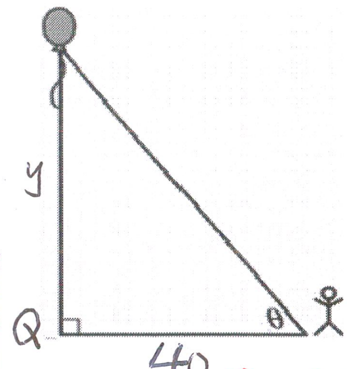
Calculus AB 2020 Mock AP Exam #3

Key

$\boxed{3}$ 1) **Related Rates Assorted Problems** 25 minutes 15 points

a) A person stands 40 feet from point Q and watches the balloon rise vertically from point Q. The balloon is rising at a constant rate of 3 feet per second. What is the rate of change, in radians per second, of angle θ at the instant when the balloon is 30 feet above the point. → find $\frac{d\theta}{dt}$

$\frac{dy}{dt}=3$



constant value, no need for variable

*create equation using trig equation:

$\tan\theta = \frac{y}{40}$

$\tan\theta = \frac{1}{40}y$

$\sec^2\theta\left(\frac{d\theta}{dt}\right)=\frac{1}{40}\left(\frac{dy}{dt}\right)$

$y=30$

30, 40, 50, θ

$\sec\theta = \frac{50}{40}=\frac{5}{4}$

$(\sec\theta)^2\left(\frac{d\theta}{dt}\right)=\frac{1}{40}\left(\frac{dy}{dt}\right)$

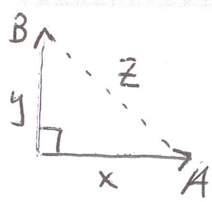
$\left(\frac{5}{4}\right)^2\left(\frac{d\theta}{dt}\right)=\frac{1}{40}(3)$

$\frac{d\theta}{dt}=\frac{1}{40}(3)\left(\frac{4}{5}\right)^2$

$\frac{d\theta}{dt}=\frac{48}{200}$

$\boxed{\frac{d\theta}{dt}=\frac{6}{25}\text{ rad/sec}}$

$\boxed{3}$ b) **Bikes A and B are traveling on perpendicular roads. At the same time, bike A is leaving the intersection at a rate of 2 feet per second and bike B is leaving the intersection at 3 feet per second. How fast is the distance, in feet per second, between them changing after 5 seconds?**



$\frac{dx}{dt}=2\text{ ft/s}$

$\frac{dy}{dt}=3\text{ ft/s}$

$\frac{dz}{dt}=$ __

start with pythagorean theorem

$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$2(10)(2)+2(15)(3)=2(5\sqrt{13})\left(\frac{dz}{dt}\right)$

$40+90=10\sqrt{13}\left(\frac{dz}{dt}\right)$

After 5 seconds

$x \to 5(2)=10$

$y \to 5(3)=15$

$x^2+y^2=z^2$

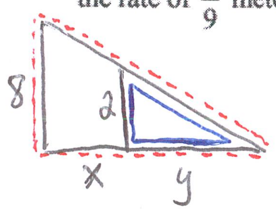
$10^2+15^2=z^2$

$z=\sqrt{325}\to 5\sqrt{13}$

$130=10\sqrt{13}\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt}=3.606\text{ ft/sec}}$

$\boxed{4}$ c) A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meters per second, at what rate, in meters per second, is the person walking?



set up similar triangle proportions

$\frac{2}{8}=\frac{y}{x+y}$

$2(x+y)=8y$

$2x+2y=8y$

$2x=6y$

$2\left(\frac{dx}{dt}\right)=6\left(\frac{dy}{dt}\right)$

$\frac{dy}{dt}=\frac{4}{9}\text{ m/s}$

$2\left(\frac{dx}{dt}\right)=6\left(\frac{4}{9}\right)$

$\boxed{\frac{dx}{dt}=\frac{4}{3}\text{ m/s}}$

$\frac{dy}{dt}$ is rate of change length of shadow

$\frac{dx}{dt}$ is rate of person walking

$\boxed{\frac{dx}{dt}+\frac{dy}{dt}=\frac{16}{9}\text{ ft/s}}$

rate of change of tip of shadow

$\boxed{5}$ d) **A beach ball is deflating at a constant rate of 10 cubic centimeters per second. When the volume of the ball is $\frac{256}{3}\pi$ cubic centimeters, what is the rate of change of the surface area? ($S=4\pi r^2$ and $V=\frac{4}{3}\pi r^3$)**

