

AP Calculus AB

Fall Semester Pre-Final Review Packet

Due Date: Beginning of class, Tuesday, December 3, 2019

- **ALL WORK SHOULD BE DONE INDIVIDUALLY. YOU MAY NOT WORK TOGETHER. HOWEVER, YOU MAY REFER TO YOUR NOTES, HOMEWORK, OR THE TEXTBOOK AS NEEDED FOR ASSISTANCE.**
- **YOU MUST SIGN THE HONOR CODE STATEMENT AT THE END OF THE DOCUMENT TO RECEIVE CREDIT FOR THE PROJECT.**
- **ALL WORK MUST BE SHOWN (WHERE NECESSARY) TO RECEIVE CREDIT.**
- **ALL PROBLEMS SHOULD BE SOLVED WITHOUT USING A CALCULATOR.**

Grading: This project will count as a test grade of 100 points. You will be graded 50% on completion and 50% on accuracy. For the accuracy portion, 10 problems will be selected at random and counted as 5 points each.

1. $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4} =$

2. $\lim_{x \rightarrow 5} \frac{3 - \sqrt{2x - 1}}{x - 5} =$

3. $\lim_{x \rightarrow -3^+} \frac{2x - 5}{x + 3} =$

4. $\lim_{x \rightarrow \infty} \frac{3x^2}{x^2 - 4x + 7} =$

5. $\lim_{x \rightarrow -\infty} \frac{5 - 4x^2}{9x - 3} =$

6. $\lim_{x \rightarrow -\infty} \frac{4x}{x^2 - 3} =$

$$7. \lim_{x \rightarrow 0} \frac{4 - 4\cos 4x}{8x} =$$

$$8. \lim_{x \rightarrow 0} \frac{5x}{\sin 3x} =$$

9. Suppose $h(x)$, $f(x)$, and $g(x)$ are continuous functions on $[a, b]$ such that $\lim_{x \rightarrow c} f(x) = 3$, $\lim_{x \rightarrow c} g(x) = 3$, and $f(x) \leq h(x) \leq g(x)$ for all x on $[a, b]$. What can we conclude about the $\lim_{x \rightarrow c} h(x)$?

10. Determine if the function $f(x) = \begin{cases} 2x - 6 & \text{if } x \leq 4 \\ x^2 - 5x + 6 & \text{if } x > 4 \end{cases}$ is continuous. If the function is not continuous, explain why not. You must use limits to JYA (justify your answer)

11. Determine if the function $f(x) = \begin{cases} \frac{x^3 + 8}{x^2 - 4} & \text{if } x \neq -2 \\ 3 & \text{if } x = -2 \end{cases}$ is continuous. If the function is not continuous, explain why not. You must use limits to JYA.

12. State the three requirements for the continuity of $f(x)$ at some value c .

13. State the function whose derivative is represented by $\lim_{\Delta x \rightarrow 0} \frac{((x + \Delta x)^3 + 1) - (x^3 + 1)}{\Delta x}$

Find $\frac{dy}{dx}$ for each of the following and simplify.

14. $y = \frac{3x^2}{x-7}$

15. $y = (2x^2 - 1)(x^3 - x)^5$

16. $y = \frac{3x^2 - 12x}{x - 4}$

17. $y^2 - 3xy + 7x = 2$

18. $y = 2\sin x - 4\cos x + x$

19. $y = 5\sec x \tan x$

20. Find the value of $f'\left(\frac{\pi}{6}\right)$ if $f(x) = \sin x$

21. If $f(x) = x^2 \cos^2 x$, then $f'(\pi) =$

22. What is the slope of the line tangent to the curve $y = 4x^3 - 3x + 2$ at the point $(1, 3)$?

23. Find the equation of the line tangent to the graph of $f(x) = 3x^2 - 6x + 12$ at the point where $x = 0$.

24. Find the equation of the line tangent to the curve $3x^2 - xy + y^2 = 5$ at the point $(1, 2)$.

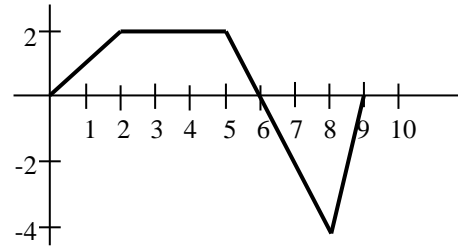
25. A 15-foot ladder is leaning against a house. If the tip of the ladder begins to slide down the house at a rate of 5 feet per second, what is the rate of change of the base of the ladder when the base is 9 feet from the house?

26. Suppose a pebble has been dropped into a pond forming a circular ripple. If the rate of change of the circumference of the circle is 4π in/s, what is the rate of change of the area of the circle when the radius is 10 inches?

27. The width of a rectangle is decreasing at 4 cm/s and the height is increasing at 2 cm/s. Determine the rate of change of the area of the rectangle when the width is 50 cm and the height is 5 cm.

The graph to the right represents the velocity of an object moving along the x -axis for $0 \leq t \leq 9$.

Use the graph to answer questions 28 – 33.



28. When is the particle at rest? Why?
29. At what time interval(s) is the particle moving to the left? Why?
30. What is the particle's acceleration at $t = 6$?
31. On what interval(s) is the particle's velocity increasing? Why?
32. On what interval(s) is the particle's speed increasing? Why?
33. During what time interval(s) does the object have the greatest acceleration?

34. Find the absolute maximum and minimum values of $f(x)$ on the interval $[0, 4]$ if $f(x) = x^2 - 3x - 4$

35. Determine if Rolle's Theorem can be applied to $f(x) = \sin x$ on $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$. If so, find c .

36. What can you conclude from the following information: $f(6) = 7, f'(6) = 0, f''(6) = -3$?

37. Determine the intervals on which the function $f(x) = \frac{x}{x^2 + x - 2}$ is decreasing. JYA

38. Determine the intervals on which the function $f(x) = x^4 - 8x^3 - 72x^2 + 24x$ is concave up. JYA

39. Determine the point(s) of inflection on the graph of $f(x) = e^{-x} \sin x$ on the interval $[0, \pi]$. JYA

40. A farmer has decided to build a pen for his horse alongside a drainage canal. If the pen is to be rectangular in shape and he has 400 feet of fencing available, what is the area of the largest possible pen he can create? He doesn't need to have fencing on the canal side of the pen.

41. A right circular cylinder is to be designed to hold 24 cubic inches of a soft drink. The cost for the material for the top and bottom of the can is \$0.02 per square inch and the cost for the material of the sides is \$0.01 per square inch. If the surface area of the can uses the formula, $S = 2\pi rh + 2\pi r^2$, and the volume of the can uses the formula $V = \pi r^2 h$, find the radius that will minimize the cost.

Find $\frac{dy}{dx}$ for each of the following:

42. $y = \ln \frac{\sqrt{x}}{5-x}$

43. $y = 4 \log_3 x$

44. $\ln(xy) = x + y$

45. $y = \frac{x^4}{4^x}$

46. $y = e^{-\frac{5}{x}}$

47. $y = x^{2x-3}$

48. If $f(x) = xe^x$, find $f''(x)$

49. Suppose $f(x) = \frac{8}{x^7 - 3x + 2}$ and $g(x)$ is the inverse of $f(x)$. Find $g'(2)$ given that $g(2) = -1$.

50. **HONOR CODE:** I confirm that all work contained in this project is my own. I have not collaborated with or asked questions of any other students or adults, with the exception of my teacher.

Signature

Date