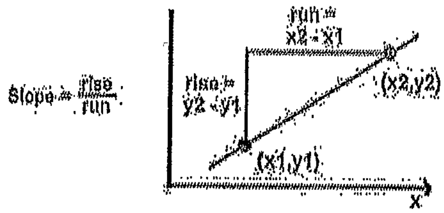


Differential Calculus vs Integral Calculus Summary Sheet

Differential Calculus (Derivative) Explores rates of change	Integral Calculus (Antiderivative) Explores the accumulation of change

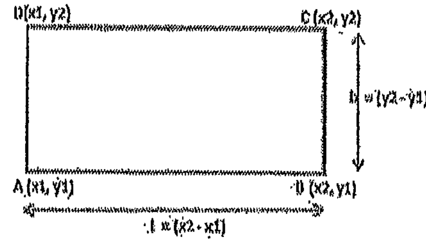
Differential Calculus (Derivative)
Explores rates of change

Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$



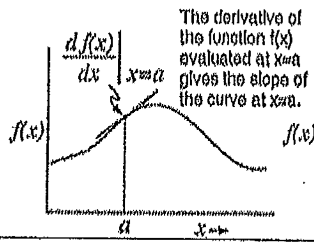
Integral Calculus (Antiderivative)
Explores the accumulation of change

Area $= (x_2 - x_1) \times (y_2 - y_1)$



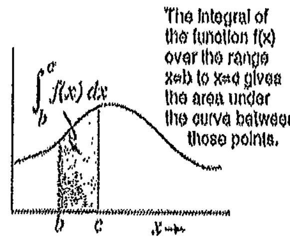
Derivative

$$\frac{d\,f(x)}{dx}$$



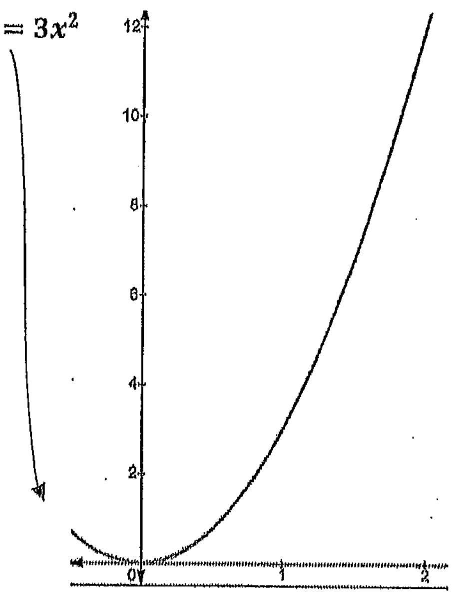
Integral

$$\int f(x)\,dx$$



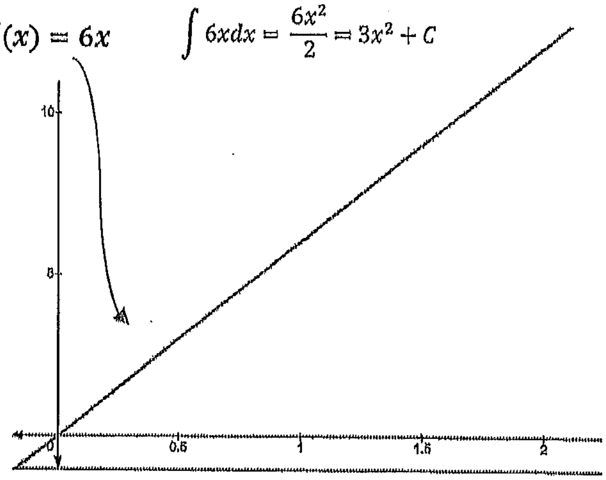
$f(x) = 3x^2$

$$\frac{d}{dx}3x^2 = 6x$$



$f'(x) = 6x$

$$\int 6x\,dx = \frac{6x^2}{2} = 3x^2 + C$$



* *The* ***area under*** *the derivative graph is equal to the* ***rise in height*** *of the antiderivative graph*

* *The average slope of the antiderivative graph is equal to the average height of region under derivative graph*

$$Avg\ value\ f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

Second Fundamental Theorem of Calculus (SFTC)

$$\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right] = f(p(x)) * p'(x)$$

First Fundamental Theorem of Calculus (FFTC)

$$\int_a^b f'(x)dx = f(a) - f(b)$$

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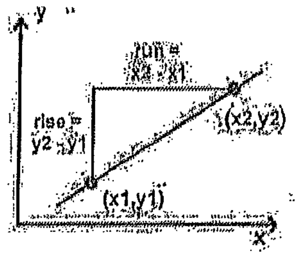
Key

