Logs Unit
Quiz Review WS#4

Key

1) Find $\frac{dy}{dx}$ $\quad y = \log_5\left(\frac{4}{x^2\sqrt{1-x}}\right)$

$$y = \log_5 4 - \log_5 x^2 - \log_5 (1-x)^{1/2}$$

$\frac{d}{dx}\log_a u = \left(\frac{1}{\ln a}\right)\cdot\frac{u'}{u}$

$$y = \log_5 4 - 2\log_5 x - \frac{1}{2}\log_5(1-x)$$

$$y' = 0 - 2\cdot\frac{1}{\ln 5}\cdot\frac{1}{x} - \frac{1}{2}\cdot\frac{1}{\ln 5}\cdot\frac{-1}{1-x}$$

$\frac{d}{dx}\log_a u = \frac{1}{\ln a}\cdot\frac{u'}{u}$

$$\boxed{\frac{dy}{dx} = \frac{-2}{x\ln 5} + \frac{1}{2\ln 5(1-x)}}$$

Find $\frac{dy}{dx}$ $\quad y = \sqrt[7]{(x^2-\ln x)^x}$

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$$y = (x^2-\ln x)^{x/7}$$

$$y = (x^2-\ln x)^{\frac{1}{7}x}$$

$* \ln a^n = n \cdot \ln a$

$$\ln y = \ln(x^2-\ln x)^{\frac{1}{7}x}$$

$$\ln y = \overbrace{\tfrac{1}{7}x}^{f}\ \overbrace{\ln(x^2-\ln x)}^{g}$$

$\frac{d}{dx}\ln u = \frac{u'}{u}$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \overbrace{\tfrac{1}{7}}^{f'} \cdot \overbrace{\ln(x^2-\ln x)}^{g} + \overbrace{\tfrac{1}{7}x}^{f} \cdot \overbrace{\frac{2x-\frac{1}{x}}{x^2-\ln x}}^{g'}$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{7}\ln(x^2-\ln x) + \frac{2x^2-1}{7(x^2-\ln x)}$$

$$\frac{dy}{dx} = y\left[\frac{1}{7}\ln(x^2-\ln x) + \frac{2x^2-1}{7(x^2-\ln x)}\right]$$

$$\boxed{\frac{dy}{dx} = \sqrt[7]{(x^2-\ln x)^x}\left[\frac{1}{7}\ln(x^2-\ln x) + \frac{2x^2-1}{7(x^2-\ln x)}\right]}$$

3) Find $\frac{d}{dx} f^{-1}(2)$ given $f(x) = x^3 + 2x - 1$

$f(\ ) = 2$	$(f^{-1})(2) = \_\_$
	$(f^{-1})'(2) = \_\_$

$2 = x^3 + 2x - 1$

$0 = x^3 + 2x - 3$

test x-values

$x = 0, 1, -1, 2, -2, \ldots$

$x = 1$

$f(1) = 2$	$(f^{-1})(2) = \underline{1}$
$f'(1) = \underline{5}$	$(f^{-1})'(2) = \_\_$

$f'(x) = 3x^2 + 2$

$f'(1) = 3(1)^2 + 2$

$f'(1) = 5$

$$\boxed{(f^{-1})'(2) = \frac{1}{5}}$$

4) Find $\frac{dy}{dx}$ for $\ln(xy) + xy = 50$

* expand log expression
* implicit diff
* product rule

$$\ln x + \ln y + \overset{f}{x}\overset{g}{y} = 50$$

$$\frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) + \overset{f'}{1}\overset{g}{y} + \overset{f}{x}\overset{g'}{\left(\frac{dy}{dx}\right)} = 0$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) + x\left(\frac{dy}{dx}\right) = -\frac{1}{x} - y$$

$$\frac{dy}{dx}\left(\frac{1}{y} + x\right) = -\frac{1}{x} - y$$

$$\frac{dy}{dx} = \frac{-\frac{1}{x} - y}{\frac{1}{y} + x} \quad xy$$

$$\boxed{\frac{dy}{dx} = \frac{-y - xy^2}{x + x^2y}}$$

$$\frac{dy}{dx} = \frac{-y(1+xy)}{x(1+xy)} = \boxed{\frac{-y}{x}}$$

5) Find tangent line equation for

$f(x) = e^{-x} \ln x$ at $(1,0)$

$* \frac{d}{dx} e^u = e^u \cdot u'$

$\frac{d}{dx} \ln u = \frac{u'}{u}$

$$f'(x) = \overbrace{e^{-x}(-1)}^{f'} \cdot \overbrace{\ln x}^{g} + \overbrace{e^{-x}}^{f} \cdot \overbrace{\left(\frac{1}{x}\right)}^{g'} = \frac{-\ln x}{e^x} + \frac{1}{xe^x}$$

