Name: ______________________________ Period: ____________

AB Calculus

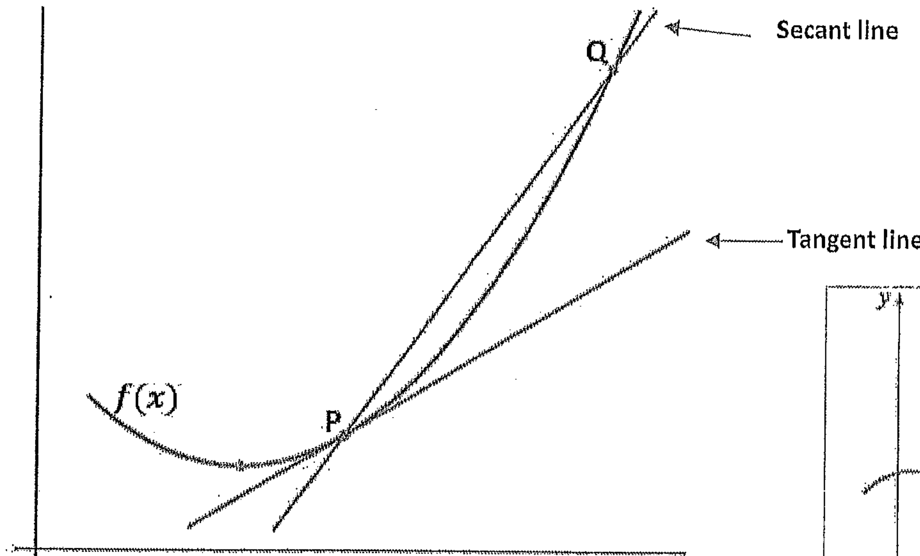
Unit 2

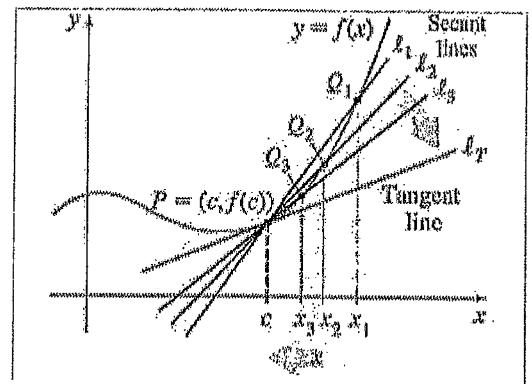
Differentiation Part 1

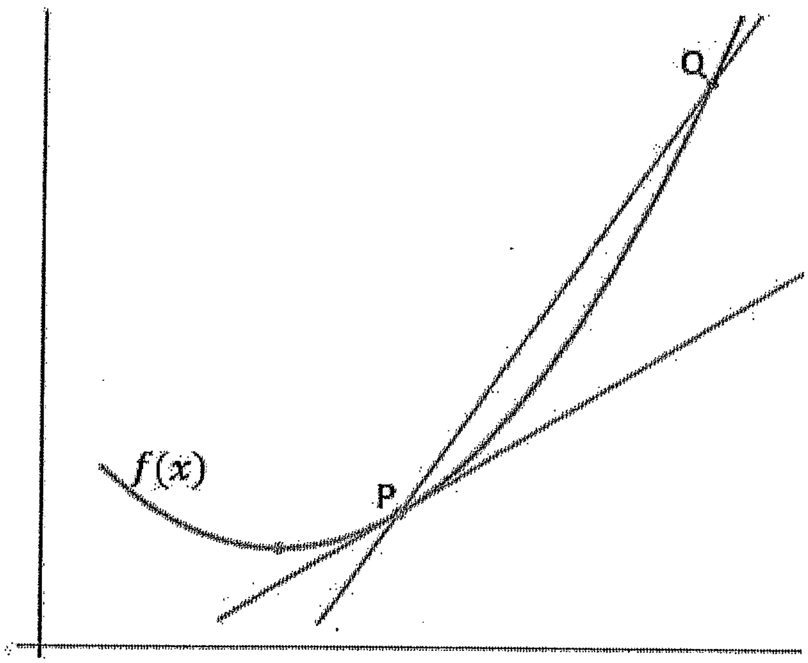
(Derivatives & Definition)

AP Calculus – 2.2 Notes - Limit Definition of a Derivative

Goal: To discover a formula to calculate the slope (steepness) of all tangent lines to a curved graph.







General Limit Definition of the Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Alternate Limit Definition of a derivative

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

f '(x) is "f prime of x": This is the notation for the derivative function.

Derivative is the slope (steepness) of a curve at a single point

*The derivative function is a **slope-finding formula** for a curved graph, where the slope is of the curve is ever-changing.

General Limit Definition of the Derivative:

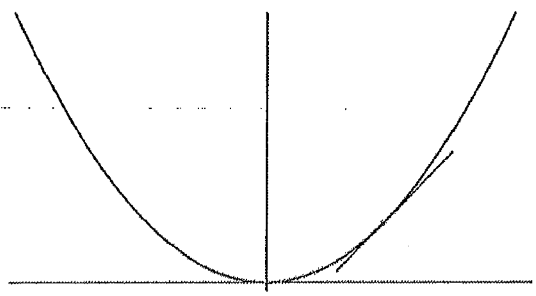
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Alternate Limit Definition of a derivative:

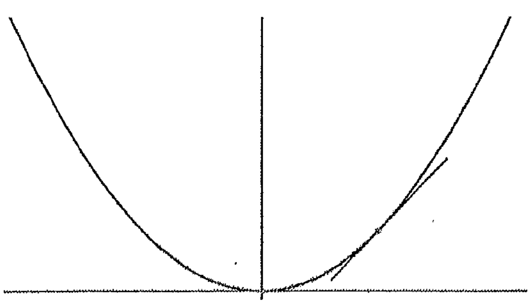
$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Example 1: (a) Find the general derivative of $f(x) = x^2$

(b) Write the equation of the tangent line to $f(x)$ at $x = 1$ (point-slope form: $y - y_1 = m(x - x_1)$)



(c) Write the equation of the tangent line to $f(x)$ at $x = -5$



To Recap:

* $f(x)$ is the **height-finding formula** (finds the y-value of graph at that point)

*Since $f(1) = 1$, this tells us that when x = 1, the height of the graph has a y-value of 1

*$f'(x)$ is the **slope-finding formula** for the $f(x)$ graph.

*Since $f'(1) = 2$, this tells us that when x = 1, the slope of the tangent line to $f(x)$ has a slope (steepness) of 2.

General Limit Definition of the Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Alternate Limit Definition of a derivative:

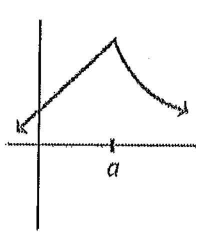
$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Example 2: (a) Find the general derivative of $f(x) = \sqrt{x}$

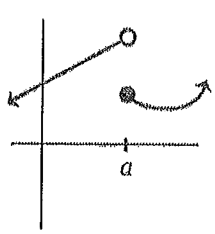
(b) Write the equation of the tangent line to $f(x)$ at $x = 2$ (point-slope form: $y - y_1 = m(x - x_1)$)

Example 3: Use the alternative derivative definition to find slope of $f(x) = \sqrt{x}$ at x = 2.

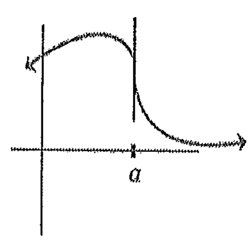
Differentiability: In order for a function to be **differentiable** (smooth curve) at a point a, then the graph must be continuous at that point, cannot contain a sharp turn & cannot have a vertical tangent at the point.



Cusp / Corner



Discontinuous



Vertical Tangent

④

Classwork Examples:

Find the derivative using limits

General Limit Definition of the Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

1. $f(x) = 7 - 6x$

2. $y = 5x^2 - x$

3. $y = \sqrt{5x + 2}$

4. $f(x) = \frac{1}{x-2}$

5

1) Use the Limit Definition of a derivative to find f'(x) if f(x) = $2x^2 - 3x + 1$

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

2) Use the Alternative definition of the derivative to find $f'(2)$ if $f(x) = \sqrt{2-x}$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

3) Use the Limit Definition of a Derivative to find f'(x) if $f(x) = \sqrt{2x-1}$

(6) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

4) Use the Limit Definition of a derivative to find f '(3) if $f(x) = \frac{2}{5-x}$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2x - 3x^2$ at $x = -1$. $y - y_1 = m(x - x_1)$

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EXAMPLE 8 **Approximating the Derivative of a Function Defined by a Table**

The table below lists several values of a function $y = f(x)$ that is continuous on the interval $[-1, 5]$ and has a derivative at each number in the interval $(-1, 5)$. Approxima the derivative of f at 2.

x	0	1	2	3	4
$f(x)$	0	3	12	33	72

Solution

There are several ways to approximate the derivative of a function defined by a tab Each uses an average rate of change to approximate the rate of change of f at 2, whi is the derivative of f at 2.

- Using the average rate of change from 2 to 3, we have

$$\frac{f(3) - f(2)}{3 - 2} = \frac{33 - 12}{1} = 21$$

With this choice, $f'(2)$ is approximately 21.

- Using the average rate of change from 1 to 2, we have

$$\frac{f(2) - f(1)}{2 - 1} = \frac{12 - 3}{1} = 9$$

With this choice, $f'(2)$ is approximately 9.

- A third approximation can be found by averaging the above two approximations. Then $f'(2)$ is approximately $\frac{21 + 9}{2} = 15$. ■

NOW WORK Problem 51 and AP® Practice Problem 8.

2.1 Assess Your Understanding

Concepts and Vocabulary

1. *True or False* The derivative is used to find instantaneous velocity.
2. *True or False* The derivative can be used to find the rate of change of a function.
3. The notation $f'(c)$ is read f ____ of c; $f'(c)$ represents the ____ of the tangent line to the graph of f at the point ____.
4. *True or False* If it exists, $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$ is the derivative of the function f at 3.
5. If $f(x) = 6x - 3$, then $f'(3) =$ ________.
6. The velocity of an object, the slope of a tangent line, and the rate of change of a function are three different interpretations of the mathematical concept called the ________.

Skill Building

In Problems 7–16,

(a) *Find an equation for the tangent line to the graph of each function at the indicated point.*

(b) *Find an equation of the normal line to each function at the indicated point.*

(c) *Graph the function, the tangent line, and the normal line at the indicated point on the same set of coordinate axes.*

7. $f(x) = 3x^2$ at $(-2, 12)$
8. $f(x) = x^2 + 2$ at $(-1, 3)$
9. $f(x) = x^3$ at $(-2, -8)$
10. $f(x) = x^3 + 1$ at $(1, 2)$
11. $f(x) = \frac{1}{x}$ at $(1, 1)$
12. $f(x) = \sqrt{x}$ at $(4, 2)$
13. $f(x) = \frac{1}{x+5}$ at $\left(1, \frac{1}{6}\right)$
14. $f(x) = \frac{2}{x+4}$ at $\left(1, \frac{2}{5}\right)$
15. $f(x) = \frac{1}{\sqrt{x}}$ at $(1, 1)$
16. $f(x) = \frac{1}{x^2}$ at $(1, 1)$

In Problems 17–20, find the rate of change of f at the indicated numbers.

17. $f(x) = 5x - 2$ at (a) $c = 0$, (b) $c = 2$
18. $f(x) = x^2 - 1$ at (a) $c = -1$, (b) $c = 1$
19. $f(x) = \frac{x^2}{x+3}$ at (a) $c = 0$, (b) $c = 1$
20. $f(x) = \frac{x}{x^2 - 1}$ at (a) $c = 0$, (b) $c = 2$

In Problems 21–30, find the derivative of each function at the given number.