Differential Calculus vs Integral Calculus Summary Sheet

Differential Calculus (Derivative)	Integral Calculus (Antiderivative)

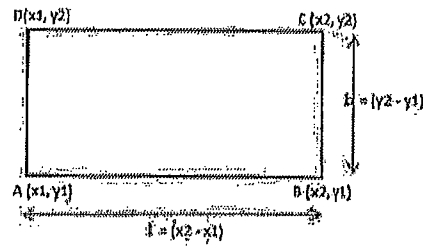
Differential Calculus (Derivative)

$Slope\ m = \frac{y_2 - y_1}{x_2 - x_1}$



Integral Calculus (Antiderivative)

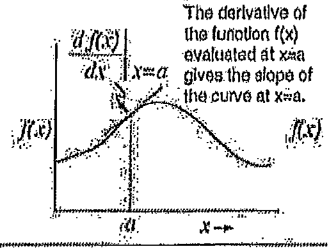
$Area = (x_2 - x_1) \times (y_2 - y_1)$



Derivative

$\frac{d\,f(x)}{dx}$

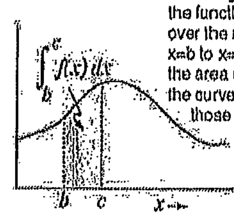
The derivative of the function f(x) evaluated at x=a gives the slope of the curve at x=a.



Integral

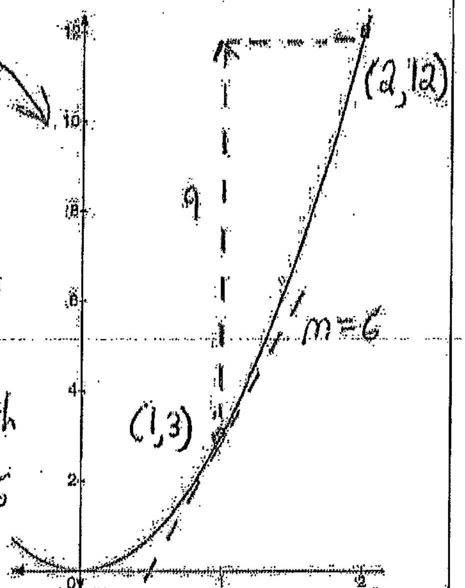
$\int f(x)\,dx$

The integral of the function f(x) over the range x=b to x=c gives the area under the curve between those points.



$f(x) = 3x^2$

$\frac{d}{dx}3x^2 = 6x$

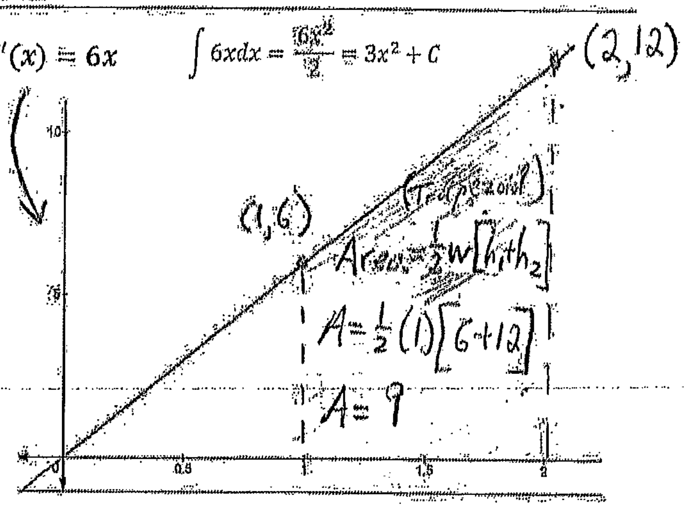


• The derivative function can find the slope of any point on this graph

ex: $f'(1) = 6(1) = 6$

$f(2) - f(1) = 12 - 3 = \boxed{9}$

$f'(x) = 6x$ $\quad \int 6x\,dx = \frac{6x^2}{2} = 3x^2 + C$



• The Integral of this function can be used to find the area under this graph

$\int_1^2 6x\,dx = \frac{6x^2}{2} \Rightarrow 3x^2\Big]_1^2 = 3(2)^2 - 3(1)^2 = \boxed{9}$

