AB Calculus Digital Learning Assignment (Friday 3/9). Due Wed (3/14). Show evidence of effort. Make corrections in red ink and score FRQs

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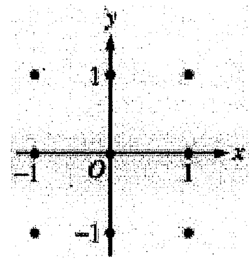
2006 SCORING GUIDELINES (Form B)

Question 5

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated. (Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

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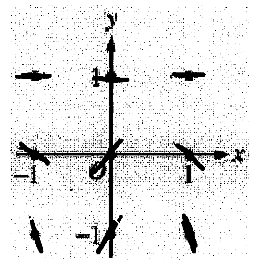

(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

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b) * horizontal line occurs when slope $=0$ $\left(\frac{dy}{dx}=0\right)$: when $y=1$,

$\frac{dy}{dx} = (1-1)^2\cos(\pi x) = 0$ ✓ so $\boxed{c=1}$

c) * Solve differential equation by separating variables, take antiderivative, solve for C, write specific equation.

$$\frac{dy}{dx} = (y-1)^2\cos(\pi x)$$

$$dy = (y-1)^2\cos(\pi x)\,dx$$

$$\frac{dy}{(y-1)^2} = \cos(\pi x)\,dx$$

$$\int \frac{dy}{(y-1)^2} = \int \cos(\pi x)\,dx$$

$u = y-1$, $\frac{du}{dy} = 1$, $dy = du$

$u = \pi x$, $\frac{du}{dx} = \pi$, $dx = \frac{du}{\pi}$

$$\int \frac{du}{u^2} = \int u^{-2}\,du = \frac{u^{-1}}{-1} = \frac{-1}{u} = \frac{-1}{y-1}$$

$$\int \cos u \cdot \frac{du}{\pi} = \frac{1}{\pi}\int \cos u\,du = \frac{1}{\pi}\sin u = \frac{1}{\pi}\sin(\pi x)$$

$\frac{-1}{y-1} = \frac{1}{\pi}\sin(\pi x) + C$ ← solve for C (plug in (1,0))

$$\frac{-1}{0-1} = \frac{1}{\pi}\sin(\pi) + C$$

$$+1 = \frac{1}{\pi}(0) + C$$

$$1 = C$$

$$\left(\frac{-1}{y-1} = \frac{1}{\pi}\sin(\pi x) + 1\right)\pi$$

$$\frac{-\pi}{y-1} = \frac{\sin(\pi x) + \pi}{1}$$

$$(y-1)(\sin(\pi x) + \pi) = -\pi$$

$$y - 1 = \frac{-\pi}{\sin(\pi x) + \pi}$$

$$\boxed{y = \frac{-\pi}{\sin(\pi x) + \pi} + 1}$$

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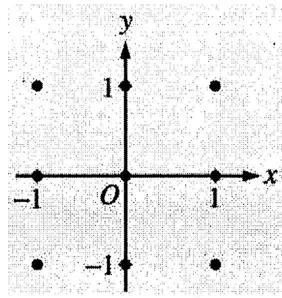

(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

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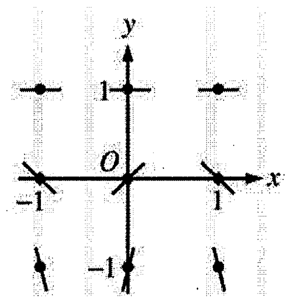
(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

(b) The line $y = 1$ satisfies the differential equation, so $c = 1$.

$1 : c = 1$

(c) $\dfrac{1}{(y-1)^2}\, dy = \cos(\pi x)\, dx$

$-(y-1)^{-1} = \dfrac{1}{\pi}\sin(\pi x) + C$

$\dfrac{1}{1-y} = \dfrac{1}{\pi}\sin(\pi x) + C$

$1 = \dfrac{1}{\pi}\sin(\pi) + C = C$

$\dfrac{1}{1-y} = \dfrac{1}{\pi}\sin(\pi x) + 1$

$\dfrac{\pi}{1-y} = \sin(\pi x) + \pi$

$y = 1 - \dfrac{\pi}{\sin(\pi x) + \pi}$ for $-\infty < x < \infty$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

