Continuity/Particle Motion/Curve Sketch FRQ Practice Problems

1) Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

2) Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1}\, for\ 0 \le x < 3 \\ x - 2 \quad for \quad x = 3 \\ 5 - x\, for \quad 3 < x \le 5 \end{cases}$$

a) $Is\ f\ continuous\ at\ x = 3?\ \ Justify\ using\ continuity\ conditions$
b) $If\ not\ continuous, determine\ type\ of\ discontinuity\ and\ justify.$
c) $Find\ tangent\ line\ to\ the\ curve\ of\ f\ at\ x = 2$

Question 3

A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

~~(c)~~ Find the position of the particle at time $t = 2$.

~~(d)~~ Find the average speed of the particle over the interval $0 \le t \le 2$.

d) Is velocity increasing or decreasing at $t = 1$?

e) Is speed increasing or decreasing at t = 3? Justify your answer

4) a) Given the function $\boxed{y = x - 2\cos x}$ on the interval $[-\pi, \pi]$, find intervals increasing, decreasing, relative extrema (ordered pairs!), Points of Inflection (ordered pairs!), intervals of concave up/down. Justify your answers!

b) Create one sign line with all critical points from $f'(x)$ and $f''(x)$

c) Find Absolute max and Absolute minimum value on closed interval $[-\pi, \pi]$

5) A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

b) If not continuous, describe type of discontinuity and justify.

6)

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \ne 2 \\ 1 & \text{if } x = 2 \end{cases}$$

Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

III. f is differentiable at $x = 2$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

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* continuity conditions

i) $f(c)$ exists

ii) $\lim_{x\to c} f(x)$ exists $\left[\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)\right]$

iii) $f(c) = \lim_{x\to c} f(x)$

1) Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

i) $f(0) = 1 - 2\sin(0) = 1$

ii) $\lim_{x\to 0^-} 1 - 2\sin x = 1$ $\lim_{x\to 0^+} e^{-4x} = e^{0} = 1$, $\lim_{x\to 0} f(x) = 1$

iii) $f(0) = \lim_{x\to 0} f(x) = 1$; therefore f is continuous at $x = 0$

2) Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} \; for\; 0 \le x < 3 \\ x - 2 \quad for \quad x = 3 \\ 5 - x \; for \quad 3 < x \le 5 \end{cases}$$

a) *Is f continuous at $x = 3$? Justify using continuity conditions*
b) *If not continuous, determine type of discontinuity and justify.*
c) *Find tangent line to the curve of f at $x = 2$*

a) i) $f(3) = 3 - 2 = 1$

ii) $\lim_{x\to 3^-} \sqrt{x+1} = \sqrt{4} = 2$ $\lim_{x\to 3^+} 5 - x = 5 - 3 = 2$, $\lim_{x\to 3} f(x) = 2$

iii) $f(3) \ne \lim_{x\to 3} f(x)$. Function not continuous at $x = 3$

b) Removable discontinuity at $x = 3$ since $\lim_{x\to 3} f(x)$ exists but $f(3) \ne \lim_{x\to 3} f(x)$ (3rd condition for continuity fails)

c) $f(2) = \sqrt{2+1} = \sqrt{3}$ $f(x) = (x+1)^{1/2}$

$f'(x) = \frac{1}{2}(x+1)^{-1/2}$ | $f'(2) = \frac{1}{2\sqrt{3}}$

$f'(x) = \frac{1}{2\sqrt{x+1}}$

point: $(2, \sqrt{3})$ slope: $m = \frac{1}{2\sqrt{3}}$

$$\boxed{y - \sqrt{3} = \frac{1}{2\sqrt{3}}(x - 2)}$$

1. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

~~(c)~~ Find the position of the particle at time $t = 2$.

