

Key

AP Calculus AB First Semester Final Exam Review Packet

1. The domain of $g(x) = \frac{\sqrt{x-2}}{x^2-x}$ is *combine restrictions from numerator and denominator to find domain

D $g(x) = \frac{\sqrt{x-2}}{x(x-1)} \rightarrow x \geq 2$ $\rightarrow x \neq 0, 1$ Domain: $[2, \infty)$

- a. $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$ b. $(-\infty, 0) \cup (0, 1) \cup (1, 2]$ c. $(-\infty, 2]$ d. $[2, \infty)$ e. $(2, \infty)$

2. Which of the following functions is not odd?

C Odd: $f(-x) = -f(x)$

b) $\sin(-2x) = -\sin(2x)$ odd

- a) $\sin x$: $\sin(-x) = -\sin x$ (odd) $|c)(x)^3 + 1 \neq -(x^3 + 1)$ (not odd)
 a. $f(x) = \sin x$ b. $f(x) = \sin 2x$ c. $f(x) = x^3 + 1$ d. $f(x) = \frac{\sqrt{-x}}{(x-x)^2+1} = -\left(\frac{x}{x^2+1}\right)$ e. $f(x) = \sqrt[3]{-2x} = -\sqrt[3]{2x}$

3. Which of the following is a reflection of the graph of $y = f(x)$ over the x-axis?

- A $(x\text{-axis reflection})$ $(y\text{-axis reflection})$ transformed graph all exists above x-axis only displays portion of graph to right of y-axis 180° rotation about origin
- a. $y = -f(x)$ b. $y = f(-x)$ c. $y = |f(x)|$ d. $y = f(|x|)$ e. $y = -f(-x)$

C 4. $\lim_{x \rightarrow 3} \frac{x-3}{x^2-2x-3} = \frac{3-3}{9-6-3} = \frac{0}{0}$ $\lim_{x \rightarrow 3} \frac{(x-3)}{(x-3)(x+1)} = \lim_{x \rightarrow 3} \frac{1}{x+1} = \boxed{\frac{1}{4}}$

a. 0 b. 1 c. $\frac{1}{4}$ d. ∞ e. none of these

A 5. $\lim_{x \rightarrow 0} \frac{x}{x} = \boxed{1}$

a. 1 b. 0 c. $\frac{0}{0}$ d. -1 e. nonexistent

D 6. $\lim_{x \rightarrow 2} \frac{x^3-8}{x^2-4} = \frac{8-8}{4-4} = \frac{0}{0}$ $\lim_{x \rightarrow 2} \frac{x^3-2^3}{x^2-4} = \lim_{x \rightarrow 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \frac{2^2+2(2)+4}{2+2} = \frac{12}{4} = \boxed{3}$

* $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ (S.O.A.P.)

- a. 4 b. 0 c. 1 d. 3 e. ∞

B 7. $\lim_{x \rightarrow \infty} \frac{4-x^2}{4x^2-x-2} = \boxed{-\frac{1}{4}}$

* compare degrees between numerator and denominator

a. -2 b. $-\frac{1}{4}$ c. 1 d. 2 e. nonexistent

Since Degrees match, take ratio of leading coefficients

compare degrees, since $N > D$, $\lim_{x \rightarrow \infty} f(x) = +\infty$ or $-\infty$

A 8. $\lim_{x \rightarrow \infty} \frac{50x^3+27}{20x^3+10x+9} = \frac{+}{+} = \boxed{\infty}$

- a. ∞ b. $\frac{1}{4}$ c. 3 d. 0 e. 1

E 9. $\lim_{x \rightarrow \infty} \frac{3x^2+27}{(x^3)-27} = *N < D$, $\lim_{x \rightarrow \infty} f(x) = \boxed{0}$

- a. 3 b. ∞ c. 1 d. -1

e. 0

B 10. $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \frac{0}{0}$ $\lim_{x \rightarrow 0} \tan x \cdot \frac{1}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = \lim_{x \rightarrow 0} \frac{1}{\cos x} = \boxed{1}$

- a. 0 **b. 1** c. π d. ∞ e. nonexistent

D 11. $\lim_{x \rightarrow 0} \frac{|x|}{x} = \frac{|x|}{x} = \begin{cases} \frac{x}{x}, x \geq 0 \\ \frac{-x}{x}, x < 0 \end{cases} = \begin{cases} 1, x \geq 0 \\ -1, x < 0 \end{cases}$ Since $\lim_{x \rightarrow 0^+} f(x) = 1$ and $\lim_{x \rightarrow 0^-} f(x) = -1$
 $\lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x)$ **limit DNE**