Second Fundamental Theorem of Calculus (SFTC)

$$\frac{d}{dx}\left[\int_a^{p(x)} f(t)\,dt\right] = f(p(x)) * p'(x)$$

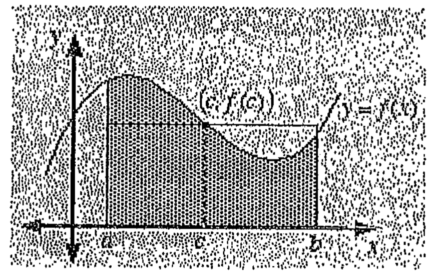
First Fundamental Theorem of Calculus (FFTC)

$$\int_a^b f'(x)\,dx = f(a) - f(b)$$

Calculus **Chapter 4.4b** **Average Value Theorem (MVT for Integrals)**

If function f is integrable on the closed interval [a, b], then the average value of f on the interval is

$$f(c)=\frac{1}{b-a}\int_a^b f(x)dx$$



*There exists a rectangle such that the area of the rectangle is the same as the area under the curve (shaded region). f (c) is the height of the rectangle

Example 1: a) Find the average value of $f(x) = x^2 + 1$ on [2, 5]. b) find the c value

6

<u>2nd Fundamental Theorem of Calculus (SFTC)</u>

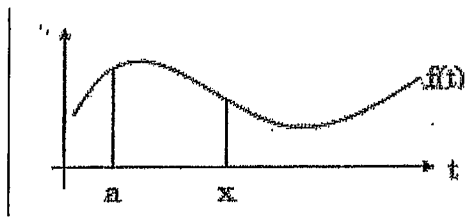
Definite Integral as a Function

To recap, we've covered:

1) Indefinite General Integrals (Area-finding functions)
2) Definite Integrals (Finds Area between 2 x-values)

There is also now a function that is the integral itself. Instead of going from a constant to another constant, we are going from a constant to a moving value of x.

Consider: $f(x)=\int_a^x f(t)dt$



<u>2nd Fundamental Theorem of Calculus</u> **Very Important**

Applies the concept that derivative and integrals are inverse operations of each other.

1) $\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right]=f(p(x))\cdot p'(x)$ (a is a constant)

2) $\frac{d}{dx}\left[\int_{q(x)}^{p(x)} f(t)dt\right]=f(p(x))\cdot p'(x)-f(q(x))\cdot q'(x)$

Example 2:

a) $\frac{d}{dx}\left[\int_{-3}^{x}\sqrt{t^2+4}dt\right]=$

b) $\frac{d}{dx}\left[\int_{3}^{x^2}\sqrt{t-1}dt\right]=$

c) $\frac{d}{dx}\left[\int_{10}^{x^2}\sqrt{t-1}dt\right]=$

d) $\frac{d}{dx}\left[\int_{3x}^{0}\frac{1}{t+2}dt\right]=$

e) $\frac{d}{dx}\left[\int_{x}^{x^2}(2t+3)dt\right]=$

Practice Problem:

Avg. value theorem: $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

#47) $f(x) = 4 - x^2$ $[-2, 2]$ a) Find avg. value
b) find c-value

(6c)

Ex. 2 Find $\frac{d}{dx}\left[\int_{2x^3}^{5} \frac{2t}{5-t^2}\,dt\right]$

$$* \quad \frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\,p'(x)$$

Calculus **Chapter 4.5a** **U-Substitution Method for Indefinite Integrals**

U-Substitution is the reverse of the derivative chain rule. We want to look for a function and its derivative to be in the integral.

Example 1:

Suppose $f(x) = \sin(3x)$

$f'(x) = \cos(3x) \cdot 3$
$f'(x) = 3\cos(3x)$

This means that:

$\int 3\cos(3x)dx = \sin(3x) + C$

U-Substitution Steps:

1. Assign the 'u' value to the expression inside the parentheses.
2. Find the derivative of u: $\frac{du}{dx}$
3. Solve for dx.
4. Rewrite the integral in terms of u and du. (Check to make sure no 'x' or 'dx' remains)
5. Evaluate the Integral
6. Write the answer in terms of x.
7. **Not all Integral problems require U-Substitution. Check first to see if expansion/rewriting problem will allow problem to only need the Power Rule**

Ex. 2: $\int x(x^2+1)^{15}dx$

Ex. 3: $\int x^2 \sec^2(2x^3)dx$

Ex. 4: $\int x^3\sqrt{5-x^4}\,dx$

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Ex. 5: $\int \tan^5 x \sec^2 x \, dx$

Ex. 6: $\int (3-y)\left(\frac{1}{\sqrt{y}}\right)dy$

Change of Variable U-Substitution Method:

