AP Calculus AB 2020 Mock AP Exam #1

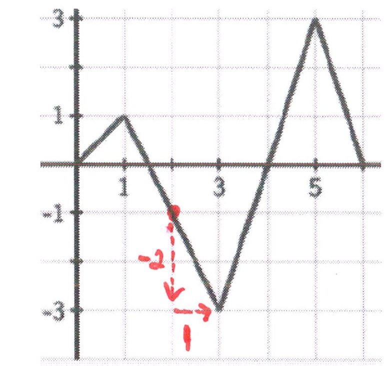
1) (25 Minutes) (15 points)

Let f be a differentiable function. The table gives values of f and its derivative f' at selected values of x.

Let $g(x) = \sin(3x) - e^{\cos(2x)}$

Let $h(x) = \int_6^x j(x)dx$

x	f(x)	f'(x)
-1	4	-3
0	2	-1
1	3	1
2	-1	3
3	5	-2
4	-2	4
5	4	-2
6	3	2



graph of $j(x)$

2 a) Is there a value of c for $1 < x < 5$ such that $f''(c) = -\frac{3}{4}$? Provide an explanation for your answer.

2 b) Let k be the function defined by $k(x) = f(j(x))$. Write an equation for the line tangent to the graph of k at x = 2

2 c) Find the slope of the tangent line to the graph of g at $x = \pi$

2 d) *Find* $h(4)$ *and* $h'(4)$

2 e) *On what interval is h increasing* and concave down?

3 f) *Find the absolute minimum, absolute* max *value of the h on the interval* $0 \le x \le 6$. Justify your answers.

2 g) *Evaluate* $\int_1^3 f''(2x)\,dx$

$\boxed{2}$ a) * This is an application of MVT: Since $f'(c) = \frac{f(b)-f(a)}{b-a}$, then also $f''(c) = \frac{f'(b)-f'(a)}{b-a}$

$f''(c) = \frac{f'(5)-f'(1)}{5-1} = \frac{-2-(1)}{5-1} = \frac{-3}{4}$. By MVT, since $f'(x)$ is continuous on $[1,5]$ and differentiable on $(1,5)$, then there is a point where $f''(c) = -3/4$.

$\boxed{2}$ b) * Apply chain Rule: Recall that $\frac{d}{dx}f[g(x)] = f'[g(x)]\cdot g'(x)$

$k(x) = f[j(x)]$

$k'(x) = f'[j(x)]\cdot j'(x)$

$k'(2) = f'[j(2)]\cdot j'(2)$

$k'(2) = f'(-1)\cdot j'(2)$

$k'(2) = (-3)\cdot(-2)$ ← graph of j at x = 2 has slope $= \frac{-2}{1} = -2$

$k'(2) = 6$ ← slope

$k(x) = f[j(x)]$

$k(2) = f[j(2)]$

$k(2) = f[-1]$

$k(2) = 4$

point ↗

point: $(2,4)$
slope: $m = 6$

$y - y_1 = m(x - x_1)$

$\boxed{y - 4 = 6(x-2)}$

$\boxed{2}$ c) $g(x) = \sin(3x) - e^{\cos(2x)}$ * Apply chain Rule

$g'(x) = \cos(3x)\cdot 3 - e^{\cos(2x)}\cdot(-\sin(2x))\cdot 2$

$g'(\pi) = \cos(3\pi)\cdot 3 - e^{\cos 2\pi}\cdot -\sin(2\pi)\cdot 2$

$g'(\pi) = (-1)(3) - e^{1}(0)\cdot 2$

$g'(\pi) = -3 - 0$

$\boxed{g'(\pi) = -3}$

d)–g) continued on next page

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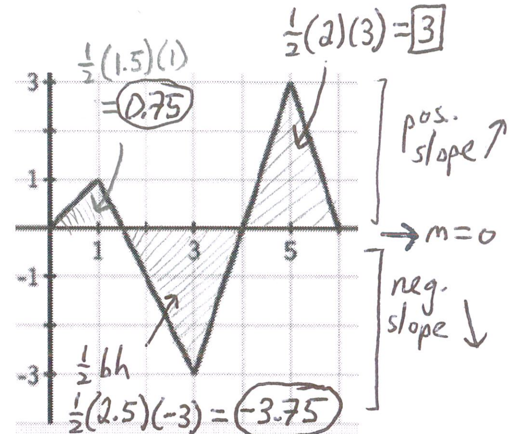

graph of $j(x)$ $= h'(x)$

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Area of triangle: $\frac{1}{2}(b)(h)$