- a. 0 b. 1 c. -1 **d. nonexistent** e. none of these

E 12. $\lim_{x \rightarrow \infty} \sin x =$ 

- a. 0 b. -1 or 1 c. oscillates between -1 and 1 d. ∞ **e. nonexistent**

C 13. $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 4x} = \frac{0}{0}$ $\rightarrow \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 4x} \cdot \frac{x}{x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \cdot \frac{x}{\sin 4x} = \frac{3}{1} \cdot \frac{1}{4} = \boxed{\frac{3}{4}}$
 $* \lim_{x \rightarrow 0} \frac{\sin ax}{ax} = \frac{a}{a}$

- a. 0 b. 1 **c. $\frac{3}{4}$** d. $\frac{4}{3}$ e. nonexistent

E 14. $\lim_{x \rightarrow 0} \frac{1-\cos x}{x} = 0$ $* \lim_{x \rightarrow 0} \frac{1-\cos(ax)}{ax} = 0$

- a. 1 b. 2 c. ∞ d. nonexistent

e. none of these

15. Let $f(x) = \begin{cases} x^2 - 1, & x \neq 1 \\ 4, & x = 1 \end{cases}$

Which of the following statements are true?

- C** ✓ I. $\lim_{x \rightarrow 1} f(x)$ exists ✓ II. $f(1)$ exists X III. f is continuous at $x = 1$

- a. I only b. II only **c. I and II** d. I, II and III e. none of them

* Step through continuity conditions

i) $f(1) = 4$ **iii) $f(1) \neq \lim_{x \rightarrow 1} f(x)$**

ii) $\lim_{x \rightarrow 1} x^2 - 1 = 1^2 - 1 = 0$ * Fails 3rd condition, Removable Discontinuity

B 16. If $f(x) = \frac{x^2-x}{2x}$ for $x \neq 0$, $f(0) = k$ and f is continuous at $x = 0$, then $k =$

continuity conditions

$$\text{i)} f(0) = k \quad \text{ii)} \lim_{x \rightarrow 0} \frac{x^2-x}{2x} = \frac{0}{0} = \frac{x(x-1)}{2x} = \lim_{x \rightarrow 0} \frac{x-1}{2} = -\frac{1}{2} \quad \text{iii)} f(0) = \lim_{x \rightarrow 0} f(x) \text{ when } k = -\frac{1}{2}$$

a. -1

b. $-\frac{1}{2}$

c. 0

d. $\frac{1}{2}$

e. 1

* Test continuity conditions for both $x=1$ and $x=2$. 17. If $f(x) = \frac{3x(x-1)}{x^2-3x+2}$ for $x \neq 1, 2$ and $f(1) = -3$ and $f(2) = 4$, then $f(x)$ is continuous

$$\begin{cases} \text{At } x=1: \\ \text{i)} f(1) = -3 \\ \text{ii)} \lim_{x \rightarrow 1} \frac{3x(x-1)}{(x-2)(x-1)} = \frac{3}{-1} = -3 \end{cases}$$

$$\text{iii)} f(1) = \lim_{x \rightarrow 1} f(x) \vee$$

$$\begin{cases} \text{At } x=2: \\ \text{i)} f(2) = 4 \\ \text{ii)} \lim_{x \rightarrow 2} \frac{3x(x-1)}{(x-2)(x-1)} = \frac{6}{0} \text{ DNE} \end{cases}$$

a. except at $x = 1$

b. except at $x = 2$

c. except at $x = 1$ or 2

d. except at $x = 0, 1, \text{ or } 2$

e. at all real numbers

* Test continuity conditions for both $x=1$ and $x=2$. 18. If $y = \frac{x}{\sqrt{1-x^2}}$, find $\frac{dy}{dx}$

$$\begin{cases} \text{quotient rule} \\ \text{chain rule} \end{cases} \quad \frac{f'g - fg'}{g^2}$$

$$y = \frac{x}{(1-x^2)^{1/2}}$$

a. $\frac{1-2x^2}{(1-x^2)^{3/2}}$

b. $\frac{1-2x^2}{(1-x^2)^{1/2}}$

c. $\frac{1}{\sqrt{1-x^2}}$

d. $\frac{1}{1-x^2}$

e. none of these

$$\begin{aligned} y' &= \frac{1((1-x^2)^{1/2}) - x \cdot \frac{1}{2}(1-x^2)^{-1/2}(-2x)}{((1-x^2)^{1/2})^2} \\ &= \left(\frac{\sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}}}{1-x^2} \right) \left(\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} \right) \\ &= \frac{1}{(1-x^2)^{3/2}} \end{aligned}$$