21. $f(x) = 2x + 3$ at 1
22. $f(x) = 3x - 5$ at 2
23. $f(x) = x^2 - 2$ at 0
24. $f(x) = 2x^2 + 4$ at 1
25. $f(x) = 3x^2 + x + 5$ at -1
26. $f(x) = 2x^2 - x - 7$ at -1
27. $f(x) = \sqrt{x}$ at 4
28. $f(x) = \frac{1}{x^2}$ at 2
29. $f(x) = \frac{2 - 5x}{1 + x}$ at 0
30. $f(x) = \frac{2 + 3x}{2 + x}$ at 1

8

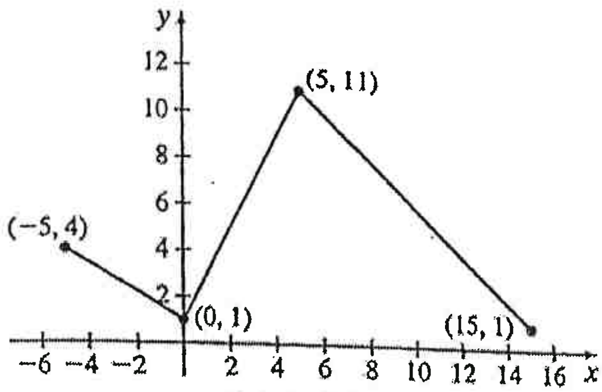
2.1 AP Practice Problems (p.171) – Rates of Change and the Derivative

1. The line $x + y = 5$ is tangent to the graph of $y = f(x)$ at the point where $x = 2$. The values $f(2)$ and $f'(2)$ are:

(A) $f(2) = 2;\ f'(2) = -1$ (B) $f(2) = 3;\ f'(2) = -1$

(C) $f(2) = 2;\ f'(2) = 1$ (D) $f(2) = 3;\ f'(2) = 2$

2. The graph of the function f, given below, consists of three line segments. Find $f'(3)$.



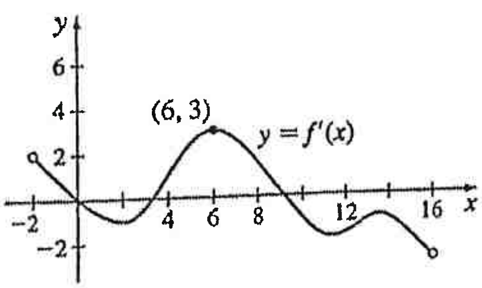
Graph of f

(A) 1 (B) 2 (C) 3 (D) $f'(3)$ does not exist

3. What is the instantaneous rate of change of the function $f(x) = 3x^2 + 5$ at $x = 2$?

(A) 5 (B) 7 (C) 12 (D) 17

4. The function f is defined on the closed interval $[-2, 16]$. The graph of the derivative of f, $y = f'(x)$, is given below.



The point $(6, -2)$ is on the graph of $y = f(x)$. An equation of the tangent line to the graph of f at $(6, -2)$ is

(A) $y = 3$ (B) $y + 2 = 6(x + 3)$

(C) $y + 2 = 6x$ (D) $y + 2 = 3(x - 6)$

5. If $x - 3y = 13$ is an equation of the normal line to the graph of f at the point (2, 6), then $f'(2) =$

(A) $-\frac{1}{3}$ (B) $\frac{1}{3}$ (C) -3 (D) $-\frac{13}{3}$

6. If f is a function for which $\lim_{x \to -3} \frac{f(x) - f(-3)}{x + 3} = 0$, then which of the following statements must be true?

(A) $x = -3$ is a vertical asymptote of the graph.
(B) The derivative of f at $x = -3$ exists.
(C) The function f is continuous at $x = 3$.
(D) f is not defined at $x = -3$.

7. If the position of an object on the x-axis at time t is $4t^2$, then the average velocity of the object over the interval $0 \leq t \leq 5$ is

(A) 5 (B) 20 (C) 40 (D) 100

8. A tank is filled with 80 liters of water at 7 a.m. ($t = 0$). Over the next 12 hours the water is continuously used and no water is added to replace it. The table below gives the amount of water $A(t)$ (in liters) remaining in the tank at selected times t, where t measures the number of hours after 7 a.m.

t	0	2	5	7	9	12
$A(t)$	80	71	66	60	54	50

Use the table to approximate $A'(5)$.

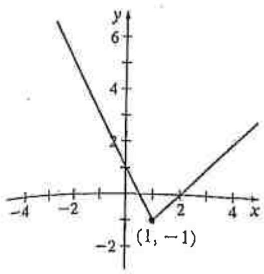


Figure 22 $g(x)=\begin{cases}1-2x & \text{if } x\le 1\\ x-2 & \text{if } x>1\end{cases}$

(b) See Figure 22. The function g is continuous at 1, which you should verify. To determine whether g is differentiable at 1, examine the one-sided limits at 1 using Form (1).

For $x<1$,

$$\lim_{x\to 1^-}\frac{g(x)-g(1)}{x-1}=\lim_{x\to 1^-}\frac{(1-2x)-(-1)}{x-1}=\lim_{x\to 1^-}\frac{2-2x}{x-1}$$
$$=\lim_{x\to 1^-}\frac{-2(x-1)}{x-1}=\lim_{x\to 1^-}(-2)=-2$$

For $x>1$,

$$\lim_{x\to 1^+}\frac{g(x)-g(1)}{x-1}=\lim_{x\to 1^+}\frac{(x-2)-(-1)}{x-1}=\lim_{x\to 1^+}\frac{x-1}{x-1}=1$$

The one-sided limits are not equal, so $\lim_{x\to 1}\frac{g(x)-g(1)}{x-1}$ does not exist. That is, g is not differentiable at 1. ■

Notice in Figure 21 the tangent lines to the graph of f turn smoothly around the origin. On the other hand, notice in Figure 22 the tangent lines to the graph of g change abruptly at the point $(1,-1)$, where the graph of g has a corner.

NOW WORK Problem 41 and AP® Practice Problems 3, 4, 6, and 7.

2.2 Assess Your Understanding

Concepts and Vocabulary

1. *True or False* The domain of a function f and the domain of its derivative function f' are always equal.
2. *True or False* If a function is continuous at a number c, then it is differentiable at c.
3. *Multiple Choice* If f is continuous at a number c and if $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ is infinite, then the graph of f has [(a) a horizontal (b) a vertical (c) no] tangent line at c.
4. The instruction, "Differentiate f," means to find the ______ of f.

Skill Building

In Problems 5–10, find the derivative of each function f at any real number c. Use Form (1) on page 171.

5. $f(x)=10$ 6. $f(x)=-4$ 7. $f(x)=2x+3$

8. $f(x)=3x-5$ 9. $f(x)=2-x^2$ 10. $f(x)=2x^2+4$

In Problems 11–16, differentiate each function f and determine the domain of f′. Use Form (2) on page 172.

11. $f(x)=5$ 12. $f(x)=-2$

13. $f(x)=3x^2+x+5$ 14. $f(x)=2x^2-x-7$

15. $f(x)=5\sqrt{x-1}$ 16. $f(x)=4\sqrt{x+3}$

In Problems 17–22, differentiate each function f. Graph y = f(x) and y = f′(x) on the same set of coordinate axes.

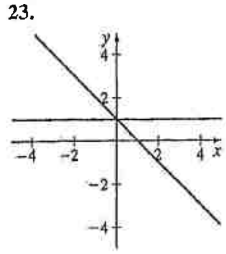
17. $f(x)=\frac{1}{3}x+1$ 18. $f(x)=-4x-5$

19. $f(x)=2x^2-5x$ 20. $f(x)=-3x^2+2$

21. $f(x)=x^3-8x$ 22. $f(x)=-x^3-8$

In Problems 23–26, for each figure determine if the graphs represent a function f and its derivative f′. If they do, indicate which is the graph of f and which is the graph of f′.

23.



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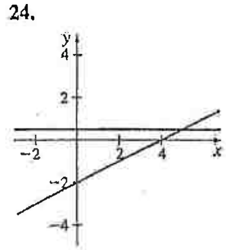

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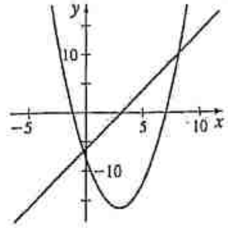

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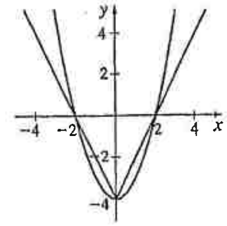

In Problems 27–30, use the graph of f to obtain the graph of f′.

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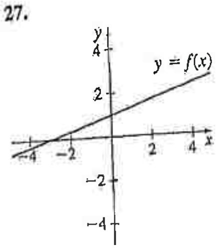

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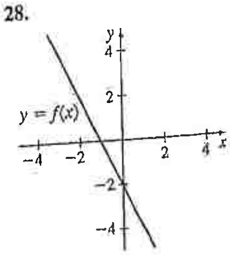
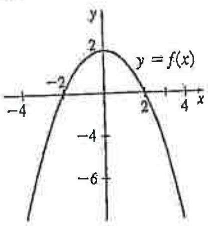


PAGE 175 **29.**



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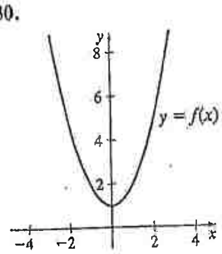
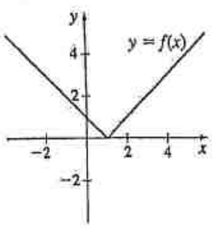


In Problems 31–34, the graph of a function f is given. Match each graph to the graph of its derivative f′ in A–D.

31.



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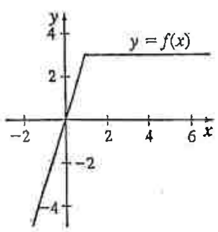

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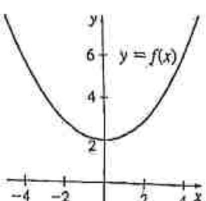

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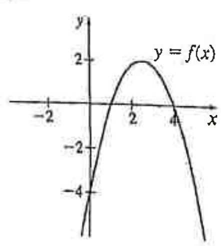
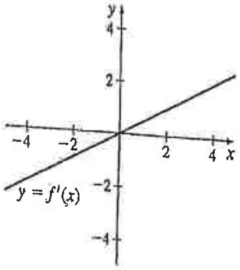
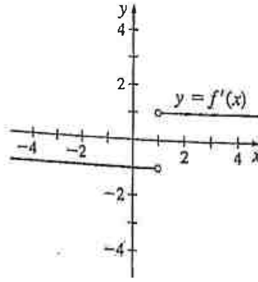
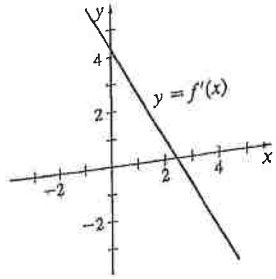




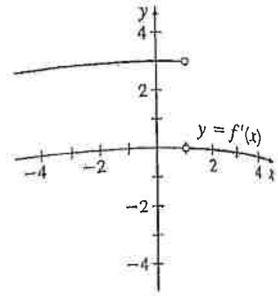
(A)



(B)



(C)



(D)

In Problems 35–44, determine whether each function f has a derivative at c. If it does, what is $f'(c)$? If it does not, give the reason why.

