Key

AP Calculus AB **Trig Quiz Review WS #4**

Write down your 6 trig derivative rules below!

1) $\frac{d}{dx}\sin u = \cos u \cdot u'$

2) $\frac{d}{dx}\cos u = -\sin u \cdot u'$

3) $\frac{d}{dx}\tan u = \sec^2 u \cdot u'$

4) $\frac{d}{dx}\cot u = -\csc^2 u \cdot u'$

5) $\frac{d}{dx}\sec u = \sec u \tan u \cdot u'$

6) $\frac{d}{dx}\csc u = -\csc u \cot u \cdot u'$

1) If $y = \sqrt{tan^5\,(7 - \pi x^2\,)}$, find $\frac{dy}{dx}$

$y = \left[\tan(7-\pi x^2)\right]^{5/2}$

*chain Rule
out: $[\ \ ]^{5/2}$
in: $\tan(7-\pi x^2)$

$y' = \frac{5}{2}\left[\tan(7-\pi x^2)\right]^{3/2} \cdot \sec^2(7-\pi x^2) \cdot -2\pi x$

$$\boxed{y' = -5\pi x\left[\tan(7-\pi x^2)\right]^{3/2}\sec^2(7-\pi x^2)}$$

2) Given $y = \frac{\overbrace{\cot(x^5)}^{f}}{\underbrace{\sec(ex)}_{g}}$ Find y'

*quotient Rule

$$y' = \frac{\overbrace{-\csc^2(x^5)\cdot 5x^4}^{f'} \cdot \overbrace{\sec(ex)}^{g} - \overbrace{\cot(x^5)}^{f} \cdot \overbrace{\sec(ex)\tan(ex)\cdot e}^{g'}}{\underbrace{\sec^2(ex)}_{g^2}}$$

$$\boxed{y' = \frac{-5x^4\csc^2(x^5)\sec(ex) - e\cot(x^5)\sec(ex)\tan(ex)}{\sec^2(ex)}}$$

3. Given $y = \overbrace{-x}^{f}\overbrace{\csc(3-\pi x)}^{g}$ Find y'

*product Rule

$y' = \overbrace{-1}^{f'} \cdot \overbrace{\csc(3-\pi x)}^{g} + \overbrace{-x}^{f} \cdot \overbrace{-\csc(3-\pi x)\cot(3-\pi x)\cdot -\pi}^{g'}$

$$\boxed{y' = -\csc(3-\pi x) - \pi x\csc(3-\pi x)\cot(3-\pi x)}$$

4. $\overset{f}{y}\,\overset{g}{\cos y} = \csc y - y + 4x^2 - 5$ find $\frac{dy}{dx}$

* implicit differentiation
* product Rule

$$\underbrace{1\left(\frac{dy}{dx}\right)}_{f'} \cdot \underbrace{\cos(y)}_{g} + \underbrace{y}_{f} \cdot \underbrace{-\sin(y)\left(\frac{dy}{dx}\right)}_{g'} = -\csc y \cot y\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) + 8x - 0$$

$$\frac{dy}{dx}(\cos y) - y\sin y\left(\frac{dy}{dx}\right) + \csc y \cot y\left(\frac{dy}{dx}\right) + 1\left(\frac{dy}{dx}\right) = 8x$$

$$\frac{dy}{dx}\left(\cos y - y\sin y + \csc y\cot y + 1\right) = 8x$$

$$\boxed{\frac{dy}{dx} = \frac{8x}{\cos y - y\sin y + \csc y\cot y + 1}}$$

5) If the position of a particle is x(t) = 3cot x

a) Find a(t) * acceleration is a(t)

$$v(t) = -3\csc^2 x = -3[\csc x]^2$$

$a(t) =$ * chainRule
out: $-3[\ ]^2$
in: $\csc x$

$$a(t) = -6[\csc x] \cdot -\csc x \cot x(1)$$

$$\boxed{a(t) = 6[\csc x]^2 \cot x}$$

b) find acceleration at t = $\pi/4$

$\pi/4 \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$$a(\pi/4) = 6[\csc(\pi/4)]^2 \cot(\pi/4)$$

$$= 6\left[\frac{2}{\sqrt{2}}\right]^2[1]$$

$$= \boxed{12}$$

6. Find the tangent line equation for $f(x) = 2\cot^3(2x)$ at $x = \frac{\pi}{6}$

$$y = 2[\cot(2x)]^3$$

*chain Rule
out: $2[\ ]^3$
in: $\cot(2x)$

$$y' = 6[\cot(2x)]^2 \cdot -\csc^2(2x) \cdot 2$$

$$y' = -12[\cot(2x)]^2[\csc(2x)]^2$$

$$y'(\pi/6) = -12\left[\cot\left(2 \cdot \frac{\pi}{6}\right)\right]^2\left[\csc\left(2 \cdot \frac{\pi}{6}\right)\right]^2$$

$\frac{\pi}{3}\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$y'(\pi/6) = -12[\cot(\pi/3)]^2\left[\csc\left(\frac{\pi}{3}\right)\right]^2$$

$$= -12\left[\frac{1}{\sqrt{3}}\right]^2\left[\frac{2}{\sqrt{3}}\right]^2 = -12\left[\frac{1}{3}\right]\left[\frac{4}{3}\right]$$

$$y'(\pi/6) = -\frac{48}{9} = \boxed{-\frac{16}{3}}$$

$$y(\pi/6) = 2\left[\cot\left(2 \cdot \pi/6\right)\right]^3 = 2\left(\frac{1}{\sqrt{3}}\right)^3$$

$$y(\pi/6) = \boxed{\frac{2}{3\sqrt{3}}}$$

point: $\left(\pi/6, \frac{2}{3\sqrt{3}}\right)$

slope: $m = \frac{-16}{3}$

$$\boxed{y - \frac{2}{3\sqrt{3}} = \frac{-16}{3}\left(x - \frac{\pi}{6}\right)}$$