19. If $y = \cos x^2$, find y'

$$y = \cos(x^2)$$

$$y' = -\sin(x^2) \cdot 2x = \boxed{-2x \sin(x^2)}$$

* chain rule

a. $2x \sin x^2$

b. $-\sin x^2$

c. $-2 \sin x \cos x$

d. $-2x \sin x^2$

e. $\sin 2x$

20. If $y = \sin^2 3x + \cos^2 3x$, find $\frac{dy}{dx}$

* chain rule

$$y = [\sin 3x]^2 + [\cos 3x]^2$$

$$y' = 2[\sin 3x]^1 \cos(3x) \cdot 3 + 2[\cos 3x]^1 (-\sin 3x) \cdot 3 =$$

$$6\sin 3x \cos 3x - 6\sin 3x \cos 3x = \boxed{0}$$

* Also, since $\sin^2 3x + \cos^2 3x = 1$
then $\frac{dy}{dx} = \boxed{0}$

a. $-6 \sin 6x$

b. 0

c. $12 \sin 3x \cos 3x$

d. $6(\sin 3x + \cos 3x)$

e. 1

21. If $y = \cos^2 x$, find y'

$$y = [\cos x]^2$$

$$y' = 2[\cos x]^1 (-\sin x)(1) = -2 \sin x \cos x = \boxed{-\sin 2x}$$

a. $-\sin^2 x$

b. $2 \sin x \cos x$

c. $-\sin 2x$

d. $2 \cos x$

e. $-2 \sin x$

22. If $y = \frac{1}{2 \sin 2x}$, find $\frac{dy}{dx}$

$$y = \frac{\csc(2x)}{2} = \frac{1}{2} \csc(2x)$$

- a. $-\csc 2x \cot 2x$ b. $\frac{1}{4 \cos 2x}$ c. $-4 \csc 2x \cot 2x$ d. $\frac{\cos 2x}{2\sqrt{\sin 2x}}$ e. $-\csc^2 2x$

$$\begin{aligned} y' &= \frac{1}{2} (-\csc 2x \cot 2x) \cdot 2 \\ &= -\csc 2x \cot 2x \end{aligned}$$

* $\frac{d}{dx} \csc u = -\csc u \cot u \cdot u'$

23. If $x + \cos(x+y) = 0$, find $\frac{dy}{dx}$ * chain rule, implicit differentiation

$$1 + -\sin(x+y) \left[1 + \frac{dy}{dx} \right] = 0$$

$$1 - \sin(x+y) - \frac{dy}{dx} \sin(x+y) = 0$$

a. $\csc(x+y) - 1$

b. $\csc(x+y)$

$$-\frac{dy}{dx} \sin(x+y) = \sin(x+y) - 1$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\sin(x+y) - 1}{-\sin(x+y)} = \frac{\sin(x+y)}{-\sin(x+y)} + \frac{-1}{-\sin(x+y)} \\ &= -1 + \csc(x+y) \end{aligned}$$

c. $\frac{x}{\sin(x+y)}$

d. $\frac{1}{\sqrt{1-x^2}}$

e. $\frac{1-\sin x}{\sin y}$

24. If $x^3 - xy + y^3 = 1$, find $\frac{dy}{dx}$ * implicit differentiation, product Rule

$$3x^2 - \left[1y + x \frac{dy}{dx} \right] + 3y^2 \left(\frac{dy}{dx} \right) = 0$$

$$3x^2 - y - x \left(\frac{dy}{dx} \right) + 3y^2 \left(\frac{dy}{dx} \right) = 0$$

a. $\frac{3x^2}{x-3y^2}$

b. $\frac{3x^2-1}{1-3y^2}$

c. $\frac{y-3x^2}{3y^2-x}$

d. $\frac{3x^2+3y^2-y}{x}$

e. $\frac{3x^2+3y^2}{x}$

$$\frac{dy}{dx} (3y^2 - x) = y - 3x^2$$

$$\frac{dy}{dx} = \frac{y-3x^2}{3y^2-x}$$

25. If $f(x) = 16\sqrt{x}$, then $f'''(4) =$

$f(x) = 16x^{1/2}$

$f'(x) = 16 \cdot \frac{1}{2} x^{-1/2} = 8x^{-1/2}$

a. $\frac{3}{16}$

b. -4

c. $-\frac{1}{2}$

d. 0

e. 6

$$f''(x) = 8 \cdot \frac{1}{2} x^{-3/2} = -4x^{-3/2}$$

$$f'''(x) = -4 \cdot \frac{3}{2} x^{-5/2} = 6x^{-5/2}$$

$$f'''(x) = \frac{6}{x^{5/2}}$$

$$f'''(4) = \frac{6}{(4)^{5/2}} = \frac{6}{2^5} = \frac{6}{32} = \frac{3}{16}$$

* Implicit Diff.