$$f'(1) = \frac{-\ln 1}{e^1} + \frac{1}{1(e^1)} = 0 + \frac{1}{1(e)} = \boxed{\frac{1}{e}}$$

point: $(1,0)$

slope: $m = \frac{1}{e}$

$y - y_1 = m(x - x_1)$

$$\boxed{y - 0 = \frac{1}{e}(x-1)}$$

6) Find $\frac{dy}{dx}$ $\quad y=(3-5x)^{2^x}$

*log differentiation

$\ln y = \ln(3-5x)^{2^x}$

$\ln y = 2^x \cdot \ln(3-5x)$

$* \frac{d}{dx}\ln u = \frac{u'}{u}$

$\frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \underbrace{(\ln 2)(2^x)(1)}_{f'} \cdot \underbrace{\ln(3-5x)}_{g} + \underbrace{2^x}_{f} \cdot \underbrace{\left(\frac{-5}{3-5x}\right)}_{g'}$$

$$\frac{dy}{dx} = y \cdot \left[(\ln 2)2^x \ln(3-5x) + \frac{2^x(-5)}{3-5x}\right]$$

$$\boxed{\frac{dy}{dx} = (3-5x)^{2^x}\left[\ln 2(2^x)\ln(3-5x) - \frac{2^x(5)}{3-5x}\right]}$$

7) Find y' for $y = 3\log_7\left(\frac{x}{e^{2x}\sqrt{1-3x^2}}\right)$

*expand first

$$y = 3\log_7 x - 3\log_7 e^{2x} - 3\log_7(1-3x^2)^{1/2}$$

$$y = 3\log_7 x - 3\log_7 e^{2x} - 3\cdot\frac{1}{2}\log_7(1-3x^2)$$

$*\ \frac{d}{dx}\log_a u = \frac{1}{\ln a}\left(\frac{u'}{u}\right)$

$$y' = \frac{3}{\ln 7}\left(\frac{1}{x}\right) - 3\cdot\frac{1}{\ln 7}\left(\frac{\cancel{e^{2x}}(2)}{\cancel{e^{2x}}}\right) - \frac{3}{2}\cdot\frac{1}{\ln 7}\cdot\left(\frac{-6x}{1-3x^2}\right)$$

$\frac{d}{dx}e^u = e^u \cdot u'$

$$\boxed{y' = \frac{3}{x\ln 7} - \frac{6}{\ln 7} + \frac{9x}{\ln 7(1-3x^2)}}$$

8) $\ln\left(\frac{\sqrt[3]{y}}{x^5}\right) = 3x^2y - y + 5x - 3$

Find $\left(\frac{dy}{dx}\right)$.

$\ln\sqrt[3]{y} - \ln x^5 = 3x^2y - y + 5x - 3$

* Implicit differentiation
* expansion property
* product rule

$\ln y^{1/3} - \ln x^5 = 3x^2y - y + 5x - 3$

$\frac{1}{3}\ln y - 5\ln x = \overbrace{3x^2}^{f}\overbrace{y}^{g} - y + 5x - 3$

$\frac{1}{3}\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) - 5\left(\frac{1}{x}\right) = \overbrace{6x}^{f'} \cdot \overbrace{y}^{g} + \overbrace{3x^2}^{f} \cdot \overbrace{\left(\frac{dy}{dx}\right)}^{g'} - 1\left(\frac{dy}{dx}\right) + 5$

$\frac{1}{3y}\left(\frac{dy}{dx}\right) - 3x^2\left(\frac{dy}{dx}\right) + 1\left(\frac{dy}{dx}\right) = 6xy + \frac{5}{x} + 5$

$\frac{dy}{dx}\left(\frac{1}{3y} - 3x^2 + 1\right) = 6xy + \frac{5}{x} + 5$

$$\boxed{\frac{dy}{dx} = \frac{6xy + \frac{5}{x} + 5}{\frac{1}{3y} - 3x^2 + 1}}$$