Key

AP Calculus AB 2020 Mock AP Exam #2

1. (25 mins) 15 points

Two particles move along the x-axis. Particle Q's position, velocity , and acceleration are given in the table below:

2 4 4

t (minutes)	0	1	2	3	6	7	10
Q(t)	5	7	8	6	5	4	7
$Q'(t) = v_Q(t)$	2	3	0	-2	-1	0	4
$Q''(t) = a_Q(t)$	-1	-2	-3	-1	0	1	2

Particle P's **velocity** is given by the piecewise function P(t)

$$P(t) = \begin{cases} 3 + 2t - t^2 & \text{for } 0 \le t \le 3 \\ 2te^{2-t} & \text{for } 3 < t \le 10 \end{cases}$$

1 a) *Find the average acceleration of particle Q in the interval* $0 \le t \le 10$

2 b) For Particle Q, explain the meaning of the definite integral $\int_0^{10} |v(t)|dt$. Approximate the value of $\int_0^{10} |v(t)|dt$ using Trapezoid approximation with 3 subintervals indicated in the table.

2 c) For Particle Q, evaluate $\int_0^{10} a(t)dt$ and explain the meaning of this value.

2 d) At t = 1, are the particles P and Q speeding up or slowing down? Show work for each to justify answer.

2 e) Find $\lim_{x\to 3^-} \dfrac{1-e^{3-x}}{2P(t)}$

3 f) *Let* $h(t) = \frac{Q(t)}{3-t^2}$ *Find* $h'(1)$

3 g) Do Particles Q and P both change directions in the interval $0 \le t \le 4$? Show work to justify your answer

$\boxed{1}$ a) * avg. acceleration = $\dfrac{\text{change in velocity}}{\text{change in time}} \rightarrow \dfrac{v(10)-v(0)}{10-0} = \dfrac{4-2}{10-0} = \dfrac{2}{10} = \dfrac{1}{5}$

$\boxed{2}$ b) $\int_0^{10} |v(t)|dt$ is total distance traveled by particle Q in time interval $0 \le t \le 10$

note the positive 1's

Trapezoid Approximation

$A = \frac{w}{2}(h_1 + h_2)$

$$\int_0^{10} |v(t)|dt \approx \frac{2}{2}[2+0] + \frac{4}{2}[0+1] + \frac{4}{2}[1+4] = 2+2+10 = \boxed{14}$$

Apply FFTC: $\int_a^b f'(x)dx = f(b) - f(a)$

$\boxed{2}$ c) $\int_0^{10} a(t)dt = v(10) - v(0)$ represents the change in velocity in the time interval $0 \le t \le 10$

$= 4 - 2 = \boxed{2}$

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velocity function

$P(t) = \begin{cases} 3 + 2t - t^2 & \text{for } 0 \le t \le 3 \\ 2te^{2-t} & \text{for } 3 < t \le 10 \end{cases}$

$P'(t) = \begin{cases} 2 - 2t & 0 \le t \le 3 \\ 2e^{2-t} + 2te^{2-t}(-1) & 3 < t \le 10 \end{cases}$

acceleration function

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2 c) For Particle Q, evaluate $\int_0^{10} a(t)dt$ and explain the meaning of this value.

2 d) At t = 1, are the particles P and Q speeding up or slowing down? Show work for each to justify answer.

2 e) *Find* $\lim_{t \to 3^-} \frac{1 - e^{3-t}}{2P(t)}$

3 f) *Let* $h(t) = \frac{Q(t)}{3 - t^2}$ *Find* $h'(1)$

3 g) Do Particles Q and P both change directions in the interval $0 \le t \le 4$? Show work to justify your answer

$\boxed{2}$ d) *To determine whether particle is speeding up/slowing down, compare signs of velocity and acceleration.

$V_Q(1) = 3$

$a_Q(1) = -2$

particle Q is slowing down since signs of acceleration and velocity are opposite.

$V_P(1) = 3 + 2(1) - 1^2 = 4$

$a_P(t) = 2 - 2t$

$a_P(1) = 0$

particle P is neither speeding up nor slowing down.