26. If $x^2 + y^2 = 25$, then at $(0, 5)$ $\frac{d^2y}{dx^2} =$

$2x + 2y \left(\frac{dy}{dx} \right) = 0$

$2y \left(\frac{dy}{dx} \right) = -2x$

a. 0

b. $\frac{1}{5}$

c. -5

$$\frac{d^2y}{dx^2} = \frac{(-1)(y) - (-x)(\frac{dy}{dx})}{y^2}$$

$$= \frac{-y + x \left(\frac{-x}{y} \right)}{y^2}$$

$$= \frac{-y - x^2/y}{y^2}$$

$$= \frac{-y^2 - x^2}{y^3}$$

$$= \frac{-5^2 - 0}{5^3} = \frac{-25}{125} = -\frac{1}{5}$$

B 27. If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

a. $\frac{-\sqrt{x}}{(x^2+1)^2}$

b. $-(x+1)^{-2}$

c. $\frac{-2x}{(x^2+1)^2}$

d. $\frac{1}{(x+1)^2}$

e. $\frac{1}{2\sqrt{x}(x+1)}$

$$f(g(x)) = \frac{1}{(\sqrt{x})^2 + 1} = \frac{1}{x+1} = (x+1)^{-1}$$

$$\frac{d}{dx} f(g(x)) = -1(x+1)^{-2}(1) = \boxed{\frac{-1}{(x+1)^2} \text{ or } -(x+1)^{-2}}$$

limit definition $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

C 28. $\lim_{h \rightarrow 0} \frac{(1+h)^6 - 1}{h} =$ → This is asking for $f'(1)$ for $f(x) = x^6 \rightarrow f'(x) = 6x^5$

$$f'(1) = 6(1)^5 = 6$$

- a. 0 b. 1 **c. 6** d. ∞ e. nonexistent

B 29. $\lim_{h \rightarrow 0} \frac{\sqrt[3]{8+h} - 2}{h} =$ * limit definition Given $f(x) = \sqrt[3]{x}$, find $f'(8)$:
of derivative

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$$

- a. 0 **b. $\frac{1}{12}$** c. 1 d. 192 e. ∞

$$f'(8) = \frac{1}{3(8)^{\frac{2}{3}}} = \frac{1}{3(2)^2} = \frac{1}{12}$$

D 30. Suppose $\lim_{x \rightarrow 0} \frac{g(x) - g(0)}{x} = 1$. It follows that

In the form of

limit definition of derivative:

Alternative

Form of Derivative.

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

- a. g is not defined at $x = 0$ b. g is not continuous at $x = 0$
c. The limit of $g(x)$ as x approaches 0 equals 1 **d. $g'(0) = 1$** e. $g'(1) = 0$

E 31. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value

Theorem because

$f(x)$ continuous on $[-8, 8] \checkmark$ $f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3x^{\frac{1}{3}}}$ $x \neq 0$ slope does not exist at $x = 0$

* check differentiability: $f(x)$ not differentiable at $x = 0$ so

- a. $f(0)$ is not defined b. $f(x)$ is not continuous on $[-8, 8]$
c. $f'(-1)$ does not exist d. $f(x)$ is not defined for $x < 0$
e. $f'(0)$ does not exist

$$f'(0) = \text{DNE}$$

32. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

- Does not pass Rolle's Theorem
 ✓ i) $f(x)$ continuous on $[a, b]$
 ✗ ii) $f(x)$ differentiable on (a, b)
 ✓ iii) $f(a) = f(b)$

B



- a. $f(x)$ must be identically zero (**F**) b. **T** $f'(x)$ may be different from zero for all x on $[a, b]$
 (F) c. there exists at least one number c , $a < c < b$, such that $f'(c) = 0$ (not necessarily, can be sharp point)
 (F) d. $f'(x)$ must exist for every x on (a, b) (not necessarily) e. none of the preceding is true

$f(x)$ continuous on $[0, \sqrt{3}]$, $f(x)$ differentiable on $(0, \sqrt{3})$, $f(\sqrt{3}) = 0$] ✓

33. If c is the number defined by Rolle's Theorem, then for $f(x) = 2x^3 - 6x$ on the interval

A

$$\begin{array}{l} [0, \sqrt{3}], c \text{ is } * \text{ set } f'(x) = 0 \\ f'(x) = 6x^2 - 6 \end{array} \quad \left| \begin{array}{l} 6x^2 - 6 = 0 \\ 6(x^2 - 1) = 0 \\ 6(x+1)(x-1) = 0 \end{array} \right| \quad \left| \begin{array}{l} x = 1, x = -1 \\ c = 1 \text{ on interval } (0, \sqrt{3}) \end{array} \right.$$

