Calculus AB **Logs and Exponentials Derivatives Unit Quiz Review WS 2**

Key

1. Find $\frac{dy}{dx}$ $\quad y = \ln\sqrt[5]{\frac{1-x^3}{4x^2-x}}$

$y = \ln\left(\frac{1-x^3}{4x^2-x}\right)^{1/5} = \frac{1}{5}\ln\left(\frac{1-x^3}{4x^2-x}\right)$

$y = \frac{1}{5}\ln(1-x^3) - \frac{1}{5}\ln(4x^2-x)$

$y' = \frac{1}{5}\cdot\frac{-3x^2}{1-x^3} - \frac{1}{5}\cdot\frac{8x-1}{4x^2-x} \rightarrow \boxed{y' = \frac{-3x^2}{5(1-x^3)} - \frac{8x-1}{5(4x^2-x)}}$

2. Find $\frac{dy}{dx}$ $\quad y = \sqrt[x]{(3+4x^2)^5}$

$y = (3+4x^2)^{5/x}$

$\ln y = \ln(3+4x^2)^{5x^{-1}}$

$\ln y = \underbrace{(5x^{-1})}_{f}\cdot\underbrace{\ln(3+4x^2)}_{g}$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \overbrace{-5x^{-2}}^{f'}\cdot\overbrace{\ln(3+4x^2)}^{g} + \overbrace{5x^{-1}}^{f}\cdot\overbrace{\frac{8x}{3+4x^2}}^{g'}$

$\frac{dy}{dx} = y\left[\frac{-5\ln(3+4x^2)}{x^2} + \frac{40x}{x(3+4x^2)}\right]$

$\boxed{\frac{dy}{dx} = \sqrt[x]{(3+4x^2)^5}\left[\frac{-5\ln(3+4x^2)}{x^2} + \frac{40}{3+4x^2}\right]}$

3. Given that $f(g(x)) = x$.
Find $f'(6)$ if $g(6) = 3$, $g(3) = 6$, $g'(3) = -\frac{1}{4}$ and $g'(6) = 2$

$g(3) = 6 \mid f(6) = 3$ $\quad \boxed{f'(6) = -4}$

$g'(3) = \frac{-1}{4} \mid f'(6) = -4$

4. f(x) = $x^3 - 2x^2 + 1$ $\quad$ Find $(f^{-1})'(-15)$

$f(2) = -15 \mid (f^{-1})(-15) = 2$

$f'(2) = 20 \mid (f^{-1})'(-15) = \frac{1}{20}$

$-15 = x^3 - 2x^2 + 1$

$0 = x^3 - 2x^2 + 16$

$x = 2$ ✓

$f'(x) = 3x^2 - 4x$

$f'(2) = 3(2)^2 - 4(2)$

$f'(2) = 12 + 8 = 20$

Find $\frac{dy}{dx}$ *for the following*

5.

$$y = 2\log\left(\frac{\sqrt[4]{(3x-2x^4)^3}}{2x^3}\right)$$

$y = 2\log_{10}(3x-2x^4)^{3/4} - 2\log_{10}(2x^3)$

$y = 2\cdot\frac{3}{4}\log_{10}(3x-2x^4) - 2\log_{10}(2x^3)$

$y' = \frac{3}{2}\cdot\frac{1}{\ln 10}\cdot\frac{3-8x^3}{3x-2x^4} - 2\cdot\frac{1}{\ln 10}\cdot\frac{6x^2}{2x^3}$

$y' = \frac{3(3-8x^3)}{2\ln 10(3x-2x^4)} - \frac{2(6x^2)}{2x^3(\ln 10)}$

$\boxed{y' = \frac{3(3-8x^3)}{2\ln 10(3x-2x^4)} - \frac{6}{x(\ln 10)}}$

6. $f(x) = 3^{4x}\left(\log_4(5-\sqrt[3]{x})\right)$ ← $\log_4(5-x^{1/3})$

f g

$f'(x) = (\ln 3)3^{4x}(4)\cdot\log_4(5-\sqrt[3]{x}) + 3^{4x}\cdot\frac{1}{\ln 4}\cdot\frac{-\frac{1}{3}x^{-2/3}}{5-\sqrt[3]{x}}$

f' g f g'

$$f'(x) = 4(\ln 3)3^{4x}\cdot\log_4(5-\sqrt[3]{x}) - \frac{3^{4x}}{(\ln 4)\,3x^{2/3}(5-\sqrt[3]{x})}$$

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7. Use Log differentiation to find the derivative dy/dx

$$y = \frac{x^2\sqrt{3x-2}}{(x+1)^2}$$

$\ln y = \ln\left[\frac{x^2(3x-2)^{1/2}}{(x+1)^2}\right]$

$\ln y = \ln x^2 + \ln(3x-2)^{1/2} - \ln(x+1)^2$

$\ln y = 2\ln x + \frac{1}{2}\ln(3x-2) - 2\ln(x+1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = 2\left(\frac{1}{x}\right) + \frac{1}{2}\left(\frac{3}{3x-2}\right) - 2\left(\frac{1}{x+1}\right)$

$\frac{dy}{dx} = y\left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x+1}\right]$

$$\frac{dy}{dx} = \frac{x^2\sqrt{3x-2}}{(x+1)^2}\left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x+1}\right]$$

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8. Find dy/dx $4xy + \ln x^2y = 7$

$4xy + \ln x^2 + \ln y = 7$

$4xy + 2\ln x + \ln y = 7$

f g

f' g + f g'

$4\cdot y + 4x\cdot dy/dx + 2\left(\frac{1}{x}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = 0$

$4x\left(\frac{dy}{dx}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = -4y - \frac{2}{x}$

$\frac{dy}{dx}\left(4x + \frac{1}{y}\right) = -4y - \frac{2}{x}$

$$\frac{dy}{dx} = \frac{-4y - \frac{2}{x}}{4x + \frac{1}{y}}$$

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9) Find the tangent line equation for the function below at the given point:

$y = xe^x - e^x, \quad (1, 0)$

f g

*product Rule

f' g + f g'

$y' = 1e^x + xe^x - e^x(1)$

$y'(1) = e^1 + 1e^1 - e^1$

$y'(1) = e$

point: (1, 0)

slope: $m = e$

$$y - 0 = e(x-1)$$