Key

Calculus AB **Logs and Exponentials Derivatives Unit Quiz Review WS 3**

Show all work. No calculators.

1. Find $\frac{dy}{dx}$ $\quad y = ln\sqrt[5]{\left(\frac{e^{5x}}{5x^3-9x}\right)^7}$

$y = \ln\left(\frac{e^{5x}}{5x^3-9x}\right)^{7/5}$

$y = \frac{7}{5}\ln(e^{5x}) - \frac{7}{5}\ln(5x^3-9x)$

$y' = \frac{7}{5}\cdot\frac{e^{5x}(5)}{e^{5x}} - \frac{7}{5}\cdot\frac{15x^2-9}{5x^3-9x}$

$y' = \frac{7}{5}(5) - \frac{7(15x^2-9)}{5(5x^3-9x)} \rightarrow \boxed{7 - \frac{7(15x^2-9)}{5(5x^3-9x)}}$

2. Find $\frac{dy}{dx}$ $\quad y = \sqrt{(3e^{2x}+5x)^x}$

$y = (3e^{2x}+5x)^{x/2}$ $\quad$ *Log differentiation

$\ln y = \ln(3e^{2x}+5x)^{\frac{1}{2}x}$

$\frac{d}{dx}e^u = e^u \cdot u'$

$\frac{d}{dx}3e^{2x} = 3e^{2x}\cdot 2$

$\ln y = \underbrace{\frac{1}{2}x}_{f}\,\underbrace{\ln(3e^{2x}+5x)}_{g}$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \underbrace{\frac{1}{2}}_{f'}\cdot\underbrace{\ln(3e^{2x}+5x)}_{g} + \underbrace{\frac{1}{2}x}_{f}\cdot\underbrace{\frac{6e^{2x}+5}{3e^{2x}+5x}}_{g'}$

$\frac{dy}{dx} = y\left[\frac{1}{2}\ln(3e^{2x}+5x) + \frac{x(6e^{2x}+5)}{2(3e^{2x}+5x)}\right]$

$\boxed{\frac{dy}{dx} = \sqrt{3e^{2x}+5x}\left[\frac{1}{2}\ln(3e^{2x}+5x) + \frac{x(6e^{2x}+5)}{2(3e^{2x}+5x)}\right]}$

3. Given that $f(g(x)) = x$.
Find $g'(9)$ if $f(9) = 1$, $f(2) = 9$, $f'(2) = -3$ and $f'(9) = -7$

$f(2) = 9$	$g(9) = 2$
$f'(2) = -3$	$g'(9) = \boxed{-1/3}$

4. $f(x) = x^3 - 2x + 10$ $\quad$ Find $(f^{-1})'(-11)$

$-11 = x^3 - 2x + 10$

$0 = x^3 - 2x + 21$

$x = -3$ ✓

$f(-3) = -11$	$(f^{-1})(-11) = -3$
$f'(-3) = 25$	$(f^{-1})'(-11) = \boxed{1/25}$

$f'(x) = 3x^2 - 2$

$f'(-3) = 3(-3)^2 - 2$

$f'(-3) = 27 - 2 = 25$

Find $\frac{dy}{dx}$ *for the following*

5.

$$y = 4\,log\left(\frac{\sqrt{x^7}}{\sqrt[4]{(3x-2x^4)^3}}\right)$$

$y = 4\log(x^{7/2}) - 4\log(3x-2x^4)^{3/4}$

$y = 4\cdot\frac{7}{2}\log_{10}x - 4\cdot\frac{3}{4}\log_{10}(3x-2x^4)$

$y' = 14\left(\frac{1}{\ln 10}\right)\cdot\frac{1}{x} - 3\left(\frac{1}{\ln 10}\right)\cdot\frac{3-8x^3}{3x-2x^4}$