- a. 1 b. -1 c. $\sqrt{2}$ d. 0 e. $\sqrt{3}$

34. The slope of the curve $y^3 - xy^2 = 4$ at the point where $y = 2$ is * Find $\frac{dy}{dx}$ and plug in point
Find point: at $y=2$, $2^3 - x(2)^2 = 4$ * Implicit diff, product rule
 $8 - 4x = 4$ | $x = 1$ | $3y^2 \left(\frac{dy}{dx}\right) - (y^2 + x \cdot 2y \left(\frac{dy}{dx}\right)) = 0$ | plug in $(1, 2)$ find slope
 $-4x = -4$ | point: $(1, 2)$ | $3y^2 \left(\frac{dy}{dx}\right) - y^2 - 2xy \left(\frac{dy}{dx}\right) = 0$ | $3(2)^2 \left(\frac{dy}{dx}\right) - 2^2 - 2(1)(2) \frac{dy}{dx} = 0$
 $12 \frac{dy}{dx} - 4 - 4 \frac{dy}{dx} = 0$ | $8 \frac{dy}{dx} - 4 = 0$ | $\frac{dy}{dx} = \frac{4}{8} = \frac{1}{2}$

D

- a. -2 b. $\frac{1}{4}$ c. $-\frac{1}{2}$ d. $\frac{1}{2}$

$$\boxed{d. \frac{1}{2}}$$

$$\begin{array}{l} e. 2 \frac{dy}{dx} - 4 - 4 \frac{dy}{dx} = 0 \\ 8 \frac{dy}{dx} - 4 = 0 \\ \frac{dy}{dx} = \frac{4}{8} = \frac{1}{2} \end{array}$$

E

$$\begin{array}{l} 35. \text{ The equation of the line tangent to the curve } y = x \sin x \text{ at the point } \left(\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ is} \\ * \text{product rule} \\ y = x \sin x + x \cos x \end{array} \quad \left| \begin{array}{l} y'(\frac{\pi}{2}) = \sin(\frac{\pi}{2}) + \frac{\pi}{2} \cos(\frac{\pi}{2}) \\ = 1 + 0 = 1 \end{array} \right| \quad \left| \begin{array}{l} \text{point: } (\frac{\pi}{2}, \frac{\pi}{2}) \\ \text{slope: } M = 1 \end{array} \right| \quad \left| \begin{array}{l} y - \frac{\pi}{2} = 1(x - \frac{\pi}{2}) \\ y = x \end{array} \right|$$

- a. $y = x - \pi$ b. $y = \frac{\pi}{2}$

$$c. y = \pi - x$$

$$d. y = x + \frac{\pi}{2}$$

* point where slope is steepest

D

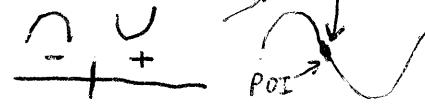
36. The minimum value of the slope of the curve $y = x^5 + x^3 - 2x$ is

* this is basically POI where concavity changes from concave down to up | $y' = 5x^4 + 3x^2 - 2$ | $0 = 2x(10x^2 + 3)$
| $y'' = 20x^3 + 6x$ | $x = 0, x^2 = -\frac{3}{10}$ | $y'(0) = 5(0) + 3(0)^2 - 2$

- a. 0 b. 2 c. 6

$$d. -2$$

e. none of these



$$\boxed{y'(0) = -2}$$

C

$$\begin{array}{l} * \text{Find } \frac{dy}{dx}, \text{ implicit differentiation} \\ 2x - 2y \left(\frac{dy}{dx}\right) = 0 \end{array} \quad \left| \begin{array}{l} \frac{dy}{dx} = \frac{x}{y} \\ -2y \left(\frac{dy}{dx}\right) = -2x \end{array} \right. \quad \left| \begin{array}{l} \frac{dy}{dx}(4, 2) = \frac{4}{2} = 2 \\ \text{point: } (4, 2) \\ \text{slope: } m = 2 \end{array} \right. \quad \left| \begin{array}{l} y - y_1 = m(x - x_1) \\ y - 2 = 2(x - 4) \\ y - 2 = 2x - 8 \end{array} \right. \quad \boxed{y = 2x - 6}$$

- a. $x - 2y + 6 = 0$ b. $y = 2x$ c. $y = 2x - 6$

$$d. y = \frac{x}{2}$$

$$e. x + 2y = 6$$

D

38. The line tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

* Find value(s) that will cause denominator of $\frac{dy}{dx}$ to be zero, therefore undefined slope.

* Implicit differentiation, product rule

$$2y \left(\frac{dy}{dx}\right) - [ly + x \left(\frac{dy}{dx}\right)] = 0$$

- a. $y = 0$ b. $y = \pm\sqrt{3}$

$$\frac{dy}{dx} = \frac{y}{2y-x}$$

c. $y = \frac{1}{2}$

* plug into original equation, solve for y
 $y^2 - (2y)y + 9 = 0$

$$\boxed{d. y = \pm 3}$$

e. none of these

$$2y \left(\frac{dy}{dx}\right) - y - x \left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}(2y - x) = y$$

* set $2y - x = 0$

$$x = 2y$$

$$y^2 - 2y^2 + 9 = 0$$

$$-y^2 = -9$$

$$\boxed{y = \pm 3}$$

(6)

* find $f'(x)$, create sign line, test intervals and count number of sign changes.