PAGE 176 **35.** $f(x) = x^{2/3}$ at $c = -8$

36. $f(x) = 2x^{1/3}$ at $c = 0$

37. $f(x) = |x^2 - 4|$ at $c = 2$

38. $f(x) = |x^2 - 4|$ at $c = -2$

PAGE 176 **39.** $f(x) = \begin{cases} 2x + 3 & \text{if } x < 1 \\ x^2 + 4 & \text{if } x \geq 1 \end{cases}$ at $c = 1$

40. $f(x) = \begin{cases} 3 - 4x & \text{if } x < -1 \\ 2x + 9 & \text{if } x \geq -1 \end{cases}$ at $c = -1$

PAGE 179 **41.** $f(x) = \begin{cases} -4 + 2x & \text{if } x \leq \frac{1}{2} \\ 4x^2 - 4 & \text{if } x > \frac{1}{2} \end{cases}$ at $c = \frac{1}{2}$

42. $f(x) = \begin{cases} 2x^2 + 1 & \text{if } x < -1 \\ -1 - 4x & \text{if } x \geq -1 \end{cases}$ at $c = -1$

PAGE 178 **43.** $f(x) = \begin{cases} 2x^2 + 1 & \text{if } x < -1 \\ 2 + 2x & \text{if } x \geq -1 \end{cases}$ at $c = -1$

44. $f(x) = \begin{cases} 5 - 2x & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$ at $c = 2$

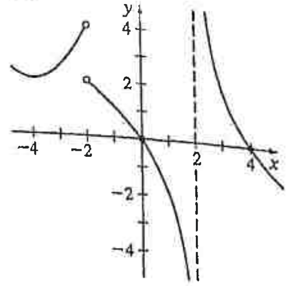
In Problems 45–48, each function f is continuous for all real numbers, and the graph of $y = f'(x)$ is given.

(a) *Does the graph of f have any horizontal tangent lines? If yes, explain why and identify where they occur.*

(b) *Does the graph of f have any vertical tangent lines? If yes, explain why, identify where they occur, and determine whether the point is a cusp of f.*

(c) *Does the graph of f have any corners? If yes, explain why and identify where they occur.*

PAGE 177 **45.**



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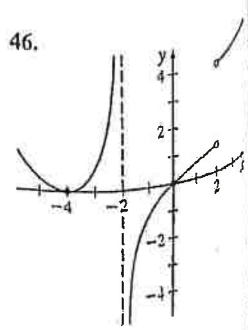

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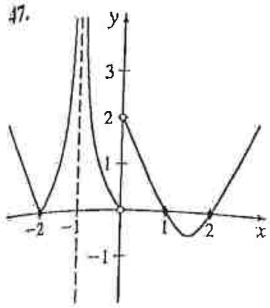
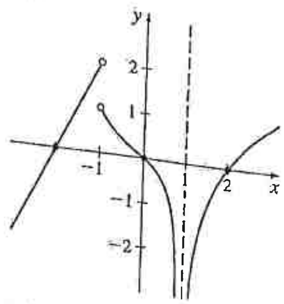


48.

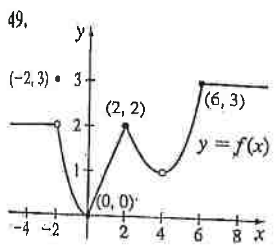


In Problems 49 and 50, use the given points $(c, f(c))$ on the graph of the function f.

(a) *For which numbers c does $\lim_{x \to c} f(x)$ exist but f is not continuous at c?*

(b) *For which numbers c is f continuous at c but not differentiable at c?*

49.



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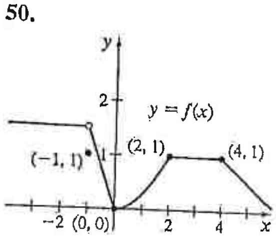


In Problems 51–54, find the derivative of each function.

51. $f(x) = mx + b$

52. $f(x) = ax^2 + bx + c$

53. $f(x) = \dfrac{1}{x^2}$

54. $f(x) = \dfrac{1}{\sqrt{x}}$

Applications and Extensions

In Problems 55–66, each limit represents the derivative of a function f at some number c. Determine f and c in each case.

55. $\lim_{h \to 0} \dfrac{(2+h)^2 - 4}{h}$

56. $\lim_{h \to 0} \dfrac{(2+h)^3 - 8}{h}$

57. $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

58. $\lim_{x \to 1} \dfrac{x^4 - 1}{x - 1}$

59. $\lim_{h \to 0} \dfrac{\sqrt{9+h} - 3}{h}$

60. $\lim_{h \to 0} \dfrac{(8+h)^{1/3} - 2}{h}$

61. $\lim_{x \to \pi/6} \dfrac{\sin x - \frac{1}{2}}{x - \frac{\pi}{6}}$

62. $\lim_{x \to \pi/4} \dfrac{\cos x - \frac{\sqrt{2}}{2}}{x - \frac{\pi}{4}}$

63. $\lim_{x \to 0} \dfrac{2(x+2)^2 - (x+2) - 6}{x}$

64. $\lim_{x \to 0} \dfrac{3x^3 - 2x}{x}$

65. $\lim_{h \to 0} \dfrac{(3+h)^2 + 2(3+h) - 15}{h}$

66. $\lim_{h \to 0} \dfrac{3(h-1)^2 + h - 3}{h}$

67. **Units** The volume V (in cubic feet) of a balloon is expanding according to $V = V(t) = 4t$, where t is the time (in seconds). Find the rate of change of the volume of the balloon with respect to time. What are the units of $V'(t)$?

68. **Units** The area A (in square miles) of a circular patch of oil is expanding according to $A = A(t) = 2t$, where t is the time (in hours). At what rate is the area changing with respect to time? What are the units of $A'(t)$?

69. **Units** A manufacturer of precision digital switches has a daily cost C (in dollars) of $C(x) = 10{,}000 + 3x$, where x is the number of switches produced daily. What is the rate of change of cost with respect to x? What are the units of $C'(x)$?

70. **Units** A manufacturer of precision digital switches has daily revenue R (in dollars) of $R(x) = 5x - \dfrac{x^2}{2000}$, where x is the number of switches produced daily. What is the rate of change of revenue with respect to x? What are the units of $R'(x)$?

71. $f(x) = \begin{cases} x^3 & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$

(a) Determine whether f is continuous at 0.
(b) Determine whether $f'(0)$ exists.
(c) Graph the function f and its derivative f'.

72. For the function $f(x) = \begin{cases} 2x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$

(a) Determine whether f is continuous at 0.
(b) Determine whether $f'(0)$ exists.
(c) Graph the function f and its derivative f'.

73. **Velocity** The distance s (in feet) of an automobile from the origin at time t (in seconds) is given by the position function

$$s = s(t) = \begin{cases} t^3 & \text{if } 0 \le t < 5 \\ 125 & \text{if } t \ge 5 \end{cases}$$

(This could represent a crash test in which a vehicle is accelerated until it hits a brick wall at $t = 5$ s.)

(a) Find the velocity just before impact (at $t = 4.99$ s) and just after impact (at $t = 5.01$ s).
(b) Is the velocity function $v = s'(t)$ continuous at $t = 5$?
(c) How do you interpret the answer to (b)?

74. **Population Growth** A simple model for population growth states that the rate of change of population size P with respect to time t is proportional to the population size. Express this statement as an equation involving a derivative.

75. **Atmospheric Pressure** Atmospheric pressure p decreases as the distance x from the surface of Earth increases, and the rate of change of pressure with respect to altitude is proportional to the pressure. Express this law as an equation involving a derivative.

76. **Electrical Current** Under certain conditions, an electric current I will die out at a rate (with respect to time t) that is proportional to the current remaining. Express this law as an equation involving a derivative.

77. **Tangent Line** Let $f(x) = x^2 + 2$. Find all points on the graph of f for which the tangent line passes through the origin.

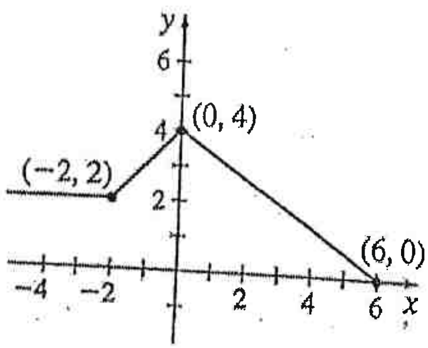
78. **Tangent Line** Let $f(x) = x^2 - 2x + 1$. Find all points on the graph of f for which the tangent line passes through the point $(1, -1)$.

2.2 AP Practice Problems (p.182) – Derivative as a function & differentiability

1. The function $f(x) = \begin{cases} x^2 - ax & \text{if } x \le 1 \\ ax + b & \text{if } x > 1 \end{cases}$, where a and b are constants. If f is differentiable at $x = 1$, then $a + b =$

(A) -3 (B) -2 (C) 0 (D) 2

2. The graph of the function f, given below, consists of three line segments. Find $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$.



(A) -1 (B) $-\frac{2}{3}$ (C) $-\frac{3}{2}$ (D) does not exist

3. If $f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & \text{if } x \ne 5 \\ 5 & \text{if } x = 5 \end{cases}$

which of the following statements about f are true?

I. $\lim_{x \to 5} f$ exists.
II. f is continuous at $x = 5$.
III. f is differentiable at $x = 5$.

(A) I only (B) I and II only
(C) I and III only (D) I, II, and III

(14)

4. Suppose f is a function that is differentiable on the open interval $(-2, 8)$. If $f(0) = 3$, $f(2) = -3$, and $f(7) = 3$, which of the following must be true?

I. f has at least 2 zeros.

II. f is continuous on the closed interval $[-1, 7]$.

III. For some c, $0 < c < 7$, $f(c) = -2$.

(A) I only (B) I and II only

(C) II and III only (D) I, II, and III

5. If $f(x) = |x|$, which of the following statements about f are true?

I. f is continuous at 0.

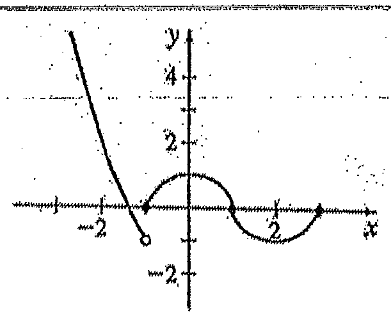
II. f is differentiable at 0.

III. $f(0) = 0$.

(A) I only (B) III only

(C) I and III only (D) I, II, and III

6. The graph of the function f shown in the figure has horizontal tangent lines at the points (0, 1) and (2, −1) and a vertical tangent line at the point (1, 0). For what numbers x in the open interval (−2, 3) is f not differentiable?



(A) −1 only (B) −1 and 1 only

(C) −1, 0, and 2 only (D) −1, 0, 1, and 2

7. Let f be a function for which $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = -3$.
Which of the following must be true?

I. f is continuous at 1.
II. f is differentiable at 1.
III. f' is continuous at 1.

(A) I only (B) II only
(C) I and II only (D) I, II, and III

8. At what point on the graph of $f(x) = x^2 - 4$ is the tangent line parallel to the line $6x - 3y = 2$?

(A) $(1, -3)$ (B) $(1, 2)$ (C) $(2, 0)$ (D) $(2, 4)$

9. At $x = 2$, the function $f(x) = \begin{cases} 4x + 1 & \text{if } x \leq 2 \\ 3x^2 - 3 & \text{if } x > 2 \end{cases}$ is

(A) Both continuous and differentiable.
(B) Continuous but not differentiable.
(C) Differentiable but not continuous.
(D) Neither continuous nor differentiable.