$\boxed{y' = \frac{14}{x\ln 10} - \frac{3(3-8x^3)}{(\ln 10)(3x-2x^4)}}$

6. $f(x) = e^{4x}\left(\log_2(5 - \sqrt[3]{x})\right)$

$\log_2(5-x^{1/3})$

$\frac{d}{dx}\log_a u = \frac{1}{\ln a}\cdot\frac{u'}{u}$

$$f'(x) = \underbrace{e^{4x}(4)}_{f'}\cdot\underbrace{\log_2(5-\sqrt[3]{x})}_{g} + \underbrace{e^{4x}}_{f}\cdot\underbrace{\frac{1}{\ln 2}\cdot\frac{-\frac{1}{3}x^{-2/3}}{5-\sqrt[3]{x}}}_{g'}$$

$$\boxed{f'(x) = 4e^{4x}\log_2(5-\sqrt[3]{x}) - \frac{e^{4x}}{3(\ln 2)x^{2/3}(5-\sqrt[3]{x})}}$$

7. Use Log differentiation to find the derivative dy/dx

$$f(x) = \frac{(x^5)\sqrt{(4x-3x^4)^5}}{(x^3-4)^5}$$

$$\ln y = \ln\left[\frac{x^5(4x-3x^4)^{5/2}}{(x^3-4)^5}\right]$$

$$\ln y = \ln x^5 + \ln(4x-3x^4)^{5/2} - \ln(x^3-4)^5$$

$$\ln y = 5\ln x + \frac{5}{2}\ln(4x-3x^4) - 5\ln(x^3-4)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = 5\left(\frac{1}{x}\right) + \frac{5}{2}\cdot\frac{4-12x^3}{4x-3x^4} - 5\cdot\frac{3x^2}{x^3-4}$$

$$\frac{dy}{dx} = y\left[\frac{5}{x} + \frac{5(4-12x^3)}{2(4x-3x^4)} - \frac{15x^2}{x^3-4}\right]$$

$$\boxed{\frac{dy}{dx} = \frac{x^5\sqrt{(4x-3x^4)^5}}{(x^3-4)^5}\left[\frac{5}{x} + \frac{5(4-12x^3)}{2(4x-3x^4)} - \frac{15x^2}{x^3-4}\right]}$$

8. Find dy/dx $\ln\left(\frac{x}{3y}\right) - xy + y = 12$

$$\ln x - \ln(3y) - (\underbrace{x}_{f}\underbrace{y}_{g}) + y = 12$$

$$\frac{1}{x} - \frac{3}{3y}\left(\frac{dy}{dx}\right) - \left(\underbrace{1}_{f'}\cdot\underbrace{y}_{g} + \underbrace{x}_{f}\cdot\underbrace{dy/dx}_{g'}\right) + 1\left(\frac{dy}{dx}\right) = 0$$

$$\frac{1}{x} - \frac{1}{y}\left(\frac{dy}{dx}\right) - y - x\left(\frac{dy}{dx}\right) + 1\left(\frac{dy}{dx}\right) = 0$$

$$1\left(\frac{dy}{dx}\right) - x\left(\frac{dy}{dx}\right) - \frac{1}{y}\left(\frac{dy}{dx}\right) = y - \frac{1}{x}$$

$$\frac{dy}{dx}\left(1 - x - \frac{1}{y}\right) = y - \frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{y-\frac{1}{x}}{1-x-\frac{1}{y}}}$$

9) Find the tangent line equation for the function below at the given point:

$$f(x) = \underbrace{x}_{f}\underbrace{(3^{3x-6})}_{g} \quad at \quad (2,2)$$

$$f'(x) = \underbrace{1}_{f'}\cdot\underbrace{3^{3x-6}}_{g} + \underbrace{x}_{f}\cdot\underbrace{(\ln 3)\cdot 3^{3x-6}\cdot(3)}_{g'}$$

$$f'(2) = 3^{6-6} + 2\ln 3(3^0)(3)$$

$$f'(2) = 3^0 + 6\ln 3$$

$$f'(2) = 1 + 6\ln 3$$

point: $(2,2)$

slope: $m = 1 + 6\ln 3$

$$\boxed{y - 2 = (1+6\ln 3)(x-2)}$$