

Key

AP Calculus AB

Trig Quiz Review WS #5

Write down your 6 trig derivative rules below!

1) $\frac{d}{dx} \sin u = \cos u \cdot u'$

2) $\frac{d}{dx} \cos u = -\sin u \cdot u'$

3) $\frac{d}{dx} \tan u = \sec^2 u \cdot u'$

4) $\frac{d}{dx} \cot u = -\csc^2 u \cdot u'$

5) $\frac{d}{dx} \sec u = \sec u \tan u \cdot u'$

6) $\frac{d}{dx} \csc u = -\csc u \cot u \cdot u'$

1) If $y = -3x \sec(5 - \pi x^9)$ find $\frac{dy}{dx}$ *product Rule

$$y' = -3 \cdot \sec(5 - \pi x^9) + -3x \cdot \sec(5 - \pi x^9) \tan(5 - \pi x^9) \cdot -9\pi x^8$$

$$y' = -3 \sec(5 - \pi x^9) + 27\pi x^8 \sec(5 - \pi x^9) \tan(5 - \pi x^9)$$

2) Given $y = \frac{11 \cot(\pi x^5)}{6 \csc(ex)}$ Find y' *quotient Rule

$$y' = \frac{-11 \csc^2(\pi x^5) \cdot 5\pi x^4 \cdot 6 \csc(ex) - 11 \cot(\pi x^5) \cdot -6 \csc(ex) \cot(ex) \cdot e}{(6 \csc(ex))^2}$$

$$y' = \frac{-330\pi x^4 \csc^2(\pi x^5) \csc(ex) + 66e \cot(\pi x^5) \csc(ex) \cot(ex)}{36 \csc^2(ex)}$$

3. Given $y = -\frac{\sqrt[3]{\csc^{11}(3e^2 - \pi^2 x)}}{7}$ Find y'

$$y = -\frac{1}{7} [\csc(3e^2 - \pi^2 x)]^{11/3}$$

*chain Rule

out: $-\frac{1}{7} []^{11/3}$

in: $\csc(3e^2 - \pi^2 x)$

$$y' = -\frac{1}{7} \cdot \frac{11}{3} [\csc(3e^2 - \pi^2 x)]^{8/3} \cdot -\csc(3e^2 - \pi^2 x) \cot(3e^2 - \pi^2 x) \cdot -\pi^2$$

$$y' = \frac{-11\pi^2}{21} [\csc(3e^2 - \pi^2 x)]^{8/3} \csc(3e^2 - \pi^2 x) \cot(3e^2 - \pi^2 x)$$

4. $y \csc y = x \cot y - y + 9y^2 - 3x + 11\pi^2$ find $\frac{dy}{dx}$

* implicit differentiation
* product Rule

$$\left(\frac{dy}{dx}\right) \cdot \csc y + y \cdot -\csc y \cot y \left(\frac{dy}{dx}\right) = 1 \cdot \cot y + x \cdot -\csc^2 y \left(\frac{dy}{dx}\right) - 1 \left(\frac{dy}{dx}\right) + 18y \left(\frac{dy}{dx}\right) - 3 + 0$$

$$\csc y \left(\frac{dy}{dx}\right) - y \csc y \cot y \left(\frac{dy}{dx}\right) + x \csc^2 y \left(\frac{dy}{dx}\right) + 1 \left(\frac{dy}{dx}\right) - 18y \left(\frac{dy}{dx}\right) = \cot(y) - 3$$

$$\frac{dy}{dx} (\csc y - y \csc y \cot y + x \csc^2 y + 1 - 18y) = \cot(y) - 3$$

$$\frac{dy}{dx} = \frac{\cot y - 3}{\csc y - y \csc y \cot y + x \csc^2 y + 1 - 18y}$$

5) If the position of a particle is $x(t) = 7 \cot(5t)$

a) Find $a(t)$

$$v(t) = -7 \csc^2(5t) \cdot 5$$

$$v(t) = -35 \csc^2(5t)$$

* chain Rule

$$v(t) = -35 [\csc(5t)]^2$$

out: $-35 []^2$
in: $\csc(5t)$

$$a(t) = -70 [\csc(5t)] \cdot -\csc(5t) \cot(5t) \cdot 5$$

$$a(t) = 350 [\csc(5t)]^2 \cot(5t)$$

b) find acceleration at $t = \pi/6$

$$a(\pi/6) = 350 \left[\csc\left(\frac{5\pi}{6}\right) \right]^2 \cot\left(\frac{5\pi}{6}\right)$$

$$= 350 \left[\frac{2}{1} \right]^2 (-\sqrt{3})$$

$$= -1400\sqrt{3}$$

$\cos x \sin x$
* $\frac{5\pi}{6} \left(-\frac{\sqrt{3}}{2}, \frac{1}{2} \right)$

6. Find the tangent line equation for $f(x) = 2 \csc^3(7x)$ at $x = \frac{\pi}{6}$

$\frac{7\pi}{6} \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2} \right)$

$$f(x) = 2 [\csc(7x)]^3$$

* chain rule
out: $2 []^3$
in: $\csc(7x)$

$$f'(x) = 6 [\csc(7x)]^2 \cdot -\csc(7x) \cot(7x) \cdot 7$$

$$f'(x) = -42 [\csc(7x)]^3 \cot(7x)$$

$$f'\left(\frac{\pi}{6}\right) = -42 \left[\csc\left(\frac{7\pi}{6}\right) \right]^3 \cot\left(\frac{7\pi}{6}\right)$$

$$= -42 [-2]^3 (\sqrt{3}) = 336\sqrt{3}$$

$$f\left(\frac{\pi}{6}\right) = 2 \left[\csc\left(\frac{7\pi}{6}\right) \right]^3 = 2 [-2]^3 = -16$$

point: $\left(\frac{\pi}{6}, -16 \right)$

slope: $m = 336\sqrt{3}$

$$y + 16 = 336\sqrt{3} \left(x - \frac{\pi}{6} \right)$$