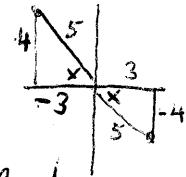
39. The total number of relative maximum and minimum points on the function whose

B derivative, for all x , is given by $f'(x) = x(x-3)^2(x+1)^4$ is

Set $f'(x) = 0$

$x=0$	-1	0	3
-	+	-	+

Only 1 minimum



- a. 0 **b. 1** c. 2 d. 3 e. none of these

- D** 40. On the closed interval $[0, 2\pi]$, find the maximum value of the function $f(x) = 4 \sin x - 3 \cos x$. (This is a bit of a challenge problem. \circlearrowleft EVT, test endpoints and critical pts.)

$$\begin{array}{ll} f(0) = 0 - 3 = -3 & f'(x) = 4 \cos x + 3 \sin x = 0 \\ f(2\pi) = 0 - 3 = -3 & 4 \cos x = -3 \sin x \\ & -4/3 = \tan x \\ & x = \tan^{-1}(-4/3) \quad \text{2nd Quad} \\ & x = \tan^{-1}(4/3) \quad \text{4th Quad} \\ \text{a. } 3 & \text{b. } 4 \\ \text{c. } \frac{24}{5} & \text{d. } 5 \\ \text{e. none of these} & \end{array}$$

Max value

41. The line $y = 3x + k$ is tangent to the curve $y = x^3$ when k is equal to

slope of 3 :
 Find where $y = x^3$ has slope of 3 .
 $y' = 3x^2$ $y = 3x + k$ $y = 3x + k$
 $3 = 3x^2$ $-1 = 3(-1) + k$ $1 = 3(1) + k$
 $x = \pm 1$ $2 = k$ $-2 = k$

- a. 1 or -1 b. 0 $f'(1) = 1$ c. 3 or -3 d. 4 or -4 **e. 2 or -2**

42. A balloon is being filled with helium at the rate of $4 \frac{\text{ft}^3}{\text{min}}$. Find the rate, in square feet per

C minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3} \text{ ft}^3$. Note, the volume of a sphere is $\frac{4}{3}\pi r^3$ and the surface area of a sphere is $4\pi r^2$.

$$\begin{array}{ll} \frac{dV}{dt} = 4 \text{ ft}^3/\text{min} & V = \frac{4}{3}\pi r^3 \\ \text{Find } \frac{ds}{dt} = & \frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt} \right) \\ \text{a. } 4\pi & 4 = 4\pi(2)^2 \left(\frac{dr}{dt} \right) \\ \text{b. } 2 & 4 = 16\pi \left(\frac{dr}{dt} \right) \\ \text{c. } 4 & \frac{4}{16\pi} = \frac{1}{4\pi} = \frac{dr}{dt} \end{array}$$

For questions 43 - 46, the motion of a particle on a straight line is given by

$$x(t) = t^3 - 6t^2 + 12t - 8.$$

$$\begin{array}{ll} v(t) = 3t^2 - 12t + 12 & t=2 \\ 0 = 3(t^2 - 4t + 4) & \text{---} \\ 0 = 3(t-2)(t-2) & \text{Increasing: } (-\infty, 2) \cup (2, \infty) \end{array}$$

- B** 43. The distance x is increasing for

- a. $t < 2$ b. all t except $t = 2$ c. $1 < t < 3$ d. $t < 1$ or $t > 3$ e. $t > 2$

- D** 44. The minimum value of the speed is

- a. 1 b. 2 c. 3 **d. 0** e. none of these

speed = |velocity|, so minimum velocity is zero.

For questions 43 – 46, the motion of a particle on a straight line is given by $x(t) = t^3 - 6t^2 + 12t - 8$.