$\boxed{2}$ d) $h(4) = \int_6^4 j(x)dx = -\int_4^6 j(x)dx = -\left[\frac{1}{2}(2)(3)\right] = \boxed{-3}$

Apply SFTC $h'(x) = \frac{d}{dx}\int_6^x j(t)dt = j(x)\cdot 1 \to h'(x) = j(x) \to h'(4) = j(4) = \boxed{0}$

$\boxed{2}$ e) Since $h'(x) = j(x)$, we can gather slope info of $h(x)$ by looking at derivative graph $j(x)$

$h'(x)$: + on (0, 1.5) ↗, − on (1.5, 4) ↘, + on (4, 6) ↗

Concavity of $h(x)$ can be found from slope of $j(x)$ (POI at $x = 1, 3, 5$)

$h''(x)$: + ∪ on (0, 1), − ∩ on (1, 3), + ∪ on (3, 5), − ∩ on (5, 6)

The graph of $h(x)$ is increasing and concave down on the intervals $1 < x < 1.5$ and $5 < x < 6$

$\boxed{3}$ f) * Apply EVT: Test critical points ($x = 1.5, 4$) and endpoints ($x = 0, 6$)

$h(0) = \int_6^0 j(x)dx = -\int_0^6 j(x)dx = -(0.75 - 3.75 + 3) = \boxed{0}$

$h(1.5) = \int_6^{1.5} j(x)dx = -\int_{1.5}^6 j(x)dx = -(-3.75 + 3) = \boxed{0.75}$

$h(4) = \int_6^4 j(x)dx = -\int_4^6 j(x)dx = -(3) = \boxed{-3}$

$h(6) = \int_6^6 j(x)dx = \boxed{0}$

By EVT, the absolute maximum value on $[0, 6]$ is 0.75 at $x = 1.5$. The absolute minimum value is -3 at $x = 4$.

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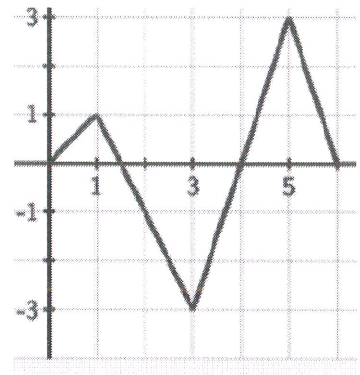

graph of $j(x)$

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$\boxed{2}$ g) $\int_1^3 f''(2x)dx$

* Apply FFTC : $\int_a^b f'(x)dx = f(b) - f(a)$

* Apply U-sub/convert bounds

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int f''(u) \cdot \frac{du}{2}$

$\frac{1}{2}\int f''(u)du$

$\frac{1}{2}f'(u) \rightarrow \frac{1}{2}f'(2x)\Big]_1^3 = \frac{1}{2}f'(6) - \frac{1}{2}f'(2) = \frac{1}{2}(2) - \frac{1}{2}(3)$

$= \frac{2}{2} - \frac{3}{2} = \boxed{\frac{-1}{2}}$

OR convert bounds

$x = 1,\ u = 2x \rightarrow u = 2(1) = 2$

$x = 3,\ u = 2x \rightarrow u = 2(3) = 6$

$\frac{1}{2}f'(u)\Big]_2^6 = \frac{1}{2}f'(6) - \frac{1}{2}f'(2)$

$= \frac{1}{2}(2) - \frac{1}{2}(3) = \boxed{-\frac{1}{2}}$

2) (15 minutes) (9 points)

Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{xy}$ with $f(1) = 2$.

2 a) Find $\frac{d^2y}{dx^2}$ at the point (1, 2)

2 b) Write an equation for the line tangent to the graph of f at (1,2) and use it to approximate f(1.1). Is the approximation for f(1.1) greater or less than f(1.1)? Explain your reasoning.