Ex. 7: $\int x\sqrt{x+3}\,dx$

Ex. 8: $\int x^2\sqrt{2-x}\,dx$

U-Substitution with definite integrals: Be sure the bounds matches the variable of the problem

Ex. 1: $\int_{1}^{2} 2x(x^2-2)^3\,dx$

Ex. 2: $\int_{1}^{5} \frac{x}{\sqrt{2x-1}}\,dx$

(10)

Integrals of Odd and Even Functions

Review: Suppose $\int_{10}^{3} f(x)dx = 9$ and $\int_{-1}^{3} f(x)dx = 5$, find $\int_{-1}^{10} f(x)dx$

Even/Odd Rules:

Even: $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

Odd: $\int_{-a}^{a} f(x)dx = 0$

Ex. 3: Suppose g(x) is an even function where $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.

(Sketch a possible graph using the above given information)

Ex. 4: Same as Example 3, but g(x) is an odd function: $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.

Ex. 5: If f(x) is even and $\int_{3}^{6} f(x)dx = 7$ and $\int_{-6}^{3} f(x)dx = 12$, find $\int_{0}^{6} f(x)dx$

Non-AP Calculus Chapter 4.5 Integration and U-Substitution Worksheet

1)

$$\int (5x+4)^5\,dx$$

2)

$$\int 3t^2(t^3+4)^5\,dt$$

3)

$$\int \sqrt{4x-5}\,dx$$

4)

$$\int \frac{5x^2}{\sqrt[5]{x^3-2}}dx$$

5)

$$\int \cos(2x+1)\,dx$$

6)

$$\int \sin^{10}(x)\cos(x)\,dx$$

(12)

7. $\int \frac{\sin(x)}{(\cos(x))^5}\,dx$

8) $\int \frac{2}{\sqrt{3x-7}}\,dx$

9) $\int \frac{4}{x^2}\sec\left(\frac{5}{x}\right)\tan\left(\frac{5}{x}\right)dx$

10) $\int \frac{3x^4}{\left(7-x^5\right)^6}\,dx$

11) $\int \frac{x^3(2x-1)}{\sqrt{x}}\,dx$

12) $\int 7x^2\sqrt{3-2x^3}\,dx$

Key (12b)

Non-AP Calculus Chapter 4.5 Integration and U-Substitution Worksheet

1)

$$\int (5x+4)^5\,dx$$

$u = 5x+4$

$\frac{du}{dx} = 5$

$du = 5dx$

$\frac{du}{5} = dx$

$\int u^5 \cdot \frac{du}{5}$

$\frac{1}{5}\int u^5 du = \frac{1}{5}\left(\frac{u^6}{6}\right) + c$

$= \frac{1}{30}u^6 + c$

$= \boxed{\frac{1}{30}(5x+4)^6 + c}$

2)

$$\int 3t^2(t^3+4)^5\,dt$$

$u = t^3+4$

$\frac{du}{dt} = 3t^2$

$du = 3t^2 dt$

$\frac{du}{3t^2} = dt$

$\int 3t^2 \cdot u^5 \cdot \frac{du}{3t^2}$

$\int u^5 du = \frac{u^6}{6} + c$

$\boxed{\frac{1}{6}(t^3+4)^6 + c}$

3)

$$\int \sqrt{4x-5}\,dx$$

$\int (4x-5)^{1/2} dx$

$u = 4x-5$

$\frac{du}{dx} = 4$

$du = 4dx$

$\frac{du}{4} = dx$

$\int u^{1/2} \cdot \frac{du}{4}$

$\frac{1}{4}\int u^{1/2} du$

$\frac{1}{4} \cdot \frac{u^{3/2}}{3/2} + c$

$\frac{1}{4} \cdot \frac{2}{3} u^{3/2} + c$

$\frac{2}{12} u^{3/2} + c$

$\boxed{\frac{1}{6}(4x-5)^{3/2} + c}$

4)

$$\int \frac{5x^2}{\sqrt[5]{x^3-2}}\,dx$$

$\int \frac{5x^2}{(x^3-2)^{1/5}} dx$

$\int 5x^2 (x^3-2)^{-1/5} dx$

$u = x^3-2$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 dx$ $\quad \frac{du}{3x^2} = dx$