(16)

10. Oil is leaking from a tank. The amount of oil, in gallons, in the tank is given by $G(t) = 4000 - 3t^2$, where t, $0 \leq t \leq 24$ is the number of hours past midnight.

(a) Find $G'(5)$ using the definition of the derivative.

(b) Using appropriate units, interpret the meaning of $G'(5)$ in the context of the problem.

11. A rod of length 12 cm is heated at one end. The table below gives the temperature $T(x)$ in degrees Celsius at selected numbers x cm from the heated end.

x	0	2	5	7	9	12
$T(x)$	80	71	66	60	54	50

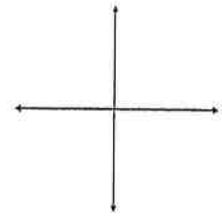
(a) Use the table to approximate $T'(8)$.

(b) Using appropriate units, interpret $T'(8)$ in the context of the problem.

AP Calculus – 2.3 Notes – Derivatives of Polynomials (Power Rule)

1. Constant Rule: If $f(x) = c$, then $f'(x) = 0$

   Example: $f(x) = 5$



2. Power Rule: If $f(x) = x^n$, then $f'(x) = n * x^{n-1}$

   Steps $a)$ Bring Exponent down as coefficient in front of the variable

   $b)$ Subtract 1 from the original exponent value

Power Rule Conditions:

$i)$ Convert radicals to rational exponents (ex: $\sqrt{x^5} = x^{\frac{5}{2}}$)

$ii)$ Bring variable to the numerator before applying power rule

$iii)$ Expand terms: resolve parentheses & fractional terms before applying Power Rule

**Important Note*: Be sure the function is in the appropriate form (all conditions met!) before applying Power Rule

Example 1: Find Derivatives of the following:

a) $y = x^7$

$b)$ $g(x) = \sqrt[3]{x}$

$c)$ $y = \dfrac{4}{x^5}$

$d)$ $y = 8x^{2/3} - \sqrt[5]{x} + \dfrac{2}{\sqrt{x}} + 0.875$

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Example 2: If $f(x) = \frac{1}{x^2}$ $\quad find\ f'(2)$

Example 3: $If\ f(x) = \sqrt[3]{x^2}$ $,write\ the\ tangent\ line\ equation\ to\ f(x) at\ x = 1$

Example 4: Find $f'(x)$ if $f(x) = \frac{x^4 - 3x^2 + 4(\sqrt[3]{x})}{\sqrt{x}}$

Example 5: Find $f'(x)$ if $f(x) = 3x(x+1)^2$

2.3 Derivative Power Rule Practice/Review Worksheet

Derivative Power Rule:

$$\frac{d}{dx}x^n = n * x^{n-1}$$

Power Rule Conditions:

i) All Radicals converted to Rational Exponents
ii) All denominator variables brought up to the numerator
iii) All parentheses resolved, all terms expanded

Finding a Derivative use the rules of differentiation to find the derivative of the function.

1) $y = x^7$

2) $y = \frac{1}{x^5}$

3) $y = \frac{3}{x^7}$

4) $f(x) = \sqrt[5]{x}$

5) $f(t) = -2t^2 + 3t - 6$

6) $y = \frac{5}{2x^2}$

7) $y = \frac{3}{2x^4}$

8) $y = \frac{6}{(5x)^3}$

(10)

Derivative Power Rule:
$\frac{d}{dx}x^n = n * x^{n-1}$

Power Rule Conditions:
i) All Radicals converted to Rational Exponents
ii) All denominator variables brought up to the numerator
iii) All parentheses resolved, all terms expanded

Find the derivative of the functions below:

9) $g(t) = t^2 - \frac{4}{t^3}$

10) $f(x) = \frac{4x^3 + 3x^2}{x}$

11) $f(x) = \frac{2x^4 - x}{x^3}$

12) $y = x^2(2x^2 - 3x)$

13) $f(x) = \sqrt{x} - 6\sqrt[3]{x}$

14) $f(t) = t^{2/3} - t^{1/3} + 4$

Finding an Equation of a Tangent Line In Exercises (a) find an equation of the tangent line to the graph of f at the given point.

Equation of tangent line:
i) Find ordered pair $((x_1, y_1)$ using $f(x)$
ii) Find slope m using $f'(x)$
iii) $y - y_1 = m(x - x_1)$

15) $y = x^4 - 3x^2 + 2$ $(1, 0)$

16) $y = x^3 - 3x$ $(2, 2)$

2.3 Derivative Power Rule Practice/Review Worksheet #2

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Derivative Power Rule:

$$\frac{d}{dx}x^n = n * x^{n-1}$$

Power Rule Conditions:

i) All Radicals converted to Rational Exponents
ii) All denominator variables brought up to the numerator
iii) All parentheses resolved, all terms expanded

Finding a Derivative **In Exercises 3–24, use the rules of differentiation to find the derivative of the function.**

1) $f(x) = 3x^5 - 4x + 156$

2) $f(x) = \dfrac{5}{3x^6}$

3) $g(x) = 3\sqrt{x^9}$

4) $f(x) = \dfrac{\sqrt{x^9}}{3}$

5) $h(t) = \dfrac{7}{5(2t)^3}$

6) $f(t) = \dfrac{7}{5(2t)^3}$

7) $f(x) = \dfrac{7}{x\sqrt{x}}$

8) $f(x) = 5\sqrt{x} - 3x^2(2 - 5x)$

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Derivative Power Rule:
$\frac{d}{dx}x^n = n * x^{n-1}$

Power Rule Conditions:

i) All Radicals converted to Rational Exponents
ii) All denominator variables brought up to the numerator
iii) All parentheses resolved, all terms expanded

Find the derivative of the functions below:

9) $f(x) = x(2-5x)^2$

10) $f(x) = \dfrac{5x^4 - 3x + 1}{x^2}$

11) $f(x) = \dfrac{3x^4 - 2x + 1}{\sqrt{x}}$

12) $f(x) = \dfrac{2x^3 - 4x^2 + 5}{\sqrt{x}}$

Finding an Equation of a Tangent Line In Exercises 53–56, (a) find an equation of the tangent line to the graph of f at the given point.

Equation of tangent line:

i) Find ordered pair (x_1, y_1) using $f(x)$
ii) Find slope m using $f'(x)$
iii) $y - y_1 = m(x - x_1)$

13) $f(x) = \dfrac{2}{\sqrt[4]{x^3}}$ $(1, 2)$

14) $y = (x-2)(x^2+3x)$ $(1, -4)$

Since $\frac{d}{dx}a^x = f'(0) \cdot a^x$, if $f(x) = e^x$, then $\frac{d}{dx}e^x = f'(0) \cdot e^x = 1 \cdot e^x = e^x$.

THEOREM Derivative of the Exponential Function $y = e^x$

The derivative of the exponential function $y = e^x$ is

$$y' = \frac{d}{dx}e^x = e^x \quad (1)$$

EXAMPLE 7 Differentiating an Expression Involving $y = e^x$

Find the derivative of $f(x) = 4e^x + x^3$.

Solution

The function f is the sum of $4e^x$ and x^3. Then

$$f'(x) = \frac{d}{dx}(4e^x + x^3) \underset{\text{Sum Rule}}{=} \frac{d}{dx}(4e^x) + \frac{d}{dx}x^3 \underset{\text{Constant Multiple Rule; Simple Power Rule}}{=} 4\frac{d}{dx}e^x + 3x^2 \underset{\text{Use (1).}}{=} 4e^x + 3x^2$$

NOTE We have not forgotten $y = \ln x$. Here is its derivative:

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

Use this result for now. We do not have the necessary mathematics to prove it until Chapter 3.

NOW WORK Problem 25 and AP® Practice Problems 4 and 9.

Now we know $\frac{d}{dx}e^x = e^x$. To find the derivative of $f(x) = a^x$, $a > 0$ and $a \neq 1$, we need more information. See Chapter 3.

2.3 Assess Your Understanding

Concepts and Vocabulary

1. $\frac{d}{dx}\pi^2 =$ ____; $\frac{d}{dx}x^3 =$ ____.
2. When n is a positive integer, the Simple Power Rule states that $\frac{d}{dx}x^n =$ ____.
3. *True or False* The derivative of a power function of degree greater than 1 is also a power function.
4. If k is a constant and f is a differentiable function, then $\frac{d}{dx}[kf(x)] =$ ____.
5. The derivative of $f(x) = e^x$ is ____.
6. *True or False* The derivative of an exponential function $f(x) = a^x$, where $a > 0$ and $a \neq 1$, is always a constant multiple of a^x.

Skill Building

In Problems 7–26, find the derivative of each function using the formulas of this section. (a, b, c, and d, when they appear, are constants.)

7. $f(x) = 3x + \sqrt{2}$
8. $f(x) = 5x - \pi$
9. $f(x) = x^2 + 3x + 4$
10. $f(x) = 4x^4 + 2x^2 - 2$
11. $f(u) = 8u^5 - 5u + 1$
12. $f(u) = 9u^3 - 2u^2 + 4u + 4$
13. $f(s) = as^3 + \frac{3}{2}s^2$
14. $f(s) = 4 - \pi s^2$
15. $f(t) = \frac{1}{3}(t^5 - 8)$
16. $f(x) = \frac{1}{5}(x^7 - 3x^2 + 2)$
17. $f(t) = \frac{t^3 + 2}{5}$
18. $f(x) = \frac{x^7 - 5x}{9}$
19. $f(x) = \frac{x^3 + 2x + 1}{7}$
20. $f(x) = \frac{1}{a}(ax^2 + bx + c), a \neq 0$
21. $f(x) = ax^2 + bx + c$
22. $f(x) = ax^3 + bx^2 + cx + d$
23. $f(x) = 4e^x$
24. $f(x) = -\frac{1}{2}e^x$
25. $f(u) = 5u^2 - 2e^u$
26. $f(u) = 3e^u + 10$

In Problems 27–32, find each derivative.

27. $\frac{d}{dt}\left(\sqrt{3}\,t + \frac{1}{2}\right)$
28. $\frac{d}{dt}\left(\frac{2t^4 - 5}{8}\right)$
29. $\frac{dA}{dR}$ if $A(R) = \pi R^2$
30. $\frac{dC}{dR}$ if $C = 2\pi R$
31. $\frac{dV}{dr}$ if $V = \frac{4}{3}\pi r^3$
32. $\frac{dP}{dT}$ if $P = 0.2T$

In Problems 33–36:
(a) Find the slope of the tangent line to the graph of each function f at the indicated point.
(b) Find an equation of the tangent line at the point.
(c) Find an equation of the normal line at the point.
(d) Graph f and the tangent line and normal line found in (b) and (c) on the same set of axes.

33. $f(x) = x^3 + 3x - 1$ at $(0, -1)$
34. $f(x) = x^4 + 2x - 1$ at $(1, 2)$
35. $f(x) = e^x + 5x$ at $(0, 1)$
36. $f(x) = 4 - e^x$ at $(0, 3)$

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In Problems 37–42:

(a) Find the points, if any, at which the graph of each function f has a horizontal tangent line.

(b) Find an equation for each horizontal tangent line.

(c) Solve the inequality $f'(x) > 0$.

(d) Solve the inequality $f'(x) < 0$.

(e) Graph f and any horizontal lines found in (b) on the same set of axes.

(f) Describe the graph of f for the results obtained in parts (c) and (d).