Find $\frac{dS}{dt}$

$\frac{dV}{dt}=-10$

$V=\frac{256}{3}\pi$

$\frac{dV}{dt}=-10\ \text{cm}^3/\text{sec}$

Find $\frac{dS}{dt}$

$V=\frac{4}{3}\pi r^3$

$\frac{dV}{dt}=\frac{4}{\cancel{3}}\pi\cancel{3}r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt}=4\pi r^2\left(\frac{dr}{dt}\right)$

$\frac{4}{3}\pi r^3=\frac{256\pi}{3}$

$4r^3=256$

$r^3=64$

$r=4$

$-10=4\pi(4)^2\left(\frac{dr}{dt}\right)$

$\frac{-5}{32\pi}=\frac{dr}{dt}$

$S=4\pi r^2$

$\frac{dS}{dt}=4\pi\cdot 2r\left(\frac{dr}{dt}\right)$

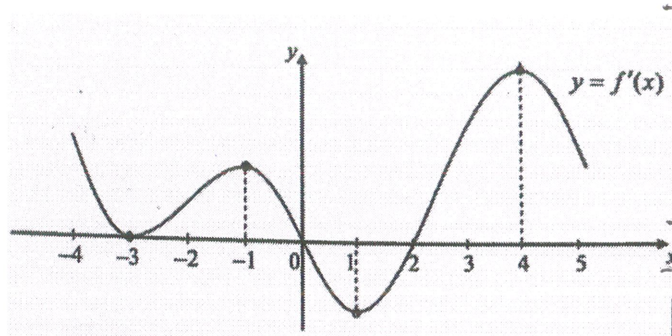
$\frac{dS}{dt}=8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt}=8\pi(4)\left(\frac{-5}{32\pi}\right)$

$\boxed{\frac{dS}{dt}=-5\ \text{cm}^2/\text{sec}}$

2) **15 minutes** **9 points**

The figure to the right shows graph of f', the derivative of the function f, for $-4 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = -3, -1, 1,$ and 4.



pos. slope for f(x) ↗

→ m=0

neg. slope of f(x) ↘

1 a) Find all the value of x, for $-4 \le x \le 5$ for which f is increasing. Justify your answer.
1 b) Find all the value of x, for $-4 \le x \le 5$ for which f has a relative maximum. Justify answer.
2 c) Find all the value of x, for $-4 \le x \le 5$, for which the graph of f is concave down.
3 d) Given $f(-4) = -2$, $f(0) = 5$, and $f(5) = 8$, sketch a possible graph of $f(x)$ on the axes below.
2 e) Sketch a possible graph of $f''(x)$

critical points are x-intercepts

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1 a) * f(x) is increasing when f′(x) graph is above x-axis.
f(x) is increasing (-4,3)∪(-3,0)∪(2,5) b/c f′(x) > 0

f′(x): + + − + at -4, -3, 0, 2, 5

1 b) Relative maximum at x=0 b/c f′(x) changes from + to −

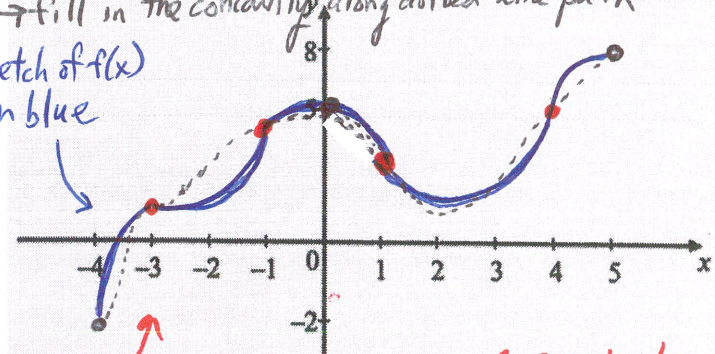
2 c) * create f″(x) sign line to visualize
→ critical points (max/mins) are POI's
→ slope of f′(x) corresponds with concavity
→ negative slope means concave down
→ positive slope means concave up

f″(x): − + − + − at -4, -3, -1, 1, 4, 5

f(x) is concave down (-4,-3)∪(-1,1)∪(4,5) b/c f″(x) < 0
(or f′(x) is decreasing)

3 d) $f(x)$ → match slope sign line first (dotted line)
→ fill in the concavity along dotted line path