$\int 5x^2 \cdot u^{-1/5} \cdot \frac{du}{3x^2}$

$\frac{5}{3}\int u^{-1/5} du$

$\frac{5}{3} \cdot \frac{u^{4/5}}{4/5} + c$

$\frac{5}{3} \cdot \frac{5}{4} u^{4/5} + c$

$\boxed{\frac{25}{12}(x^3-2)^{4/5} + c}$

5)

$$\int \cos(2x+1)\,dx$$

$u = 2x+1$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{du}{2} = dx$

$\int \cos u \cdot \frac{du}{2}$

$= \frac{1}{2}\int \cos u\, du$

$= \frac{1}{2}\sin u + c$

$\boxed{\frac{1}{2}\sin(2x+1) + c}$

6)

$$\int \sin^{10}(x)\cos(x)\,dx$$

$\int (\sin x)^{10} \cos x\, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\, dx$

$\frac{du}{\cos x} = dx$

$\int u^{10} \cdot \cos x \cdot \frac{du}{\cos x}$

$= \int u^{10} du$

$= \frac{u^{11}}{11} + c$

$= \boxed{\frac{1}{11}(\sin x)^{11} + c}$

12c

7. $\int \frac{\sin(x)}{(\cos(x))^5}\,dx$

$\int \frac{\sin x}{(\cos x)^5}\,dx$

$\int \sin x \cdot (\cos x)^{-5}\,dx$

$u = \cos x$ | $du = -\sin x\,dx$

$\frac{du}{dx} = -\sin x$ | $\frac{du}{-\sin x} = dx$

$\int \sin x \cdot u^{-5} \cdot \frac{du}{-\sin x}$

$-\int u^{-5}\,du = \frac{-u^{-4}}{-4} + C$

$\frac{1}{4u^4} + C$

$\boxed{\frac{1}{4(\cos x)^4} + C}$

8) $\int \frac{2}{\sqrt{3x-7}}\,dx$

$\int \frac{2}{(3x-7)^{1/2}}\,dx$

$\int 2(3x-7)^{-1/2}\,dx$

$u = 3x-7$

$\frac{du}{dx} = 3$

$du = 3dx \quad \frac{du}{3} = dx$

$\int 2 \cdot u^{-1/2} \cdot \frac{du}{3} = \frac{2}{3}\int u^{-1/2}\,du$

$\frac{2}{3} \cdot \frac{u^{1/2}}{1/2} + C = \frac{2}{3} \cdot \frac{2}{1} u^{1/2} + C$

$\boxed{\frac{4}{3}(3x-7)^{1/2} + C}$

9) $\int \frac{4}{x^2} \sec\left(\frac{5}{x}\right) \tan\left(\frac{5}{x}\right) dx$