$\boxed{2}$ e) $\lim_{t \to 3^-} \frac{1 - e^{3-t}}{2P(t)} \to \frac{1 - e^{3-3}}{2P(3)} = \frac{1 - e^0}{2(3+6-9)} = \frac{0}{0} \to$

Apply L'Hopital's Rule (take derivative of numerator and denominator separately)

$\xrightarrow{L'H} \lim_{t \to 3^-} \frac{0 - e^{3-t}(-1)}{2P'(t)} \to \lim_{t \to 3^-} \frac{e^{3-t}}{2(2-2t)} = \frac{e^{3-3}}{2(2-6)} = \frac{e^0}{2(-4)} = \boxed{\frac{1}{-8}}$

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$\boxed{3}$ f) $h(t) = \dfrac{\overbrace{Q(t)}^{f}}{\underbrace{3-t^2}_{g}}$ Apply Quotient Rule

$$h'(t) = \frac{\overbrace{Q'(t)}^{f'} \cdot \overbrace{(3-t^2)}^{g} - \overbrace{Q(t)}^{f} \cdot \overbrace{(-2t)}^{g'}}{\underbrace{(3-t^2)^2}_{g^2}}$$

$$h'(1) = \frac{Q'(1)(3-1^2) - Q(1)(-2)}{[3-1^2]^2}$$

$$h'(1) = \frac{3(2) - 7(-2)}{2^2} = \frac{6+14}{4} = \frac{20}{4}$$

$$\boxed{h'(1) = 5}$$

$\boxed{3}$ g) $V_p(t) = 3 + 2t - t^2$

set velocity = 0
find critical points
test intervals

$0 = -t^2 + 2t + 3$

$0 = -1(t^2 - 2t - 3)$

$0 = -1(t-3)(t+1)$

$t = 3, t = -1$

$V_p(t)$: 0 — + (1) — 3 — + (4)

No sign change for particle P at t=3, so no change in direction.

Particle Q has at least one change in direction in interval $0 \le t \le 4$. By IVT, since $V_Q(t) > 0$ at $t=0$ and $V_Q(t) < 0$ at $t = 2$ and $V_Q(t)$ is continuous then particle Q will change signs of velocity.

2) (15 minutes) 9 points

Let $f(x) = e^x - x$

2 a) Find the critical value(s) of f. Classify each of these values as a relative minimum, relative maximum, or neither. Justify your conclusion

1 b) Write the equation of the line tangent to the graph of f at the point where x = 1

3 c) Given $\int_0^a f(x)dx = f'(a)$ *find a.*

3 d) Suppose $y^2 f(x) = y - 4$. *Find* $\frac{dy}{dx}$

$\boxed{2}$ a) * find f'(x), set f'(x) = 0, put critical points on sign line, test intervals

$f'(x) = e^x - 1$

$0 = e^x - 1$

$1 = e^x$

$\ln 1 = \ln e^x$

$0 = x \cancel{\ln e}$

$x = 0$

$f'(x)$: $-$ at -1, $+$ at 1; critical point 0

Relative minimum at $x=0$ since $f'(x)$ changes from $-$ to $+$

$\boxed{1}$ b) find point: $f(1) = e^1 - 1 = e - 1$

find slope: $f'(1) = e^1 - 1 = e - 1$

point: $(1, e-1)$

slope: $m = e - 1$

$y - y_1 = m(x - x_1)$

$$\boxed{y - (e-1) = (e-1)(x-1)}$$

$\boxed{3}$ c) $\int_0^a f(x)dx = f'(a)$ $f'(x) = e^x - 1$

$\int_0^a e^x - x\,dx = e^a - 1$

$e^x - \frac{x^2}{2}\Big]_0^a = e^a - \frac{a^2}{2} - \left(e^0 - \frac{0^2}{2}\right)$

$\cancel{e^a} - \frac{1}{2}a^2 - 1 = \cancel{e^a} - 1$

$-\frac{1}{2}a^2 = 0$

$a^2 = 0$

$$\boxed{a = 0}$$

$\boxed{3}$ d) * Implicit differentiation

* product rule

$y^2 f(x) = y - 4$

$\underbrace{2y\left(\tfrac{dy}{dx}\right)}_{f'} \cdot \underbrace{f(x)}_{g} + \underbrace{y^2}_{f} \underbrace{f'(x)}_{g'} = 1\left(\frac{dy}{dx}\right) - 0$

$2y\left(\frac{dy}{dx}\right)f(x) - 1\left(\frac{dy}{dx}\right) = -y^2 f'(x)$

$\frac{dy}{dx}\left(2yf(x) - 1\right) = -y^2 f'(x)$

$\frac{dy}{dx} = \frac{-y^2 f'(x)}{2yf(x) - 1}$

$\frac{dy}{dx} = \frac{-y^2(e^x - 1)}{2y(e^x - x) - 1}$

$$\boxed{\frac{dy}{dx} = \frac{-y^2 e^x + y^2}{2ye^x - 2xy - 1}}$$