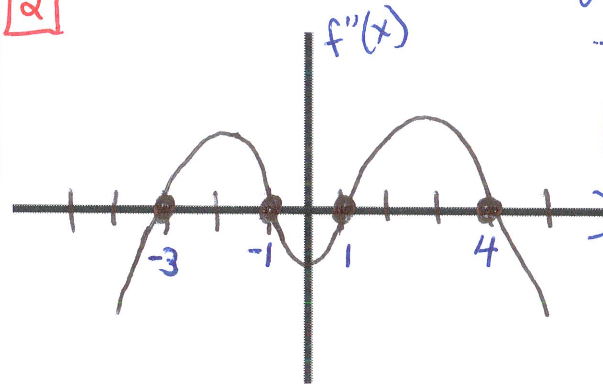
sketch of f(x) in blue



marked estimated locations of POI's to create targets where concavity changes

e) $f''(x)$ → base graph off of f″(x) sign line

2

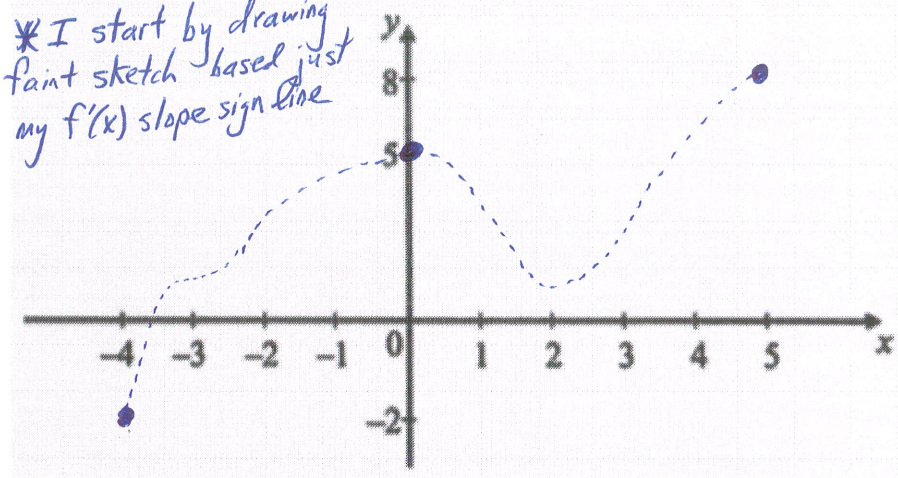


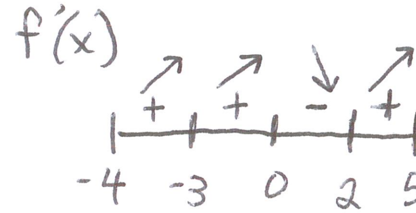
f″(x) > 0
← POI
f″(x) < 0

* the below graphs show the progression of reaching to my final sketch of f(x)

d) $f(x)$

*I start by drawing a faint sketch based just on my f'(x) slope sign line

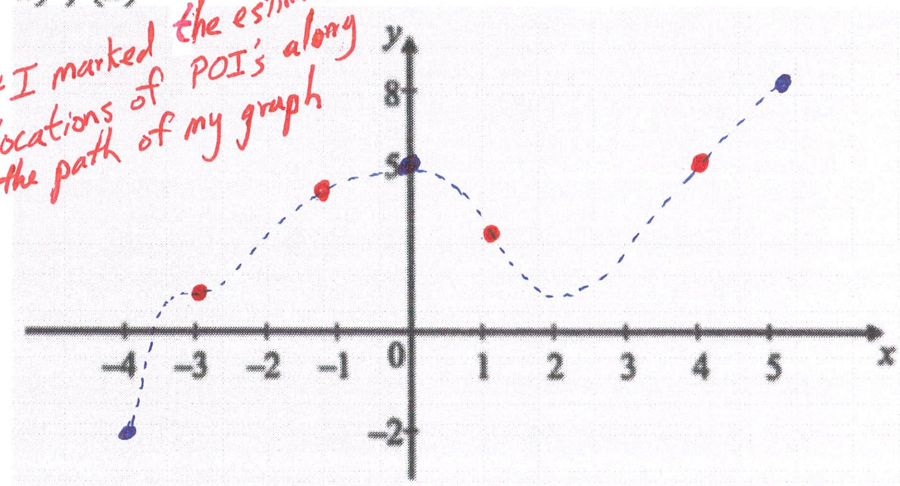


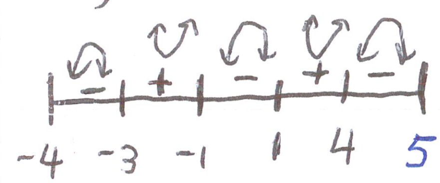


slope 0 occurs at x=-3,0,2

d) $f(x)$

*I marked the estimated locations of POI's along the path of my graph





POI's at x=-3,-1,1,4

d) $f(x)$

*connect the dots with the appropriate curvature indicated by f''(x) concavity sign line