Key

AP Calculus AB 2020 Mock AP Exam #4

1) 25 minutes (15 points)

The function g has derivative g′ where g′ is dereasing and twice-differentiable. Selected values of g′ are given in the table. It is given that g(1) = 2

2 1 6

x	1	3	4	10
g′(x)	9	7	5	0
g″(x)	4	1	2	6

2 a) What can we conclude using mean value theorem in the interval [1, 10]

2 b) *Use left Riemann Sum with 3 subintervals indicated in table to approximate* $\int_1^{10} g'(x)dx$

Is this an over or underapproximation of $\int_1^{10} g'(x)dx$ *? Provide support for your answer.*

3 c) *Evaluate* $\int_{-1}^{-3} g''(1-3x)dx$ *. Show the work that leads to your answer.*

2 d) *Evaluate* $\lim_{x\to 1} \frac{2e^{(9-g'(x))}-2}{g'(4x)-5}$ *Show work to support your answer.*

2 e) The function w *is defined by* $w(x) = 3x^2(g'(2x))$. *Find* $w'(2)$.

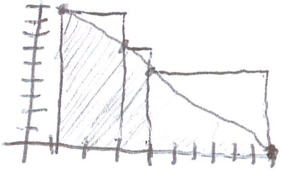
4 f) *Given the differential equation* $y' = (1-2y)g''(x)$. *Let* $y = k(x)$ *be the particular solution with initial condition of* $k(1) = 0$. *Then use expression to find* $k(3)$

MVT adapted from $g'(c) = \frac{g(b)-g(a)}{b-a}$

$\boxed{2}$ a) Since $g'(x)$ is differentiable on $(1,10)$ and continuous $[1,10]$ then by MVT, $g''(c) = \frac{g'(b)-g'(a)}{b-a}$

$g''(c) = \frac{g'(10)-g'(1)}{10-1} \rightarrow \frac{0-9}{10-1} = \frac{-9}{9} \rightarrow \boxed{g''(c) = -1 \text{ on } [1,10]}$

$\boxed{2}$ b) $\int_1^{10} g'(x)dx \approx 2(9) + 1(7) + 6(5) = \boxed{55}$



Since g' is decreasing on interval $[1,10]$ left Riemann Sum is an overapproximation of actual area.

$\boxed{3}$ c) $\int_{-1}^{-3} g''(\underline{1-3x})\underline{dx}$

$u = 1-3x$

$\frac{du}{dx} = -3$

$dx = \frac{du}{-3}$

$\int g''(u)\frac{du}{-3}$

* Apply U-substitution

* Apply FFTC

$\int_a^b g''(u)du = g'(b) - g'(a)$

*convert bounds

if $x=-1$, $u = 1-3x \rightarrow u = 1-3(-1) = 4$

if $x=-3$, $u = 1-3(-3) \rightarrow u = 1+9 = 10$

$\frac{-1}{3}\int_4^{10} g''(u)du \rightarrow \frac{-1}{3}[g'(10) - g'(4)]$

$\rightarrow \frac{-1}{3}[0-5] = \frac{-1}{3}(-5) = \boxed{\frac{5}{3}}$

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c) Evaluate $\int_{-1}^{-3} g''(1-3x)dx$. Show the work that leads to your answer.

d) Evaluate $\lim_{x\to1}\frac{2e^{(9-g'(x))}-2}{g'(4x)-5}$ Show work to support your answer.

e) The function w is defined by $w(x) = 3x^2(g'(2x))$. Find $w'(2)$

f) Given the differential equation $y' = (1-2y)g''(x)$. Let $y = k(x)$ be the particular solution with initial condition of $k(1) = 0$. Then use expression to find $k(3)$

$\boxed{2}$ d) $\lim_{x\to1}\frac{2e^{9-g'(x)}-2}{g'(4x)-5} = \frac{2e^{9-g'(1)}-2}{g'(4(1))-5} = \frac{2e^{9-9}-2}{5-5} = \frac{0}{0}$ → Apply L'Hopital's Rule $\lim_{x\to c}\frac{f'(c)}{g'(c)} = L$

plug in first

$\xrightarrow{L'H}\lim_{x\to1}\frac{2e^{9-g'(x)}\cdot g''(x)-0}{g''(4x)\cdot4-0} = \frac{2e^{9-g'(1)}\cdot g''(1)}{g''(4(1))\cdot4} = \frac{2e^{9-9}\cdot(4)}{2(4)} = \frac{2(4)}{2(4)} = \frac{8}{8} = \boxed{1}$