$u = \frac{5}{x} = 5x^{-1}$

$\frac{du}{dx} = -5x^{-2}$

$\frac{du}{dx} = \frac{-5}{x^2}$

$x^2 du = -5dx$

$\frac{x^2 du}{-5} = dx$

$\int \frac{4}{x^2} \sec u \tan u \frac{x^2 du}{-5}$

$-\frac{4}{5}\int \sec u \tan u\,du$

$= -\frac{4}{5}\sec u = \boxed{-\frac{4}{5}\sec\left(\frac{5}{x}\right) + C}$

10) $\int \frac{3x^4}{(7-x^5)^6}\,dx$

$\int 3x^4 (7-x^5)^{-6}\,dx$

$u = 7-x^5$

$\frac{du}{dx} = -5x^4$

$du = -5x^4\,dx$

$\frac{du}{-5x^4} = dx$

$\int 3x^4 \cdot u^{-6} \cdot \frac{du}{-5x^4}$

$-\frac{3}{5}\int u^{-6}\,du$

$= -\frac{3}{5} \cdot \frac{u^{-5}}{-5} + C$

$\frac{3}{25}(7-x^5)^{-5} + C$

$= \boxed{\frac{3}{25(7-x^5)^5} + C}$

11) $\int \frac{x^3(2x-1)}{\sqrt{x}}\,dx$

$\int (2x^4 - x^3)x^{-1/2}\,dx$

$\int 2x^{7/2} - x^{5/2}\,dx$

$\frac{2x^{9/2}}{9/2} - \frac{x^{7/2}}{7/2} + C$

$2 \cdot \frac{2}{9}x^{9/2} - \frac{2}{7}x^{7/2} + C$

$\boxed{\frac{4}{9}x^{9/2} - \frac{2}{7}x^{7/2} + C}$

12) $\int 7x^2\sqrt{3-2x^3}\,dx$

$\int 7x^2(3-2x^3)^{1/2}\,dx$

$u = 3-2x^3$

$\frac{du}{dx} = -6x^2$

$du = -6x^2\,dx$

$\frac{du}{-6x^2} = dx$

$\int 7x^2 \cdot u^{1/2} \cdot \frac{du}{-6x^2}$

$-\frac{7}{6}\int u^{1/2}\,du$

$-\frac{7}{6} \cdot \frac{u^{3/2}}{3/2}$

$-\frac{7}{6} \cdot \frac{2}{3} u^{3/2} + C$

$\frac{-14}{18} u^{3/2} + C$

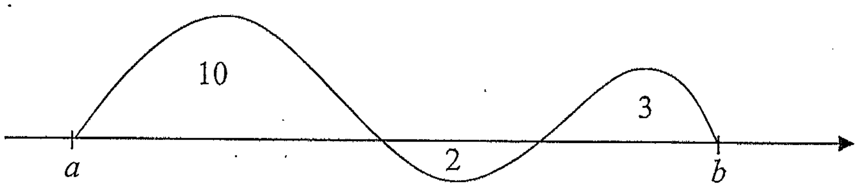
$= \boxed{\frac{-7}{9}(3-2x^3)^{3/2} + C}$

AP CALCULUS AB   Integral Unit   Additional Notes

Displacement is how far you are from where you started. displacement = $\int_a^b v(t)dt$ where $v(t)$ is the velocity function. (*displacement = integral of velocity*)

Distance is the total amount a particle has traveled. distance = $\int_a^b |v(t)|dt$.
(distance = integral of absolute value of velocity)

Ex) Suppose the graph below represents an object's velocity function. The numbers inside of each region represent the area of that region.



Based on this graph the object's displacement from a to b would be the integral of this function from a to b :

The object's distance traveled would be the integral of the absolute value of this function :

Position, Velocity, Acceleration

the derivative of position is velocity, the derivative of velocity is acceleration

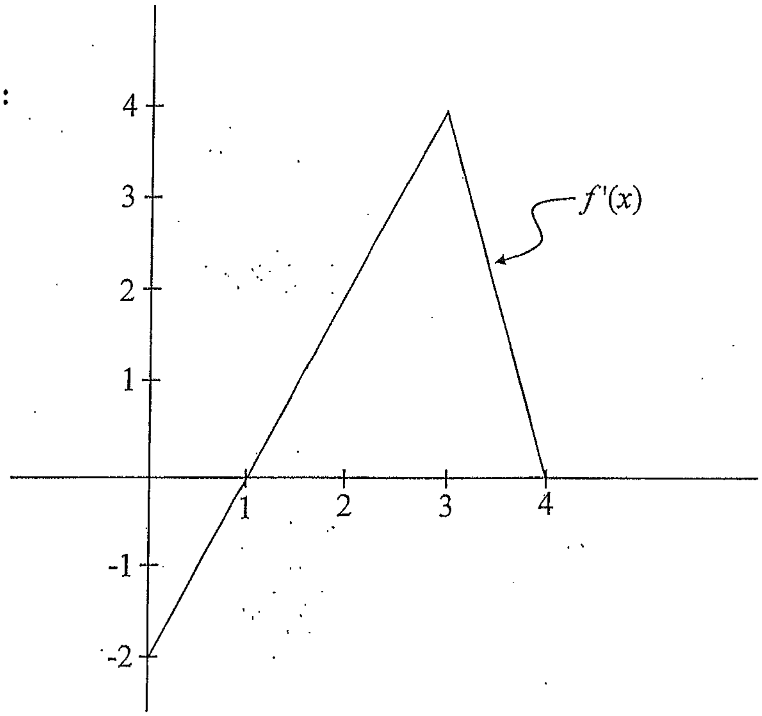
So, the antiderivative of acceleration is velocity and the antiderivative of velocity is position

Acceleration s''(t)	+	+	-	-
Velocity s'(t)	+	-	+	-
Speed	increasing	decreasing	decreasing	increasing

Remember: Speed is increasing when s''(t) and s'(t) have the same signs.
Speed is decreasing when s''(t) and s'(t) have opposite signs

(14)

Example 2:



The derivative of $f(x)$ is graphed above. Use this graph to answer the following:

a) Find the x-coordinates of all relative extrema. Justify your answer.	b) Find the x-coordinates of all points of inflection. Justify your answer.
c) If $f(0) = 1$, find $f(1), f(2), f(3)$, and $f(4)$.	d) State the absolute extrema and the x-values where they occur.
e) Find total distance traveled from $t = 0$ to $t = 4$	f) Find total displacement from $t = 0$ to $t = 4$
g) On the interval $1 < t < 3$, is speed increasing or decreasing? State reason:	h) On the interval $0 < t < 1$, is speed increasing or decreasing? State reason