189 **37.** $f(x) = 3x^2 - 12x + 4$ **38.** $f(x) = x^2 + 4x - 3$

39. $f(x) = x + e^x$ **40.** $f(x) = 2e^x - 1$

41. $f(x) = x^3 - 3x + 2$ **42.** $f(x) = x^4 - 4x^3$

43. Rectilinear Motion At t seconds, an object in rectilinear motion is s meters from the origin, where $s(t) = t^3 - t + 1$. Find the velocity of the object at $t = 0$ and at $t = 5$.

44. Rectilinear Motion At t seconds, an object in rectilinear motion is s meters from the origin, where $s(t) = t^4 - t^3 + 1$. Find the velocity of the object at $t = 0$ and at $t = 1$.

Rectilinear Motion *In Problems 45 and 46, each position function gives the signed distance s from the origin at time t of an object in rectilinear motion:*

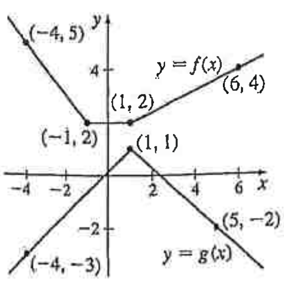
(a) Find the velocity v of the object at any time t.

(b) When is the velocity of the object 0?

45. $s(t) = 2 - 5t + t^2$ **46.** $s(t) = t^3 - \frac{9}{2}t^2 + 6t + 4$

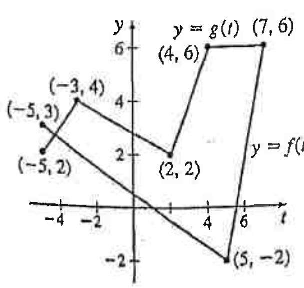
In Problems 47 and 48, use the graphs to find each derivative.

47. Let $u(x) = f(x) + g(x)$ and $v(x) = f(x) - g(x)$.



(a) $u'(0)$ (b) $u'(4)$
(c) $v'(-2)$ (d) $v'(6)$
(e) $3u'(5)$ (f) $-2v'(3)$

48. Let $F(t) = f(t) + g(t)$ and $G(t) = g(t) - f(t)$.



(a) $F'(0)$ (b) $F'(3)$
(c) $F'(-4)$ (d) $G'(-2)$
(e) $G'(-1)$ (f) $G'(6)$

In Problems 49 and 50, for each function f:

(a) Find $f'(x)$ by expanding $f(x)$ and differentiating the polynomial.

CAS *(b) Find $f'(x)$ using a CAS.*

(c) Show that the results found in parts (a) and (b) are equivalent.

49. $f(x) = (2x - 1)^3$ **50.** $f(x) = (x^2 + x)^4$

Applications and Extensions

In Problems 51–56, find each limit.

51. $\lim_{h \to 0} \frac{5\left(\frac{1}{2} + h\right)^8 - 5\left(\frac{1}{2}\right)^8}{h}$ **52.** $\lim_{h \to 0} \frac{6(2+h)^5 - 6 \cdot 2^5}{h}$

53. $\lim_{h \to 0} \frac{\sqrt{3}(8+h)^5 - \sqrt{3} \cdot 8^5}{h}$ **54.** $\lim_{h \to 0} \frac{\pi(1+h)^{10} - \pi}{h}$

55. $\lim_{h \to 0} \frac{a(x+h)^3 - ax^3}{h}$ **56.** $\lim_{h \to 0} \frac{b(x+h)^n - bx^n}{h}$

In Problems 57–62, find an equation of the tangent line(s) to the graph of the function f that is (are) parallel to the line L.

57. $f(x) = 3x^2 - x$; $L: y = 5x$

58. $f(x) = 2x^3 + 1$; $L: y = 6x - 1$

59. $f(x) = e^x$; $L: y - x - 5 = 0$

60. $f(x) = -2e^x$; $L: y + 2x - 8 = 0$

61. $f(x) = \frac{1}{3}x^3 - x^2$; $L: y = 3x - 2$

62. $f(x) = x^3 - x$; $L: x + y = 0$

63. Tangent Lines Let $f(x) = 4x^3 - 3x - 1$.

(a) Find an equation of the tangent line to the graph of f at $x = 2$.

(b) Find the coordinates of any points on the graph of f where the tangent line is parallel to $y = x + 12$.

(c) Find an equation of the tangent line to the graph of f at any points found in (b).

(d) Graph f, the tangent line found in (a), the line $y = x + 12$, and any tangent lines found in (c) on the same screen.

64. Tangent Lines Let $f(x) = x^3 + 2x^2 + x - 1$.

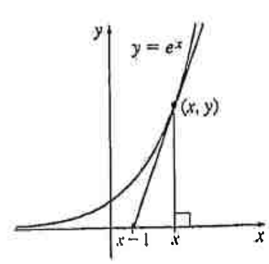
(a) Find an equation of the tangent line to the graph of f at $x = 0$.

(b) Find the coordinates of any points on the graph of f where the tangent line is parallel to $y = 3x - 2$.

(c) Find an equation of the tangent line to the graph of f at any points found in (b).

(d) Graph f, the tangent line found in (a), the line $y = 3x - 2$, and any tangent lines found in (c) on the same screen.

65. Tangent Line Show that the line perpendicular to the x-axis and containing the point (x, y) on the graph of $y = e^x$ and the tangent line to the graph of $y = e^x$ at the point (x, y) intersect the x-axis 1 unit apart. See the figure.



2.3 AP Practice Problems (p. 193) – Derivative Power Rule & exponential e^x

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1. If $g(x) = x$, then $g'(7) =$

   (A) 0 (B) 1 (C) 7 (D) $\frac{49}{2}$

2. The line $x + y = k$, where k is a constant, is a tangent line to the graph of the function $f(x) = x^2 - 5x + 2$. What is the value of k?

   (A) -1 (B) 2 (C) -2 (D) -4

3. An object moves along the x-axis so that its position at time t is $x(t) = 3t^2 - 9t + 7$. For what time t is the velocity of the object zero?

   (A) -3 (B) 3 (C) $\frac{3}{2}$ (D) 7

4. If $f(x) = e^x$, then $\ln(f'(3)) =$

   (A) 3 (B) 0 (C) e^3 (D) $\ln 3$

5. An equation of the normal line to the graph of $g(x) = x^3 + 2x^2 - 2x + 1$ at the point where $x = -2$ is

   (A) $x + 2y = 12$ (B) $x - 2y = 8$
   (C) $2x + y = -9$ (D) $x + 2y = 8$

6. The line $9x - 16y = 0$ is tangent to the graph of $f(x) = 3x^3 + k$, where k is a constant, at a point in the first quadrant. Find k.

   (A) $\frac{3}{32}$ (B) $\frac{3}{16}$ (C) $\frac{3}{64}$ (D) $\frac{9}{64}$

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7. If $f(x) = 1 + |x - 4|$, find $f'(4)$.

(A) -1 (B) 0 (C) 1 (D) $f'(4)$ does not exist.

8. The cost C (in dollars) of manufacturing x units of a product is $C(x) = 0.3x^2 + 4.02x + 3500$.
What is the rate of change of C when $x = 1000$ units?

(A) 307.52 (B) 0.60402 (C) 604.02 (D) 1020

9. $\frac{d}{dx}(5 \ln x) =$

(A) $\frac{1}{5x}$ (B) $5e^x$ (C) $-\frac{5}{\ln x}$ (D) $\frac{5}{x}$

10. For the function $f(x) = x^2 + 4$
   (a) Find $f'(1)$.
   (b) Find an equation of the tangent line to the graph of f at $x = 1$.
   (c) Find $f'(-4)$.
   (d) Find an equation of the tangent line to the graph of f at $x = -4$.
   (e) Find the point of intersection of the two tangent lines found in (b) and (d).

11. Which is an equation of the tangent line to the graph of $f(x) = x^4 + 3x^2 + 2$ at the point where $f'(x) = 2$?

(A) $y = 2x + 2$ (B) $y = 2x + 2.929$

(C) $y = 2x + 1.678$ (D) $y = 2x - 2.929$

AP Calculus Ch. 2.3b Notes: Position-Velocity-Acceleration(PVA)

Instantaneous velocity ,v(t), of the object is the derivative of the position function s(t) with respect to time

Acceleration, a(t), is the derivative of velocity with respect to time

AVERAGE rate of change of f(x) from a to b = slope of secant = $\frac{f(b)-f(a)}{b-a}$

INSTANTANEOUS rate of change of f(x) at x = c = slope of tangent = $f'(c)$

Speed = |velocity|

Displacement = how far you are from where you started
Distance = total amount you have traveled

Ex) If I travel 10 feet to the right and then turn around and travel 3 feet back to the left, my distance is 13 feet but my displacement is 7 feet.

Speed is **increasing** when velocity and acceleration have the **same** sign.
Speed is **decreasing** when velocity and acceleration have **opposite** signs.

Particle Motion

Particle motion (linear motion) describes the object moving along a line (usually along a horizontal line)

x(t) = Position function
v(t) = velocity function
a(t) = acceleration function

Positive velocity indicates ______________________________

Negative velocity indicates ______________________________

When v(t) = 0, this indicates ______________________________

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A.P. Calculus PVA **Worksheet 23b**

A ball is thrown vertically upwards from the edge of a building and it eventually hits the ground next to the building. If the height of the ball at any given time, $t \geq 0$ (seconds), is $h(t) = -16t^2 + 64t + 80$ (feet), answer the following:

1. Sketch a diagram and label values at important places

2. How tall is the building?	3. When does the ball reach maximum height?
4. What is the maximum height?	5. How long does it take to hit the ground?
6. What was the initial velocity?	7. What is the velocity at $t = 1$ second? At $t = 2$ seconds?
8. What is the height at $t = 3$ seconds?	9. What is the speed when it hits the ground?

10. What is the acceleration at $t = 1$ second? At $t = 2$ seconds?

11. Find the average velocity in [0, 2]

12. Find the average acceleration in [1, 2]

13. Is the speed increasing or decreasing at t = 1 seconds?

14. Is the velocity increasing or decreasing at t = 3 seconds?

A.P. Calculus PVA Worksheet 2.3b Linear Motion Problems

1. An object is traveling at 20 m/sec to the left. What is its speed and velocity?

2. Which has the greater speed and velocity: object A with a velocity of -20 m/sec or object B with a velocity of -10 m/sec?

3. A billiard ball is hit and travels in a straight line. If x centimeters is the distance of the ball from its initial position at t seconds, then $x(t) = 5t^2 - 4t$. If the ball hits a cushion that is 12 cm from its initial position, at what velocity does it hit the cushion?

4. If a particle moves along a line according to the equation $s(t) = t^5 - 5t^4$ for all real numbers, t, then how many times does the particle reverse its direction?

5. The position in meters of a particle moving on the x-axis is given by $x(t) = 2t^3 - 2t + 1$ at all times t, $t > 0$. Find the acceleration when the velocity is 4 m/sec.

6. If $x(t) = \dfrac{t}{t^2 + 5}$ is the position function of a moving particle for $t > 0$, at what instant of time will the particle start to reverse its direction of motion, and where is it at that instant?

7. The position function of a particle moving on a coordinate line is given by: $x(t) = 2t^3 - 21t^2 + 60t + 3$, where x is in feet and t is in seconds.

a) When is the particle at rest?

b) When does the particle reverse direction?

c) What is the velocity when the acceleration is zero?

d) What is the speed when the acceleration is 6 ft/sec?

e) What is the displacement from $t = 1$ to $t = 3$?

f) What is the total distance moved from $t = 1$ to $t = 3$?