*Recall: $\frac{d}{dx}e^u = e^u\cdot u'$

$\boxed{2}$ e) $w(x) = \overbrace{3x^2}^{f}\cdot\overbrace{g'(2x)}^{g}$ → chain rule: out: $g'(u)$ in: $2x$

*Apply product rule

$w'(x) = \overbrace{6x}^{f'}\cdot\overbrace{g'(2x)}^{g} + \overbrace{3x^2}^{f}\cdot\overbrace{g''(2x)\cdot2}^{g'}$

$w'(2) = 6(2)\cdot g'(4) + 3(2)^2\cdot g''(4)\cdot2$

$w'(2) = 12(5) + 3(4)(2)(2)$

$w'(2) = 60 + 48$

$\boxed{w'(2) = 108}$

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f) *Given the differential equation* $y' = (1-2y)g''(x)$*. Let* $y = k(x)$ *be the particular solution with initial condition of* $k(1) = 0$*. Then use expression to find* $k(3)$

$\boxed{4}$ f) $\frac{dy}{dx} = (1-2y)g''(x)$

$dy = (1-2y)g''(x)dx$

$\frac{dy}{1-2y} = g''(x)dx$

$\int \frac{dy}{1-2y} = \int g''(x)dx$

$u = 1-2y$

$\frac{du}{dy} = -2$

$dy = \frac{du}{-2}$

$\int \frac{1}{u} \cdot \frac{du}{-2} = \int g''(x)dx$

$-\frac{1}{2}\int \frac{1}{u}du = \int g''(x)dx$

$-\frac{1}{2}\ln|u| = g'(x) + C$

$-\frac{1}{2}\ln|1-2y| = g''(x)+C$

plug in (1,0) to find C

$-\frac{1}{2}\ln|1-2(0)| = g''(1) + C$

$-\frac{1}{2}\ln|1| = 4 + C$

$0 = 4 + C$

$\underline{\underline{-4 = C}}$

plug in x=3 to solve for y: (finds k(3))

$-\frac{1}{2}\ln|1-2y| = g''(x) - 4$

$-\frac{1}{2}\ln|1-2y| = g''(3) - 4$

$-\frac{1}{2}\ln|1-2y| = 1 - 4$

$-2\left(-\frac{1}{2}\ln|1-2y| = -3\right)$

$\ln|1-2y| = 6$

$e^{\ln|1-2y|} = e^6$

$1-2y = e^6$

$1-e^6 = 2y$

$\frac{1}{2}(1-e^6) = y$

$$\boxed{k(3) = \frac{1}{2}(1-e^6) \text{ or } \frac{1-e^6}{2}}$$

2. 15 minutes (9 points)

The function f is continuous on the closed interval [-4,3].

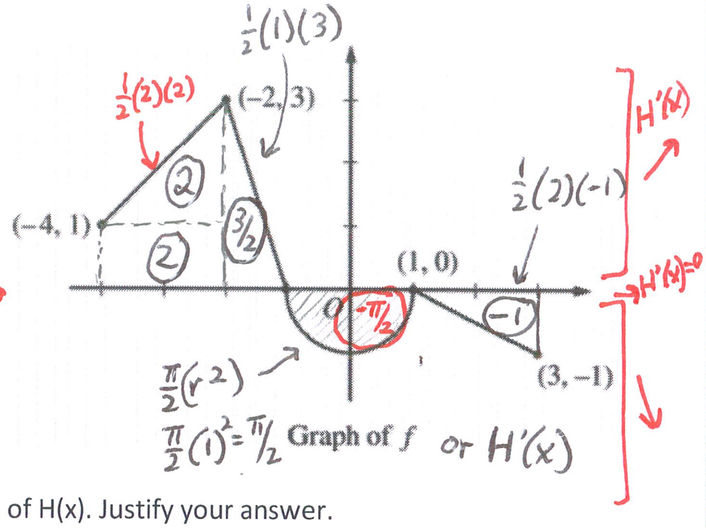
The graph of f consists of 3 line segments and semicircle.

H(x) is defined as $H(x) = \int_{-1}^{x} f(t)dt$

f(x) is derivative graph for H(x)

$$H'(x) = \frac{d}{dx}\int_{-1}^{x} f(t)dt = f(x)$$

SFTC →



Graph of *f* or H'(x)

2 a) Find the x-coordinate of each point of inflection for graph of H(x). Justify your answer.

3 b) Find the maximum value of H on the closed interval [-4, 3]. Justify your answer.

1 c) Find H''(2). Justify your answer.