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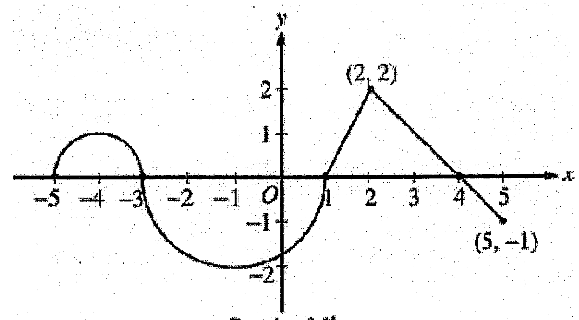
Question 4



Graph of f'

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

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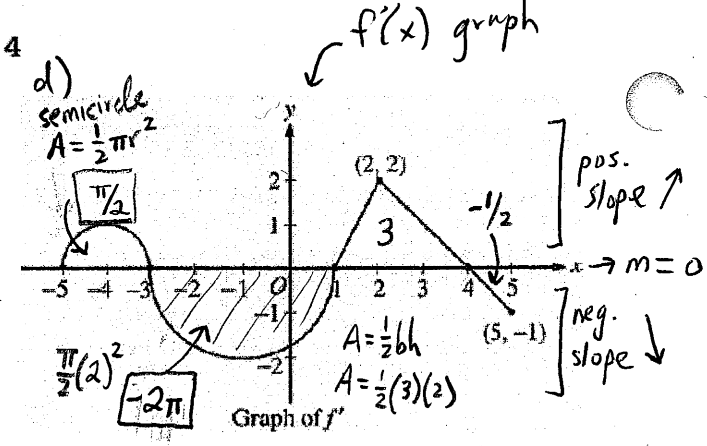

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*Create slope sign line

a) $f'(x)$: -5 | $+$ ↗ | -3 | $-$ ↘ | 1 | $+$ ↗ | 4 | $-$ ↘ | 5

Relative max. occurs at $x=-3$, $x=4$ b/c $f'(x)$ changes from + to −

b) *concavity sign line

-5 | ∪ $+$ | -4 | ∩ $-$ | -1 | ∪ $+$ | 2 | ∩ $-$ | 5

POI occurs at $x=4, -1, 2$ b/c $f''(x)$ changes signs (or $f'(x)$ changes from increase to decrease or decrease to increase)

c) Graph of $f(x)$ is concave up and positive slope in interval $-5<x<-4$ and $1<x<2$ ... or $(-5,-4)\cup(1,2)$

-5 | ↗ ∪ | -4 | ↗ ∩ | -3 | ↘ ∩ | -1 | ↘ ∪ | 1 | ↗ ∪ | 2 | ↗ ∩ | 4 | ↘ ∩ | 5

*Recall: $x(b) = x(a) + \int_a^b f'(x)\,dx$

final pos. = initial pos. + displacement

d) *Apply EVT to find absolute minimum.

i) Test endpoints

ii) Test critical points (rel. max/min)

Test candidate x-values at $x=-5, 1, 5$

Given: $f(1)=\boxed{3}$

$f(-5) = f(1) + \int_1^{-5} f'(x)\,dx$

$f(-5) = f(1) - \int_{-5}^{1} f'(x)\,dx$

$f(-5) = 3 - \left(\frac{\pi}{2} - 2\pi\right)$

$= 3 + 2\pi - \frac{\pi}{2} \approx \boxed{7.5}$

$f(5) = f(1) + \int_1^5 f'(x)\,dx$

$= 3 + \left(3 - \frac{1}{2}\right)$

$f(5) = \boxed{5.5}$

$\boxed{\text{Absolute minimum is 3 at } x=1}$

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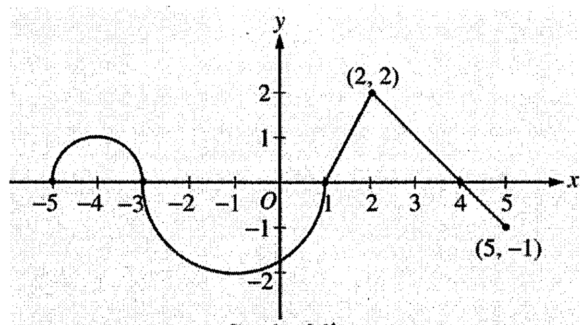

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(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3,\ 1, 4$
f' changes from positive to negative at -3 and 4.
Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1,$ and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1,$ and 2.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{explanation} \end{cases}$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

$3 : \begin{cases} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{cases}$