8. If $v(t) = (t-5)(t-3)^2(t-1)$ represents the velocity of a particle moving along a line,
   a) When will the particle be at rest?
   b) When will the particle move to the left?
   c) When will the particle change direction?

9. A ball is thrown vertically upwards from the edge at the top of a building 160 ft tall with an initial velocity of 24 ft/sec. If the height of the ball (measured from the ground) is given by the function: $h(t) = -16t^2 + bt + c$,

a) Find the values of b and c.

b) How long does it take the ball to reach its maximum height?

c) What is the maximum height of the ball?

d) How long before the ball passes the top of the building on the way down?

e) How long does it take for the ball to hit the ground?

f) What is the speed of the ball when it hits the ground?

g) What is the speed of the ball at $t = 1$ second?

2.3b PVA (Position-Velocity-Acceleration) WS Problems

(31)

Vertical Motion In Exercises 97 and 98, use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.

97. A silver dollar is dropped from the top of a building that is 1362 feet tall.

(a) Determine the position and velocity functions for the coin.

(b) Determine the average velocity on the interval [1, 2].

(c) Find the instantaneous velocities when $t = 1$ and $t = 2$.

(d) Find the time required for the coin to reach ground level.

(e) Find the velocity of the coin at impact.

Vertical Motion In Exercises 97 and 98, use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.

98. A ball is thrown straight down from the top of a 220-foot building with an initial velocity of -22 feet per second. What is its velocity after 3 seconds? What is its velocity after falling 108 feet?

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Vertical Motion In Exercises 99 and 100, use the position function $s(t) = -4.9t^2 + v_0t + s_0$ for free-falling objects.

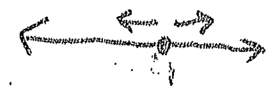
99. A projectile is shot upward from the surface of Earth with an initial velocity of 120 meters per second. What is its velocity after 5 seconds? After 10 seconds?

Vertical Motion In Exercises 99 and 100, use the position function $s(t) = -4.9t^2 + v_0t + s_0$ for free-falling objects.

100. To estimate the height of a building, a stone is dropped from the top of the building into a pool of water at ground level. The splash is seen 5.6 seconds after the stone is dropped. What is the height of the building?

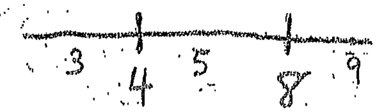
Particle Motion Problem Steps (Motion along a line)

1) Find $v(t)$ by taking the derivative of the position function $x(t)$ (use power rule)



2) Find times when object is motionless $(v(t)=0)$. Solve for t. (ex: $t=4,8$)

3) Create velocity sign line $v(t)$

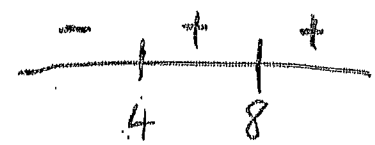


4) Pick a value in each interval to plug into $v(t)$

Ex. $v(3) = -6$ (moves left)

$v(5) = 1$ (moves right)

$v(9) = 3$ (moves right) $v(t)$



5) Find when object change directions (count the number of sign changes)

object changes direction at $t=4$ b/c $v(t)$ changes signs.

6) Determine the time intervals of object moving left and right

based on above example:

moving left: $(-\infty, 4)$ because $v(t) < 0$

moving right: $(4,8), (8,\infty)$ because $v(t) > 0$

(34) PVA Quiz Review Problem

Given:

$s(2) = 40$ m

$s(4) = 10$ m

$v(2) = -15$ m/s

$v(4) = -12$ m/s

$v(6) = -4$ m/s

$a(4) = 3$ m/s^2

$a(6) = 7$ m/s^2

Find the following:

a) Average velocity on [2,4]

b) Instantaneous velocity at $t=4$

c) Is velocity positive or negative at $t=4$?

d) Is velocity increasing or decreasing at $t=4$?

e) Is speed increasing or decreasing at $t=4$?

f) Find average acceleration on [4,6].

a) $\text{Avg. velocity} = \frac{\text{change in position}}{\text{change in time}} = \frac{s(4)-s(2)}{4-2} = \frac{10-40}{4-2} = \frac{-30}{2} = -15 \text{ m/s}$

b) $v(4) = -12$ m/s

c) $v(4) < 0$, negative

d) velocity is increasing at $t=4$ because $a(4) > 0$

e) speed is decreasing at $t=4$ because velocity and acceleration have opposite signs: $v(4) < 0$ and $a(4) > 0$.

f) $\text{avg. acceleration} = \frac{\text{change in velocity}}{\text{change in time}} = \frac{v(6)-v(4)}{6-4}$

$$= \frac{-4-(-12)}{6-4} = \frac{8}{2} = \boxed{4 \text{ m/s}^2}$$

Ch. 2.4 Notes Product and Quotient Rules

Product Rule: formula used to find the derivatives of products of two or more functions

$$* \frac{d}{dx}[f(x)\cdot g(x)] = f'(x)g(x) + f(x)g(x)'$$

"f prime g plus f g prime"

Ex. 1 $y = (3x-2x^2)(5+4x)$ Find $\frac{dy}{dx}$

$\underbrace{(3x-2x^2)}_{f(x)}\underbrace{(5+4x)}_{g(x)}$

Quotient Rule: formula for finding derivative of function that is the quotient of two other functions.

$$* \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Ex. 2 $y = \frac{3x-2x^2}{5+4x}$ Find y'

Higher order derivatives

Ex.3 $y = 2x^5 + x^4 - 3x^3 - 8x^2 + 10x - 12$. Find y''''

$y' =$

$y'' =$

$y''' =$

$y'''' =$

* Notations

Notations for 1st derivative: $f'(x)$, $g'(x)$, y', $\frac{dy}{dx}$

Notation for 2nd derivative: $f''(x)$, $y''(x)$, y'', $\frac{d^2y}{dx^2}$

*Note: This means "2nd derivati
NOT "square the 1st derivati

Notation for 3rd derivative: $f'''(x)$, y''', $\frac{d^3y}{dx^3}$

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The velocity v of the falling object is

$$v = \frac{ds}{dt} = \frac{d}{dt}(-ct^2) = -2ct$$

and its acceleration a is

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = -2c$$

which is a constant. Usually, we denote the constant $2c$ by g so $c = \frac{1}{2}g$. Then

$$a = -g \qquad v = -gt \qquad s = -\frac{1}{2}gt^2$$

The number g is called the **acceleration due to gravity.** For our planet, g is approximately 32 ft/s², or 9.8 m/s². On the planet Jupiter, $g \approx 26.0$ m/s², and on our moon, $g \approx 1.60$ m/s².

2.4 Assess Your Understanding

Concepts and Vocabulary

1. *True or False* The derivative of a product is the product of the derivatives.
2. If $F(x) = f(x)g(x)$, then $F'(x) =$ ____.
3. *True or False* $\frac{d}{dx}x^n = nx^{n+1}$, for any integer n.
4. If f and $g \neq 0$ are two differentiable functions, then $\frac{d}{dx}\frac{f(x)}{g(x)} =$ ____.
5. *True or False* $f(x) = \frac{e^x}{x^2}$ can be differentiated using the Quotient Rule or by writing $f(x) = \frac{e^x}{x^2} = x^{-2}e^x$ and using the Product Rule.
6. If $g \neq 0$ is a differentiable function, then $\frac{d}{dx}\frac{1}{g(x)} =$ ____.
7. If $f(x) = x$, then $f''(x) =$ ____.
8. When an object in rectilinear motion is modeled by the position function $s = s(t)$, then the acceleration a of the object at time t is given by $a = a(t) =$ ____.

Skill Building

In Problems 9–40, find the derivative of each function.

9. $f(x) = xe^x$ (PAGE 195)
10. $f(x) = x^2e^x$
11. $f(x) = x^2(x^3 - 1)$
12. $f(x) = x^4(x + 5)$
13. $f(x) = (3x^2 - 5)(2x + 1)$ (PAGE 196)
14. $f(x) = (3x - 2)(4x + 5)$
15. $s(t) = (2t^5 - t)(t^3 - 2t + 1)$
16. $F(u) = (u^4 - 3u^2 + 1)(u^2 - u + 2)$
17. $f(x) = (x^3 + 1)(e^x + 1)$
18. $f(x) = (x^2 + 1)(e^x + x)$
19. $g(s) = \frac{2s}{s + 1}$
20. $F(z) = \frac{z + 1}{2z}$
21. $G(u) = \frac{1 - 2u}{1 + 2u}$
22. $f(w) = \frac{1 - w^2}{1 + w^2}$
23. $f(x) = \frac{4x^2 - 2}{3x + 4}$ (PAGE 197)
24. $f(x) = \frac{-3x^3 - 1}{2x^2 + 1}$
25. $f(w) = \frac{1}{w^3 - 1}$ (PAGE 197)
26. $g(v) = \frac{1}{v^2 + 5v - 1}$
27. $s(t) = t^{-3}$
28. $G(u) = u^{-4}$
29. $f(x) = -\frac{4}{e^x}$
30. $f(x) = \frac{3}{4e^x}$
31. $f(x) = \frac{10}{x^4} + \frac{3}{x^2}$ (PAGE 198)
32. $f(x) = \frac{2}{x^5} - \frac{3}{x^3}$
33. $f(x) = 3x^3 - \frac{1}{3x^2}$
34. $f(x) = x^5 - \frac{5}{x^5}$
35. $s(t) = \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3}$
36. $s(t) = \frac{1}{t} + \frac{1}{t^2} + \frac{1}{t^3}$
37. $f(x) = \frac{e^x}{x^2}$
38. $f(x) = \frac{x^2}{e^x}$
39. $f(x) = \frac{x^2 + 1}{xe^x}$
40. $f(x) = \frac{xe^x}{x^2 - x}$

In Problems 41–54, find f' and f'' for each function.

41. $f(x) = 3x^2 + x - 2$ (PAGE 199)
42. $f(x) = -5x^2 - 3x$
43. $f(x) = e^x - 3$
44. $f(x) = x - e^x$
45. $f(x) = (x + 5)e^x$ (PAGE 200)
46. $f(x) = 3x^4e^x$
47. $f(x) = (2x + 1)(x^3 + 5)$
48. $f(x) = (3x - 5)(x^2 - 2)$
49. $f(x) = x + \frac{1}{x}$
50. $f(x) = x - \frac{1}{x}$
51. $f(t) = \frac{t^2 - 1}{t}$
52. $f(u) = \frac{u + 1}{u}$
53. $f(x) = \frac{e^x + x}{x}$
54. $f(x) = \frac{e^x}{x}$
55. Find y' and y'' for **(a)** $y = \frac{1}{x}$ and **(b)** $y = \frac{2x - 5}{x}$.
56. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for **(a)** $y = \frac{5}{x^2}$ and **(b)** $y = \frac{2 - 3x}{x}$.

Rectilinear Motion *In Problems 57–60, find the velocity $v = v(t)$ and acceleration $a = a(t)$ of an object in rectilinear motion whose signed distance s from the origin at time t is modeled by the position function $s = s(t)$.*

57. $s(t) = 16t^2 + 20t$

58. $s(t) = 16t^2 + 10t + 1$

59. $s(t) = 4.9t^2 + 4t + 4$

60. $s(t) = 4.9t^2 + 5t$

In Problems 61–68, find the indicated derivative.