2 d) Let p(x) be defined below: Is p continuous at x = 1? Show work leading to your answer.

$$p(x) = \begin{cases} f(x) + 1 & for\ x \le 1 \\ f(x+2) - f(x) & for\ x > 1 \end{cases}$$

1 e) For $-4 \le t \le 3$, a particle moves along the x-axis. The velocity v of the particle is represented by equation v(t) = f(t). Find the acceleration of the particle at $t = \frac{5}{2}$. Is the velocity of the particle increasing, decreasing, or neither. Justify your answer.

$\boxed{2}$ a) * Since f graph is the derivative of H, the points of inflections are the max/mins of derivative graph:

POI are at x = -2, x = 0, and x = 1 since H''(x) change signs.

$\boxed{3}$ b) * Apply EVT (test endpts and critical points) (just relative max)

H'(x): + on -4 to -1, − on -1 to 1, − on 1 to 3

$$H(-4) = \int_{-1}^{-4} f(t)dt = -\int_{-4}^{-1} f(t)dt = 2 + 2 + \frac{3}{2} = \boxed{5.5}$$

$$H(-1) = \int_{-1}^{-1} f(t)dt = \boxed{0}$$

$$H(3) = \int_{-1}^{3} f(t)dt = -\frac{\pi}{2} - 1 \approx \boxed{-2.5}$$

Absolute max of H(x) on [-4, 3] is 0 at x = -1

$\boxed{1}$ c) $H''(x) = f'(x)$

$H''(2) = f'(2)$

* slope of graph of f at x = 2 is $\frac{-1-0}{3-1} = \frac{-1}{2}$

$$\boxed{H''(2) = \frac{-1}{2}}$$

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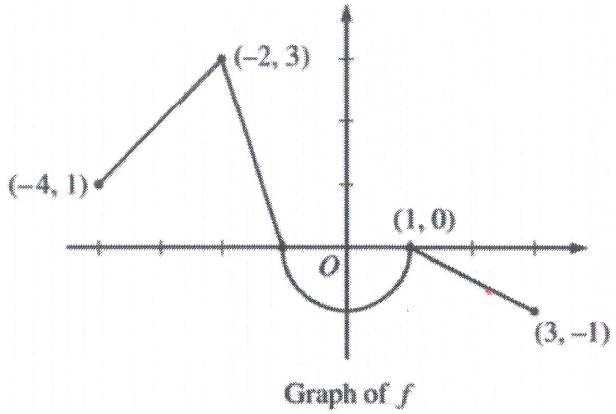

Graph of f

a) Find the x-coordinate of each point of inflection for graph of H(x). Justify your answer.

b) Find the maximum value of H on the closed interval [-4, 3]. Justify your answer.

c) Find H''(2). Justify your answer.

d) Let p(x) be defined below: Is p continuous at x = 1? Show work leading to your answer.

$$p(x) = \begin{cases} f(x) + 1 & for\ x \le 1 \\ f(x+2) - f(x) & for\ x > 1 \end{cases}$$

e) For $-4 \le t \le 3$, a particle moves along the x-axis. The velocity v of the particle is represented by equation v(t) = f(t). Find the acceleration of the particle at $t = \frac{5}{2}$. Is the velocity of the particle increasing, decreasing, or neither. Justify your answer.

This is only referring to acceleration a(t).

$\boxed{2}$

d) *Apply continuity conditions:

point exists → i) f(c) exists

limit exists → ii) $\lim_{x\to c} f(x)$ exists $\left[\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)\right]$

point and limit exists at same location → iii) $f(c) = \lim_{x\to c} f(x)$

i) $p(1) = f(1) + 1 = 0 + 1 = \boxed{1}$

ii) $\lim_{x\to 1^-} f(x) + 1 = f(1) + 1 = 0 + 1 = 1$

$\lim_{x\to 1^+} f(x+2) - f(x) = f(3) - f(1) = -1 - 0 = -1$

$\lim_{x\to 1} p(x)$ does not exist

p(x) is not continuous at x=1 since there is nonremovable discontinuity at x=1.

$\boxed{1}$

e) Since $v(t) = f(t)$ then

$a(t) = f'(t)$

$a(5/2) = f'(5/2) = \frac{-1-0}{3-1} = \boxed{-\frac{1}{2}}$

Since $a(5/2) < 0$, then velocity is decreasing since acceleration is negative.

*If problem said "is speed increasing or decreasing?", then we compare signs between v(t) and a(t)

In this case, since $v(5/2) < 0$ and $a(5/2) < 0$, they have same signs and therefore speed is increasing.