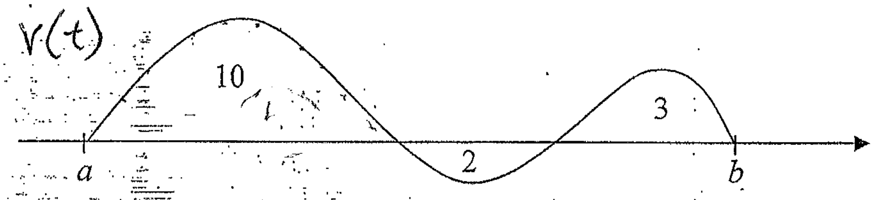
i) Using the data from parts a, b, and c, sketch a graph of $f(x)$ below

AP CALCULUS AB Integral Unit Additional Notes

Diplacement is how far you are from where you started. displacement = $\int_a^b v(t)dt$ where $v(t)$ is the velocity function. (*displacement = integral of velocity*)

Distance is the total amount a particle has traveled. distance = $\int_a^b |v(t)|dt$.
(distance = integral of absolute value of velocity)

Ex) Suppose the graph below represents an object's velocity function. The numbers inside of each region represent the area of that region.



Based on this graph the object's displacement from *a* to *b* would be the integral of this function from *a* to *b* :

$$\int v(t)dt = 10 - 2 + 3 = 11$$

The object's distance traveled would be the integral of the absolute value of this function :

$$\int |v(t)|dt = 10 + 2 + 3 = 15$$

$s(t)$ = position function
$v(t)$ = velocity function
$a(t)$ = acceleration function

Position, Velocity, Acceleration

the derivative of position is velocity, the derivative of velocity is acceleration

$\frac{d}{dx} s(t) = s'(t) = v(t)$ $\frac{d}{dx} v(t) = v'(t) = a(t)$

So, the antiderivative of acceleration is velocity and the antiderivative of velocity is position

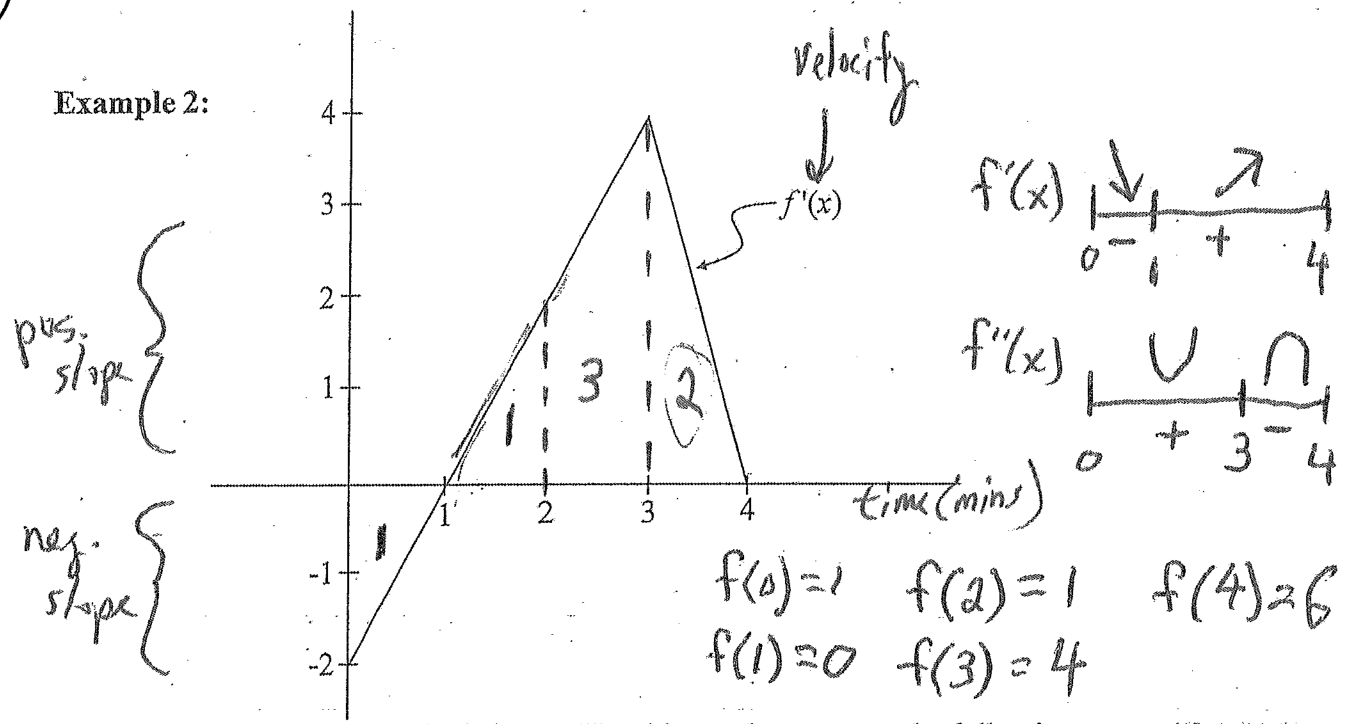
$\int a(t)dt = v(t) + c$ | $\int v(t)dt = s(t) + c$