61. $f^{(4)}(x)$ if $f(x) = x^3 - 3x^2 + 2x - 5$

62. $f^{(5)}(x)$ if $f(x) = 4x^3 + x^2 - 1$

63. $\dfrac{d^8}{dt^8}\left(\dfrac{1}{8}t^8 - \dfrac{1}{7}t^7 + t^5 - t^3\right)$

64. $\dfrac{d^6}{dt^6}(t^6 + 5t^5 - 2t + 4)$

65. $\dfrac{d^7}{du^7}(e^u + u^2)$

66. $\dfrac{d^{10}}{du^{10}}(2e^u)$

67. $\dfrac{d^5}{dx^5}(-e^x)$

68. $\dfrac{d^8}{dx^8}(12x - e^x)$

In Problems 69–72:

(a) Find the slope of the tangent line for each function f at the given point.
(b) Find an equation of the tangent line to the graph of each function f at the given point.
(c) Find the points, if any, where the graph of the function has a horizontal tangent line.
(d) Graph each function, the tangent line found in (b), and any tangent lines found in (c) on the same set of axes.

69. $f(x) = \dfrac{x^2}{x-1}$ at $\left(-1, -\dfrac{1}{2}\right)$

70. $f(x) = \dfrac{x}{x+1}$ at $(0, 0)$

71. $f(x) = \dfrac{x^3}{x+1}$ at $\left(1, \dfrac{1}{2}\right)$

72. $f(x) = \dfrac{x^2+1}{x}$ at $\left(2, \dfrac{5}{2}\right)$

In Problems 73–80:

(a) Find the points, if any, at which the graph of each function f has a horizontal tangent line.
(b) Find an equation for each horizontal tangent line.
(c) Solve the inequality $f'(x) > 0$.
(d) Solve the inequality $f'(x) < 0$.
(e) Graph f and any horizontal lines found in (b) on the same set of axes.
(f) Describe the graph of f for the results obtained in (c) and (d).

73. $f(x) = (x+1)(x^2 - x - 11)$

74. $f(x) = (3x^2 - 2)(2x + 1)$

75. $f(x) = \dfrac{x^2}{x+1}$

76. $f(x) = \dfrac{x^2+1}{x}$

77. $f(x) = xe^x$

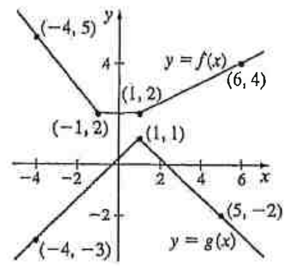
78. $f(x) = x^2e^x$

79. $f(x) = \dfrac{x^2 - 3}{e^x}$

80. $f(x) = \dfrac{e^x}{x^2+1}$

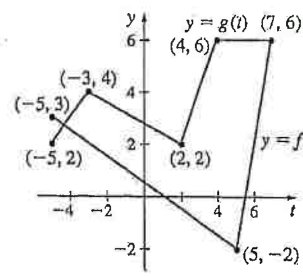
In Problems 81 and 82, use the graphs to determine each derivative.

81. Let $u(x) = f(x) \cdot g(x)$ and $v(x) = \dfrac{g(x)}{f(x)}$.



(a) $u'(0)$
(b) $u'(4)$
(c) $v'(-2)$
(d) $v'(6)$
(e) $\dfrac{d}{dx}\dfrac{1}{f(x)}$ at $x = -2$
(f) $\dfrac{d}{dx}\dfrac{1}{g(x)}$ at $x = 4$

82. Let $F(t) = f(t) \cdot g(t)$ and $G(t) = \dfrac{f(t)}{g(t)}$.



(a) $F'(0)$
(b) $F'(3)$
(c) $F'(-4)$
(d) $G'(-2)$
(e) $G'(-1)$
(f) $\dfrac{d}{dt}\dfrac{1}{f(t)}$ at $t = 3$

Applications and Extensions

83. **Vertical Motion** An object is propelled vertically upward from the ground with an initial velocity of 39.2 m/s. The distance s (in meters) of the object from the ground after t seconds is given by the position function $s = s(t) = -4.9t^2 + 39.2t$.

(a) What is the velocity of the object at time t?
(b) When will the object reach its maximum height?
(c) What is the maximum height?
(d) What is the acceleration of the object at any time t?
(e) How long is the object in the air?
(f) What is the velocity of the object upon impact with the ground? What is its speed?
(g) What is the total distance traveled by the object?

84. **Vertical Motion** A ball is thrown vertically upward from a height of 6 ft with an initial velocity of 80 ft/s. The distance s (in feet) of the ball from the ground after t seconds is given by the position function $s = s(t) = 6 + 80t - 16t^2$.

(a) What is the velocity of the ball after 2 s?
(b) When will the ball reach its maximum height?
(c) What is the maximum height the ball reaches?
(d) What is the acceleration of the ball at any time t?
(e) How long is the ball in the air?
(f) What is the velocity of the ball upon impact with the ground? What is its speed?
(g) What is the total distance traveled by the ball?

85. **Environmental Cost** The cost C, in thousands of dollars, for the removal of a pollutant from a certain lake is given by the function $C(x) = \dfrac{5x}{110 - x}$, where x is the percent of pollutant removed.

(a) What is the domain of C?
(b) Graph C.
(c) What is the cost to remove 80% of the pollutant?
(d) Find $C'(x)$, the rate of change of the cost C with respect to the amount of pollutant removed.
(e) Find the rate of change of the cost for removing 40%, 60%, 80%, and 90% of the pollutant.
(f) Interpret the answers found in (e).

86. **Investing in Fine Art** The value V of a painting t years after it is purchased is modeled by the function

$$V(t) = \frac{100t^2 + 50}{t} + 400 \quad 1 \le t \le 5$$

(a) Find the rate of change in the value V with respect to time.
(b) What is the rate of change in value after 2 years?
(c) What is the rate of change in value after 3 years?
(d) Interpret the answers in (b) and (c).

87. **Drug Concentration** The concentration of a drug in a patient's blood t hours after injection is given by the function $f(t) = \dfrac{0.4t}{2t^2 + 1}$ (in milligrams per liter).

(a) Find the rate of change of the concentration with respect to time.
(b) What is the rate of change of the concentration after 10 min? After 30 min? After 1 hour?
(c) Interpret the answers found in (b).
(d) Graph f for the first 5 hours after administering the drug.
(e) From the graph, approximate the time (in minutes) at which the concentration of the drug is highest. What is the highest concentration of the drug in the patient's blood?

88. **Population Growth** A population of 1000 bacteria is introduced into a culture and grows in number according to the formula

$$P(t) = 1000\left(1 + \frac{4t}{100 + t^2}\right), \text{ where } t \text{ is measured in hours.}$$

(a) Find the rate of change in population with respect to time.
(b) What is the rate of change in population at $t = 1$, $t = 2$, $t = 3$, and $t = 4$?
(c) Interpret the answers found in (b).
(d) Graph $P = P(t)$, $0 \le t \le 20$.
(e) From the graph, approximate the time (in hours) when the population is the greatest. What is the maximum population of the bacteria in the culture?

89. **Economics** The price-demand function for a popular e-book is given by $D(p) = \dfrac{100{,}000}{p^2 + 10p + 50}$, $4 \le p \le 20$, where $D = D(p)$ is the quantity demanded at the price p dollars.

(a) Find $D'(p)$, the rate of change of demand with respect to price.
(b) Find $D'(5)$, $D'(10)$, and $D'(15)$.
(c) Interpret the results found in (b).

90. **Intensity of Light** The intensity of illumination I on a surface is inversely proportional to the square of the distance r from the surface to the source of light. If the intensity is 1000 units when the distance is 1 m from the light, find the rate of change of the intensity with respect to the distance when the source is 10 meters from the surface.

PAGE 198 91. **Ideal Gas Law** The Ideal Gas Law, used in chemistry and thermodynamics, relates the pressure p, the volume V, and the absolute temperature T (in Kelvin) of a gas, using the equation $pV = nRT$, where n is the amount of gas (in moles) and $R = 8.31$ is the ideal gas constant. In an experiment, a spherical gas container of radius r meters is placed in a pressure chamber and is slowly compressed while keeping its temperature at 273 K.

(a) Find the rate of change of the pressure p with respect to the radius r of the chamber.

Hint: The volume V of a sphere is $V = \frac{4}{3}\pi r^3$.

(b) Interpret the sign of the answer found in (a).
(c) If the sphere contains 1.0 mol of gas, find the rate of change of the pressure when $r = \frac{1}{4}$ m.

Note: The metric unit of pressure is the pascal, Pa.

92. **Body Density** The density ρ of an object is its mass m divided by its volume V; that is, $\rho = \dfrac{m}{V}$. If a person dives below the surface of the ocean, the water pressure on the diver will steadily increase, compressing the diver and therefore increasing body density. Suppose the diver is modeled as a sphere of radius r.

(a) Find the rate of change of the diver's body density with respect to the radius r of the sphere.

Hint: The volume V of a sphere is $V = \frac{4}{3}\pi r^3$.

(b) Interpret the sign of the answer found in (a).
(c) Find the rate of change of the diver's body density when the radius is 45 cm and the mass is 80,000 g (80 kg).

Jerk and Snap *Problems 93–96 use the following discussion:* Suppose that an object is moving in rectilinear motion so that its signed distance s from the origin at time t is given by the position function $s = s(t)$. The velocity $v = v(t)$ of the object at time t is the rate of change of s with respect to time, namely, $v = v(t) = \dfrac{ds}{dt}$. The acceleration $a = a(t)$

of the object at time t is the rate of change of the velocity with respect to time,

$$a = a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$$

There are also physical interpretations of the third derivative and the fourth derivative of $s = s(t)$. The **jerk** $J = J(t)$ of the object at time t is the rate of change of the acceleration a with respect to time; that is,

$$J = J(t) = \frac{da}{dt} = \frac{d}{dt}\left(\frac{dv}{dt}\right) = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}$$

The **snap** $S = S(t)$ of the object at time t is the rate of change of the jerk J with respect to time; that is,

$$S = S(t) = \frac{dJ}{dt} = \frac{d^2a}{dt^2} = \frac{d^3v}{dt^3} = \frac{d^4s}{dt^4}$$

Engineers take jerk into consideration when designing elevators, aircraft, and cars. In these cases, they try to minimize jerk, making for a smooth ride. But when designing thrill rides, such as roller coasters, the jerk is increased, making for an exciting experience.

93. **Rectilinear Motion** As an object in rectilinear motion moves, its signed distance s from the origin at time t is given by the position function $s = s(t) = t^3 - t + 1$, where s is in meters and t is in seconds.
    (a) Find the velocity v, acceleration a, jerk J, and snap S of the object at time t.
    (b) When is the velocity of the object 0 m/s?
    (c) Find the acceleration of the object at $t = 2$ and at $t = 5$.
    (d) Does the jerk of the object ever equal 0 m/s^3?
    (e) How would you interpret the snap for this object in rectilinear motion?

94. **Rectilinear Motion** As an object in rectilinear motion moves, its signed distance s from the origin at time t is given by the position function $s = s(t) = \frac{1}{6}t^4 - t^2 + \frac{1}{2}t + 4$, where s is in meters and t is in seconds.
    (a) Find the velocity v, acceleration a, jerk J, and snap S of the object at any time t.
    (b) Find the velocity of the object at $t = 0$ and at $t = 3$.
    (c) Find the acceleration of the object at $t = 0$. Interpret your answer.
    (d) Is the jerk of the object constant? In your own words, explain what the jerk says about the acceleration of the object.
    (e) How would you interpret the snap for this object in rectilinear motion?