~~(d)~~ Find the average speed of the particle over the interval $0 \le t \le 2$.

d) Is velocity increasing or decreasing at $t=1$?

e) Is speed increasing or decreasing at t = 3? Justify your answer

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a) $v'(t) = a(t) = \dfrac{2t-3}{t^2-3t+3}$

$a(4) = \dfrac{8-3}{4^2-12+3} = \dfrac{5}{7}$

b) set $v(t) = 0$

$\ln(t^2-3t+3) = 0$

$\log_e(t^2-3t+3) = 0$

$e^0 = t^2-3t+3$

$1 = t^2-3t+3$

$0 = t^2-3t+2$

$0 = (t-2)(t-1)$

$t = 1,\ t = 2$

$+ \quad - \quad +$

$0 \quad 3/2 \quad 3$

$1 \quad 2$

particle changes direction at $t=1$ and $t=2$ since $v(t)$ changes signs.

particle travels left $1 < t < 2$ since $v(t) < 0$

e) Increasing speed since $v(4) > 0$ and $a(4) > 0$ (same signs)

d) $a(1) < 0$, so velocity is decreasing.

a) Given the function $\boxed{y = x - 2\cos x}$ on the interval $[-\pi, \pi]$, find intervals increasing, decreasing, relative extrema (ordered pairs!), Points of Inflection (ordered pairs!), intervals of concave up/down. Justify your answers! $(\sqrt{3} \approx 1.7)$

b) Create one sign line with all critical points from f'(x) and f''(x)

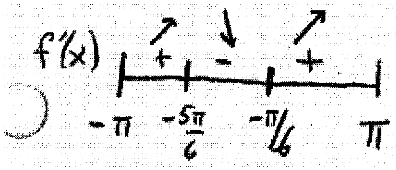
c) Find Abs max and Abs min value

$y' = 1 - 2(-\sin x) = 1 + 2\sin x$

$0 = 1 + 2\sin x$

$\sin x = -1/2$

$x = -\frac{\pi}{6}, -\frac{5\pi}{6}$

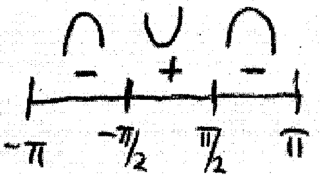


Rel. max : $\left(-\frac{5\pi}{6}, -\frac{5\pi}{6} + \sqrt{3}\right)$

Rel. min $\left(-\frac{\pi}{6}, -\frac{\pi}{6} - \sqrt{3}\right)$

$y'' = 0 + 2\cos x = 2\cos x$

$2\cos x = 0 \quad x = -\frac{\pi}{2}, \pi/2$



POI: $(-\pi/2, -\pi/2)$ and $\left(\frac{\pi}{2}, \pi/2\right)$

c) * Apply EVT: (To find Abs max/Abs min)

$f(-\pi) = -\pi - 2\cos(-\pi) \approx -\pi + 2 \approx -1.14$

$f(\pi) = \pi - 2\cos(\pi) = \pi + 2 \approx 5.14$

$f(-5\pi/6) = -\frac{5\pi}{6} + \sqrt{3}$

$f(-\pi/6) = -\frac{\pi}{6} - \sqrt{3}$

Abs min value is $-\frac{\pi}{6} - \sqrt{3}$

Abs max value is $\pi + 2$

5) A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

b) If not continuous, describe type of discontinuity and justify.

a) i) $r(5) = \frac{600(5)}{5+3} = 375$

ii) $\lim_{t\to5^-} \frac{600t}{t+3} = 375$ $\quad \lim_{t\to5^+} 1000e^{-\frac{1}{5}t} = \frac{1000}{e}$, $\lim_{t\to5} r(t)$ does not exist

b) Since $\lim_{t\to5} r(t)$ does not exist, $r(t)$ is not continuous and has nonremovable discontinuity since $\lim_{t\to5} r(t) = DNE$

6)

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \ne 2 \\ 1 & \text{if } x = 2 \end{cases}$$

Let f be the function defined above. Which of the following statements about f are true?

✓ I. f has a limit at $x = 2$.

X II. f is continuous at $x = 2$.

X III. f is differentiable at $x = 2$.

$\boxed{\text{(A) I only}}$

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

i) $f(2) = 1$

ii) $\lim_{x\to2} \frac{x^2-4}{x-2} = \frac{0}{0}$ $\quad \lim_{x\to2} \frac{(x-2)(x+2)}{\cancel{(x-2)}} = 4$

iii) $f(2) \neq \lim_{x\to2} f(x)$, so $f(x)$ not continuous (removable discontinuity)