AB/ Cirque Extra Credit Spring 2018 Assignment #2:

Directions: Create and Solve 1 original FRQ. Then answer the following 4 FRQs, make corrections (red ink) and score the FRQs. Due Mon(5/14)

1)

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

BC

2017 FRQ
Scoring Guideline

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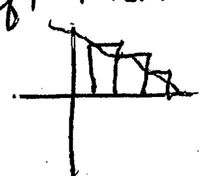
2 a) Volume $\displaystyle\int_0^{10} A(h)\,dh \approx 2(50.3) + 3(14.4) + 5(6.5) = \boxed{176.3\text{ ft}^3}$

2 b) Since $A(h)$ is decreasing, left Riemann Sum will overestimate the volume of tank.



2 c) $\displaystyle V(h) = \int_0^{10} f(h)\,dh = \int_0^{10} \frac{50.3}{e^{0.2h}+h}\,dh = \boxed{101.325\text{ ft}^3}$

3 d) $\displaystyle V = \int_0^h f(x)\,dx$

$\displaystyle \frac{dV}{dt} = \frac{d}{dt}\int_0^h f(x)\,dx$

Apply SFTC $\displaystyle \frac{d}{dx}\int^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$

$\displaystyle \frac{dV}{dt} = f(h)\cdot\frac{dh}{dt}$ ← $\begin{cases}\frac{dh}{dt} = 0.26\text{ ft/min}\\ h = 5\end{cases}$

$\displaystyle \frac{dV}{dt} = \frac{50.3}{e^{0.2h}+5}\cdot\frac{dh}{dt}$

$\displaystyle \frac{dV}{dt} = \frac{50.3}{e^{0.2(5)}+5}(0.26) = \boxed{1.694\text{ ft}^3/\text{min}}$

2)

Question

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

2)

AP® CALCULUS AB

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(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$
$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$
Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r, $1 < r < 3$, such that $h(r) = -5$.

$2: \begin{cases} 1: h(1) \text{ and } h(3) \\ 1: \text{conclusion, using IVT} \end{cases}$

(b) $\dfrac{h(3) - h(1)}{3 - 1} = \dfrac{-7 - 3}{3 - 1} = -5$
Since h is continuous and differentiable, by the Mean Value Theorem, there exists a value c, $1 < c < 3$, such that $h'(c) = -5$.

$2: \begin{cases} 1: \dfrac{h(3) - h(1)}{3 - 1} \\ 1: \text{conclusion, using MVT} \end{cases}$

(c) $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$

$2: \begin{cases} 1: \text{apply chain rule} \\ 1: \text{answer} \end{cases}$

(d) $g(1) = 2$, so $g^{-1}(2) = 1$.

$$\left(g^{-1}\right)'(2) = \frac{1}{g'\left(g^{-1}(2)\right)} = \frac{1}{g'(1)} = \frac{1}{5}$$

An equation of the tangent line is $y - 1 = \frac{1}{5}(x - 2)$.

$3: \begin{cases} 1: g^{-1}(2) \\ 1: \left(g^{-1}\right)'(2) \\ 1: \text{tangent line equation} \end{cases}$

3)

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

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The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

(a) $\int_0^{1.5} (3f'(x) + 4)\,dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$

$= 3f(x) + 4x\Big|_0^{1.5} = 3(-1 - (-7)) + 4(1.5) = 24$

2: { 1: antiderivative; 1: answer }

(b) $y = 5(x-1) - 4$

$f(1.2) \approx 5(0.2) - 4 = -3$

The approximation is less than $f(1.2)$ because the graph of f is concave up on the interval $1 < x < 1.2$.

3: { 1: tangent line; 1: computes y on tangent line at $x = 1.2$; 1: answer with reason }

(c) By the Mean Value Theorem there is a c with $0 < c < 0.5$ such that

$f''(c) = \dfrac{f'(0.5) - f'(0)}{0.5 - 0} = \dfrac{3 - 0}{0.5} = 6 = r$

2: { 1: reference to MVT for f' (or differentiability of f'); 1: value of r for interval $0 \le x \le 0.5$ }

(d) $\lim_{x\to 0^-} g'(x) = \lim_{x\to 0^-} (4x - 1) = -1$

$\lim_{x\to 0^+} g'(x) = \lim_{x\to 0^+} (4x + 1) = +1$

Thus g' is not continuous at $x = 0$, but f' is continuous at $x = 0$, so $f \ne g$.

OR

$g''(x) = 4$ for all $x \ne 0$, but it was shown in part (c) that $f''(c) = 6$ for some $c \ne 0$, so $f \ne g$.

2: { 1: answers "no" with reference to g' or g''; 1: correct reason }

AB #4

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

4) The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

AB #

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$\boxed{3}$ a) * To write tangent line equation, find point and slope:

point: $k(x) = f(g(x))$
$k(3) = f(g(3))$
$= f(6)$
$\underline{k(3) = 4}$

slope: * Use chain rule to find $k'(x)$ first.
$k(x) = f[g(x)]$
$k'(x) = f'[g(x)] \cdot g'(x)$
$k'(3) = f'[g(3)] \cdot g'(3)$

$k'(3) = f'(6) \cdot g'(3)$
$= f'(6) \cdot g'(3)$
$= 5 \cdot 2$
$\underline{k'(3) = 10}$

$\boxed{\text{Tangent line equation: } y - 4 = 10(x-3)}$

$\boxed{3}$ b) * Use quotient rule first to find $h'(x)$. Then find $h'(1)$

$h'(x) = \dfrac{g'(x)f(x) - g(x)f'(x)}{f(x)^2}$

$h'(1) = \dfrac{g'(1)f(1) - g(1)f'(1)}{f(1)^2} = \dfrac{8(-6) - 2(3)}{(-6)^2}$

$h'(1) = \dfrac{-54}{36} = \boxed{-\dfrac{3}{2}}$

$\boxed{3}$ c) * Go through u-substitution first to handle the $(2x)$

$\int_1^3 f''(2x)\,dx$

$u = 2x \quad dx = \dfrac{du}{2}$

$\dfrac{du}{dx} = 2$

$\int f''(u) \cdot \dfrac{du}{2}$

$= \dfrac{1}{2}\int f''(u)\,du$

* Apply FFTC $\int_a^b f'(x)\,dx = f(b) - f(a)$

$= \dfrac{1}{2} f'(2x)\Big]_1^3 = \dfrac{1}{2} \cdot f'(2\cdot 3) - \dfrac{1}{2} f'(2\cdot 1)$

$= \dfrac{1}{2} f'(6) - \dfrac{1}{2} f'(2)$

$= \dfrac{1}{2}(5) - \dfrac{1}{2}(-2) = \boxed{7/2}$