Acceleration s''(t)	+	+	-	-
Velocity s'(t)	+	-	+	-
Speed	increasing	decreasing	decreasing	increasing

Remember: Speed is increasing when s''(t) and s'(t) have the same signs
Speed is decreasing when s''(t) and s'(t) have opposite signs

(16)

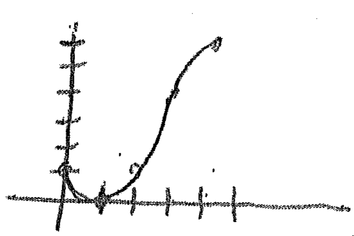
Example 2:



The derivative of $f(x)$ is graphed above. Use this graph to answer the following:

a) Find the x-coordinates of all relative extrema. Justify your answer. $x=1$ (min) b/c $f'(x)$ changes from − to +	b) Find the x-coordinates of all points of inflection. Justify your answer. $x=3$ b/c $f''(x)$ changes signs
c) If $f(0)=1$, find $f(1), f(2), f(3)$, and $f(4)$. $f(1)=f(0)+\int_0^1 f'(x)dx = 1-1=0$ $f(2)=f(1)+\int_1^2 f'(x)dx = 0+1=1$ $f(3)=f(2)+\int_2^3 f'(x)dx = 1+3=4$ $f(4)=f(3)+\int_3^4 f'(x)dx = 4+2=6$	d) State the absolute extrema and the x-values where they occur. $f(1)=0$ $f(4)=6$
e) Find total distance traveled from t = 0 to t = 4 7	f) Find total displacement from t = 0 to t = 4 5
g) On the interval $1<t<3$, is speed increasing or decreasing? State reason: increasing speed b/c $f'(x)>0$, $f''(x)>0$	h) On the interval $0<t<1$, is speed increasing or decreasing? State reason decreasing speed b/c $f'(x)<0$, $f''(x)>0$

i) Using the data from parts a, b, and c, sketch a graph of $f(x)$ below



DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
22. $\frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1+u^2}$
23. $\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2-1}}$
24. $\frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{|u|\sqrt{u^2-1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\text{sech}^2 u)u'$
28. $\frac{d}{dx}[\coth u] = -(\text{csch}^2 u)u'$
29. $\frac{d}{dx}[\text{sech}\, u] = -(\text{sech}\, u \tanh u)u'$
30. $\frac{d}{dx}[\text{csch}\, u] = -(\text{csch}\, u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2+1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2-1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1-u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1-u^2}$
35. $\frac{d}{dx}[\text{sech}^{-1} u] = \frac{-u'}{u\sqrt{1-u^2}}$
36. $\frac{d}{dx}[\text{csch}^{-1} u] = \frac{-u'}{|u|\sqrt{1+u^2}}$

Basic Integration Formulas

1. $\int kf(u)\, du = k\int f(u)\, du$
2. $\int [f(u) \pm g(u)]\, du = \int f(u)\, du \pm \int g(u)\, du$
3. $\int du = u + C$
4. $\int a^u\, du = \left(\frac{1}{\ln a}\right)a^u + C$
5. $\int e^u\, du = e^u + C$
6. $\int \sin u\, du = -\cos u + C$
7. $\int \cos u\, du = \sin u + C$
8. $\int \tan u\, du = -\ln|\cos u| + C$
9. $\int \cot u\, du = \ln|\sin u| + C$
10. $\int \sec u\, du = \ln|\sec u + \tan u| + C$
11. $\int \csc u\, du = -\ln|\csc u + \cot u| + C$
12. $\int \sec^2 u\, du = \tan u + C$
13. $\int \csc^2 u\, du = -\cot u + C$
14. $\int \sec u \tan u\, du = \sec u + C$
15. $\int \csc u \cot u\, du = -\csc u + C$
16. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$
17. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
18. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C$