95. **Elevator Ride Quality** The ride quality of an elevator depends on several factors, two of which are acceleration and jerk. In a study of 367 persons riding in a 1600-kg elevator that moves at an average speed of 4 m/s, the majority of riders were comfortable in an elevator with vertical motion given by

$$s(t) = 4t + 0.8t^2 + 0.333t^3$$

    (a) Find the acceleration that the riders found acceptable.
    (b) Find the jerk that the riders found acceptable.

Source: Elevator Ride Quality, January 2007, http://www.lift-report.de/index.php/news/176/368/Elevator-Ride-Quality

96. **Elevator Ride Quality** In a hospital, the effects of high acceleration or jerk may be harmful to patients, so the acceleration and jerk need to be lower than in standard elevators. It has been determined that a 1600-kg elevator that is installed in a hospital and that moves at an average speed of 4 m/s should have vertical motion

$$s(t) = 4t + 0.55t^2 + 0.1167t^3$$

    (a) Find the acceleration of a hospital elevator.
    (b) Find the jerk of a hospital elevator.

Source: Elevator Ride Quality, January 2007, http://www.lift-report.de/index.php/news/176/368/Elevator-Ride-Quality

97. **Current Density in a Wire** The current density J in a wire is a measure of how much an electrical current is compressed as it flows through a wire and is modeled by the function $J(A) = \frac{I}{A}$, where I is the current (in amperes) and A is the cross-sectional area of the wire. In practice, current density, rather than merely current, is often important. For example, superconductors lose their superconductivity if the current density is too high.
    (a) As current flows through a wire, it heats the wire, causing it to expand in area A. If a constant current is maintained in a cylindrical wire, find the rate of change of the current density J with respect to the radius r of the wire.
    (b) Interpret the sign of the answer found in (a).
    (c) Find the rate of change of current density with respect to the radius r when a current of 2.5 amps flows through a wire of radius $r = 0.50$ mm.

98. **Derivative of a Reciprocal, Function** Prove that if a function g is differentiable, then $\frac{d}{dx}\frac{1}{g(x)} = -\frac{g'(x)}{[g(x)]^2}$, provided $g(x) \neq 0$.

99. **Extended Product Rule** Show that if f, g, and h are differentiable functions, then

$$\frac{d}{dx}[f(x)g(x)h(x)] = f(x)g(x)h'(x) + f(x)g'(x)h(x) + f'(x)g(x)h(x)$$

From this, deduce that

$$\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x)$$

In Problems 100–105, use the Extended Product Rule (Problem 99) to find y'.

100. $y = (x^2 + 1)(x - 1)(x + 5)$
101. $y = (x - 1)(x^2 + 5)(x^3 - 1)$
102. $y = (x^4 + 1)^3$
103. $y = (x^3 + 1)^3$
104. $y = (3x + 1)\left(1 + \frac{1}{x}\right)(x^{-5} + 1)$
105. $y = \left(1 - \frac{1}{x}\right)\left(1 - \frac{1}{x^2}\right)\left(1 - \frac{1}{x^3}\right)$

106. **(Further) Extended Product Rule** Write a formula for the derivative of the product of four differentiable functions. That is, find a formula for $\frac{d}{dx}[f_1(x)f_2(x)f_3(x)f_4(x)]$. Also find a formula for $\frac{d}{dx}[f(x)]^4$.

2.4 AP Practice Problems (p. 206) – Product & Quotient Rule & Higher order derivatives

1. What is the instantaneous rate of change at $x = -2$ of the function $f(x) = \dfrac{x-1}{x^2+2}$?

   (A) $-\dfrac{1}{6}$ (B) $\dfrac{1}{9}$ (C) $\dfrac{1}{2}$ (D) -1

2. An equation of the tangent line to the graph of $f(x) = \dfrac{5x-3}{3x-6}$ at the point $(3, 4)$ is

   (A) $7x + 3y = 37$ (B) $7x + 3y = 33$

   (C) $7x - 3y = 9$ (D) $13x + 3y = 51$

3. If f, g, and h are nonzero differentiable functions of x, then $\dfrac{d}{dx}\left(\dfrac{gh}{f}\right) =$

   (A) $\dfrac{fgh' + fg'h - f'gh}{f^2}$ (B) $\dfrac{g'h' - ghf'}{f^2}$

   (C) $\dfrac{gh' + g'h}{f'}$ (D) $\dfrac{fgh' + fg'h + f'gh}{f^2}$

4. If $y = x^3e^x$, then $\dfrac{dy}{dx} =$

   (A) $3x^2e^x$ (B) $3x^2 + e^x$

   (C) $3x^2e^x(x + 1)$ (D) $x^2e^x(x + 3)$

5. $\dfrac{d}{dt}\left(t^2 - \dfrac{1}{t^2} + \dfrac{1}{t}\right)$ at $t = 2$ is

   (A) $\dfrac{7}{2}$ (B) $\dfrac{9}{2}$ (C) $\dfrac{9}{4}$ (D) 4

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6. The position of an object moving along a straight line at time t, in seconds, is given by $s(t) = 16t^2 - 5t + 20$ meters. What is the acceleration of the object when $t = 2$?

(A) 32 m/s (B) 0 m/s^2 (C) 32 m/s^2 (D) 64 m/s^2

7. If $y = \frac{x-3}{x+3}$, $x \neq -3$, the instantaneous rate of change of y with respect to x at $x = 3$ is

(A) $-\frac{1}{6}$ (B) $\frac{1}{6}$ (C) $\frac{1}{36}$ (D) 1

8. Find an equation of the normal line to the graph of the function $f(x) = \frac{x^2}{x+1}$ at $x = 1$.

(A) $8x + 6y = 11$ (B) $-8x + 6y = -5$
(C) $-3x + 4y = -1$ (D) $3x + 4y = 5$

9. If $y = xe^x$, then the nth derivative of y is

(A) e^x (B) $(x+n)e^x$ (C) ne^x (D) x^ne^x

2.2-2.4 Quiz Review

1. The velocity of a function is described by the function $v(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 2$.

a) Find the time(s) when acceleration is zero

b) Find the velocity when acceleration is zero

2. The position function of a particle moving in a straight line is $x(t) = t^3 - 9t^2 + 24t - 2$ meters for $t > 0$ seconds.

a. When is the particle at rest?

b. During what time interval is particle moving to the right?

c. During what time interval is the particle moving to the left?

3. Given function $f(x) = \frac{x}{x-3}$

a. Find the equation of tangent line to the curve at $x = 4$

b. Find the equation of the tangent line to the curve where the slope is equal to $-\frac{3}{4}$

4. Given $f(x) = \sqrt[3]{x}\,(1 - x^3)$. Find $f'(x)$

5. Given $f(x) = \frac{2}{\sqrt{x}} - 5\sqrt[4]{x} + 12x^3 - 4\pi + 6.5x$ Find $f'(x)$.

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KEY

2.2-2.4 Quiz Review

1. The velocity of a function is described by the function v(t) = $\frac{1}{3}t^3 - 2t^2 + 3t + 2$.

a) Find the time(s) when acceleration is zero

$a(t) = t^2 - 4t + 3$

$0 = (t-3)(t-1)$

$t = 1, 3$

$t = 1, 3$

b) Find the velocity when acceleration is zero

$v(3) = \frac{1}{3}(3)^3 - 2(3)^2 + 3(3) + 2 =$

$= 9 - 18 + 9 + 2 = \boxed{2}$

$v(1) = \frac{1}{3} - 2 + 3 + 2 = \boxed{\frac{16}{3}}$

2. The position function of a particle moving in a straight line is x(t) = $t^3 - 9t^2 + 24t - 2$ meters for t > 0 seconds .

a. When is the particle at rest?

$v(t) = 3t^2 - 18t + 24$

$v(t) = 3(t^2 - 6t + 8)$

$v(t) = 3(t-4)(t-2)$

$0 = 3(t-4)(t-2)$

$t = 2, 4$ secs.

b. During what time interval is particle moving to the right?

$(0,2) \cup (4, \infty)$

Sign line: x | + | − | + at 0, 2, 4

c. During what time interval is the particle moving to the left?

$(2,4)$

3. Given function f(x) = $\frac{x}{x-3}$

$f(4) = \frac{4}{4-3} = 4$ point $(4,4)$ slope: $m = -3$

$y - 4 = -3(x-4)$

a. Find the equation of tangent line to the curve at x = 4

$f'(x) = \frac{1(x-3) - x(1)}{(x-3)^2}$

$= \frac{x-3-x}{(x-3)^2} = \frac{-3}{(x-3)^2}$

$f'(x) = \frac{-3}{(x-3)^2}$

$f'(4) = \frac{-3}{(4-3)^2} = \frac{-3}{1} = -3$

b. Find the equation of the tangent line to the curve where the slope is equal to $-\frac{3}{4}$

$\frac{-3}{4} = \frac{-3}{(x-3)^2}$

$-3(x-3)^2 = -3(4)$

$(x-3)^2 = 4$

$x^2 - 6x + 9 = 4$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 1, 5$

$f(1) = \frac{1}{1-3} = -\frac{1}{2}$

$f(5) = \frac{5}{5-3} = \frac{5}{2}$

$y + \frac{1}{2} = -\frac{3}{4}(x-1)$

$y - \frac{5}{2} = -\frac{3}{4}(x-5)$

4. Given f(x) = $\sqrt[3]{x}\,(1-x^3)$. Find f'(x)

$f(x) = x^{1/3}(1-x^3)$

$f'(x) = \frac{1}{3}x^{-2/3}(1-x^3) + x^{1/3}(-3x^2)$

$= \frac{1-x^3}{3x^{2/3}} - 3x^{7/3}$

$f'(x) = \frac{1-x^3}{3\sqrt[3]{x^2}} - 3\sqrt[3]{x^7}$

5. Given f(x) = $\frac{2}{\sqrt{x}} - 5\sqrt[4]{x} + 12x^3 - 4\pi + 6.5x$ Find f'(x)

$f(x) = 2x^{-1/2} - 5x^{1/4} + 12x^3 - 4\pi + 6.5x$

$f'(x) = 2(-\frac{1}{2})x^{-3/2} - 5(\frac{1}{4})x^{-3/4} + 36x^2 + 0 + 6.5$

$= \frac{-1}{x^{3/2}} - \frac{5}{4x^{3/4}} + 36x^2 + 6.5$

$= \frac{-1}{\sqrt{x^3}} - \frac{5}{4\sqrt[4]{x^3}} + 36x^2 + 6.5$

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2-2-24 **Review WS #2**

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3(x-1) - \frac{3x^2}{11} + 4\pi x - 5\pi^4 + \sqrt[5]{x^4} + \frac{5}{\sqrt{x^7}}$

2. If $f(x) = \frac{x+4}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

3) Find the derivative of f(x) and then evaluate the slope of the graph at x = 1
$f(x) = (3x^5 - 4\sqrt{x})(2x - 5\pi + 9)$

(46)

4. Particle moves along the x-axis so that its position at time t is given $x(t) = t^3 - 9t^2 + 15t - 7$ where $x(t)$ is in feet per second and $t \geq 0$. Use this to answer the questions below. **Include units with your answers**

a) Find the velocity and acceleration function

b) What is its velocity at $t = 2$ seconds?

c) What is its acceleration at t = 4 seconds?

d) Find the average velocity of particle in [3, 8]

e) When is the particle at rest?

f) When is the particle moving right? When does particle change directions? (Create Sign Line) Give justification.

g) What is displacement of particle from t = 2 to t = 6? Show work.

h) What is the total distance of particle from t = 2 to t = 6? Show work.

i) Is the speed increasing or decreasing at t = 4? Justify.

j) Is velocity increasing or decreasing at t = 2? Justify.