4 c) Find the solution of the given differential equation that satisfies the initial condition f(1) = 2

1 d) Let g be a differential function such that $g(x) = f^{-1}(x)$ for all x. What is the value of g'(2)?

$\boxed{2}$ a) * Apply implicit differentiation

* Apply product Rule/chain rule

$$\frac{dy}{dx} = \frac{1}{xy} = (xy)^{-1}$$

$$\frac{d^2y}{dx^2} = -1(xy)^{-2}\Big[\underbrace{1}_{f'}\cdot\underbrace{y}_{g} + \underbrace{x}_{f}\cdot\underbrace{\frac{dy}{dx}}_{g'}\Big]$$

$$\frac{d^2y}{dx^2} = \frac{-\left(y + x\frac{dy}{dx}\right)}{(xy)^2} \quad \leftarrow \quad \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{1}{xy} = \frac{1}{(1)(2)} = \frac{1}{2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = \frac{-\left(2 + 1\left(\frac{1}{2}\right)\right)}{(1\cdot 2)^2} = \frac{-\frac{5}{2}}{2^2} = \frac{-5}{2}\cdot\frac{1}{4} = \boxed{\frac{-5}{8}}$$

$\boxed{2}$ b) *Linear Approximation Steps:

i) find slope using $\frac{dy}{dx}$, find point (x, y)

ii) create tangent line using point-slope: $y - y_1 = m(x - x_1)$

iii) plug decimal into tangent line equation

point: (1, 2) slope: $\left.\frac{dy}{dx}\right|_{(1,2)} \rightarrow \frac{1}{xy} = \frac{1}{1(2)}$

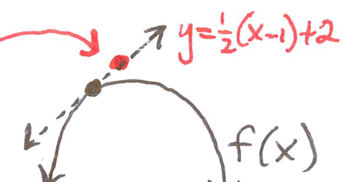
$$y - y_1 = m(x - x_1)$$
$$y - 2 = \tfrac{1}{2}(x - 1)$$
$$y = \tfrac{1}{2}(x - 1) + 2$$

Slope: $m = \frac{1}{2}$

$$y(1.1) = \tfrac{1}{2}(1.1 - 1) + 2$$
$$\boxed{y(1.1) = 2.05}$$

Since $\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = \frac{-5}{8} < 0$, the graph of f(x) is concave down.

If graph is concave down, then tangent line will sit above the curve. Therefore, the approximation y(1.1) is greater than f(1.1) resulting in an overapproximation.



2) (15 minutes) 9 points

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1 d) Let g be a differential function such that $g(x) = f^{-1}(x)$ for all x. What is the value of g'(2)?

$\boxed{4}$ c) * separation of variables

$$\frac{dy}{dx} = \frac{1}{xy}$$

$$xy\,dy = dx$$

$$y\,dy = \frac{dx}{x}$$

$$\int y\,dy = \int \frac{1}{x}\,dx$$

$$\frac{y^2}{2} = \ln|x| + C$$

solve for C
plug in (1,2)

$$\frac{(2)^2}{2} = \ln|1| + C$$

$$2 = 0 + C$$

$$2 = C$$

$$\frac{y^2}{2} = \ln|x| + 2$$

$$2\left(\frac{y^2}{2} = \ln|x| + 2\right)$$

$$y^2 = 2\ln|x| + 4$$

$$y = \pm\sqrt{2\ln|x| + 4}$$

$$y = -\cancel{\sqrt{2\ln|x|+4}} \qquad \boxed{y = \sqrt{2\ln|x| + 4}}$$

fulfills condition of ordered pair (1, 2)

$\boxed{1}$ d) *A function and its inverse will have slopes that are reciprocals of each other at their corresponding points.

Since $f(1) = 2$, then $g(2) = 1$

Since $f'(1) = \frac{1}{2}$, then $\boxed{g'(2) = 2}$

$$\left.\frac{dy}{dx}\right|_{(1,2)} \rightarrow \frac{1}{xy} = \frac{1}{1(2)} = \frac{1}{2}$$

$f(a) = b$	$g(b) = a$
$f'(a) = n$	$g'(b) = \frac{1}{n}$

$f(1) = 2$	$g(2) = 1$
$f'(1) = \frac{1}{2}$	$g'(2) = \boxed{2}$