45. The acceleration is positive when $a(t) = 6t^2 - 12$

$$0 = 6t^2 - 12$$

$$6t = 12 \quad t = 2$$

$\begin{array}{c} - \\ + \\ 2 \end{array}$

$a(t) > 0 \text{ when } t > 2$

A

- a. $t > 2$ b. $t \neq 2$ c. $t < 2$ d. $1 < t < 3$ e. $1 < t < 2$

46. The speed of the particle is decreasing for $v(t)$ $\begin{array}{c} + \\ - \\ + \end{array}$ $a(t)$ $\begin{array}{c} - \\ + \\ 2 \end{array}$ *Speed is decreasing when $v(t)$ and $a(t)$ have opposite signs when $t < 2$*

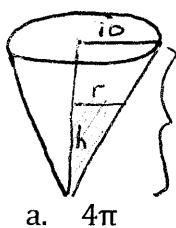
E

a. $t > 2$ b. $t < 3$ c. all t d. $t < 1$ or $t > 2$ e. none of these

47. A circular conical reservoir, vertex down, has depth 20 ft and radius of the top 10 ft.

Water is leaking out so that the surface is falling at the rate of $\frac{1}{2}$ ft/hour. The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 ft deep is $V = \frac{\pi}{3} r^2 h$

*similar triangle $h = 8$ ft.



$$\frac{dh}{dt} = -\frac{1}{2} \text{ ft/hr}$$

Find $\frac{dv}{dt} =$

$a. 4\pi$	$b. 8\pi$	$c. 16\pi$
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*Rewrite equation in terms of h

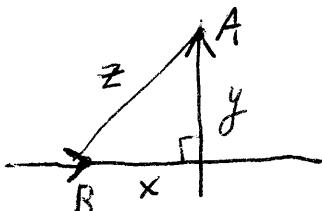
$\frac{r}{10} = \frac{h}{20}$	$V = \frac{\pi}{3} \left(\frac{h}{2}\right)^2 h$	$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \left(\frac{dh}{dt}\right)$
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$20r = 10h$

$r = \frac{1}{2}h = \frac{h}{2}$	$V = \frac{\pi}{3} \cdot \frac{h^2}{4} \cdot h$	$\frac{dV}{dt} = \frac{\pi}{4} (8)^2 (-\frac{1}{2}) = [-8\pi \text{ ft}^3/\text{hr.}]$
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$d. \frac{1}{4\pi}$	$e. \frac{1}{8\pi}$
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48. Two cars are traveling along perpendicular roads, car A at 40 mi/hour, car B at 60 mi/hour. At noon, when car A reaches the intersection, car B is 90 miles away and moving toward the intersection. At 1 pm, the distance between the cars is changing, in miles per hour, at a rate of



$$\frac{dy}{dt} = 40 \text{ mph}$$

$$\frac{dx}{dt} = -60 \text{ mph}$$

$$\frac{dz}{dt} =$$

- a. -40 b. 68 c. 4

$$x = 30 \text{ mi}$$

$$y = 40 \text{ mi}$$

$$z = 50 \text{ mi}$$

$$x^2 + y^2 = z^2$$

$$2x \left(\frac{dx}{dt}\right) + 2y \left(\frac{dy}{dt}\right) = 2z \left(\frac{dz}{dt}\right)$$

$$2(30)(-60) + 2(40)(40) = 2(50) \left(\frac{dz}{dt}\right)$$

$$-3600 + 3200 = 100 \frac{dz}{dt}$$

- d. -4 e. 40

$$-400 = 100 \left(\frac{dz}{dt}\right)$$

A

49. Which statement below is true about the curve $\frac{x^2+4}{2+7x-4x^2}$?

- T a. The line $x = -\frac{1}{4}$ is a vertical asymptote
 F b. The line $x = 1$ is a vertical asymptote
 F c. The line $y = \frac{1}{4}$ is a horizontal asymptote
 F d. The graph has no vertical or horizontal asymptotes
 F e. The line $y = 2$ is a horizontal asymptote

$$\frac{x^2+4}{-4x^2+7x+2} = \frac{x^2+4}{-(4x^2-7x-2)} = \frac{x^2+4}{-(4x+1)(x-2)}$$

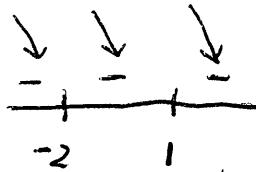
VA: $x = -\frac{1}{4}, x = 2$

$$\text{HA: } \lim_{x \rightarrow \infty} \frac{x^2+4}{-4x^2+7x+2} = \frac{1}{4}$$

$$y = \frac{1}{4}$$

$$\frac{dz}{dt} = -4 \text{ mph}$$

$$f(x) = \frac{x}{(x+2)(x-1)} \quad VA: x = -2, x = 1$$



D

50. Find all open intervals for which the function $f(x) = \frac{x}{x^2+x-2}$ is decreasing.

* Find $f'(x)$, critical pts, sign line.

$$f'(x) = \frac{1(x^2+x-2) - x(2x+1)}{(x^2+x-2)^2} = \frac{x^2+x-2-2x^2-x}{(x^2+x-2)^2} = \frac{-x^2-2}{(x^2+x-2)^2}$$

a. $(-\infty, \infty)$

b. $(-\infty, 0)$

c. $(-\infty, -2)$ and $(1, \infty)$

d. $(-\infty, -2), (-2, 1)$ and $(1, \infty)$

e. none of these

51. Find the values of x that give relative extrema for the function $f(x) = (x+1)^2(x-2)$.

* find $f'(x)$, product, chain rule:

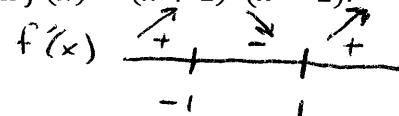
$$f'(x) = 2(x+1)(x-2) + (x+1)^2(1)$$

$$O = (x+1)[2(x-2) + (x+1)]$$

$$O = (x+1)(2x-4+x+1)$$

$$O = (x+1)(3x-3)$$

$$O = 3(x+1)(x+1) \quad x=1, -1$$



Rel. max at $x = -1$,
Rel. min at $x = 1$

a. Relative maximum at $x = -1$; relative minimum at $x = 1$

b. Relative maxima at $x = 1, 3$; relative minimum at $x = -1$

c. Relative minimum at $x = 2$

d. Relative maximum at $x = -1$; relative minimum at $x = 2$

e. None of these

52. Find all intervals on which the graph of the function $f(x) = \frac{x-1}{x+3}$ is concave upward.

* find $f''(x)$ * quotient rule

$$f'(x) = \frac{(1)(x+3) - (x-1)(1)}{(x+3)^2}$$

$$f'(x) = \frac{x+3-x+1}{(x+3)^2}$$

$$f'(x) = \frac{4}{(x+3)^2}$$

$$f''(x) = \frac{-8}{(x+3)^3}$$

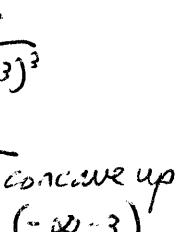
a. $(-\infty, \infty)$

b. $(-\infty, -3)$

c. $(1, \infty)$

d. $(-3, \infty)$

e. none of these



concave up

53. Let $f''(x) = 3x^2 - 4$ and let $f(x)$ have critical numbers at $-2, 0$ and 2 . Use the Second Derivative Test to determine which critical numbers, if any, give a relative maximum.

$$f''(-2) = 3(-2)^2 - 4 = 8, \text{ since } f''(-2) > 0 \text{ and } f'(-2) = 0, \text{ Rel. min at } x = -2$$

$$f''(0) = 3(0)^2 - 4 = -4, \text{ since } f''(0) < 0 \text{ and } f'(0) = 0, \text{ Rel. max at } x = 0$$

$$f''(2) = 3(2)^2 - 4 = 8, \text{ since } f''(2) > 0 \text{ (concave up) Rel. min at } x = 2$$

a. -2

b. 2

c. 0

d. -2 and 2

e. none of these

C

54. Find the coordinates of all extrema (relative and absolute) on the interval $[0, 2\pi]$ for $y = x - \cos x$. EVT: Test Endpts and critical pts

a. $\left(\frac{3\pi}{2}, \frac{3\pi}{2}\right), (2\pi, 2\pi - 1)$

b. $\left(\frac{3\pi}{2}, \frac{3\pi}{2}\right), (0, -1)$

c. $(2\pi, 2\pi - 1), (0, -1)$

d. $(0, -1), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right), (2\pi, 2\pi - 1)$

e. none of these

$$y' = 1 - (-\sin x)$$

$$f'(0) = 0 - \cos(0) = -1 \leftarrow \text{Abs. min}$$

$$y' = 1 + \sin x$$

$$f(2\pi) = 2\pi - \cos(0) = 2\pi - 1 \approx 5 \leftarrow \text{Abs. max}$$

$$O = 1 + \sin x$$

$$f\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} - \cos\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} - 0 \approx 4.5$$



Not Relative extrema (no sign change)

⑨

D

*Expand using log properties

55. Find the derivative of $f(x) = \ln \frac{\sqrt{x^2+1}}{x(2x^3-1)^2}$

$$\begin{aligned}\ln \left[\frac{(x^2+1)^{1/2}}{x(2x^3-1)^2} \right] &= \ln(x^2+1)^{1/2} - \ln x - \ln(2x^3-1)^2 \\ &= \frac{1}{2} \ln(x^2+1) - \ln x - 2 \ln(2x^3-1)\end{aligned}$$

$$\frac{d}{dx} \ln u = \frac{u'}{u}$$

$$\begin{aligned}f'(x) &= \frac{1}{2} \cdot \frac{2x}{x^2+1} - \frac{1}{x} - 2 \cdot \frac{6x^2}{2x^3-1} \\ f'(x) &= \frac{x}{x^2+1} - \frac{1}{x} - \frac{12x^2}{2x^3-1}\end{aligned}$$

a. $\frac{x}{x^2+1} - \frac{1}{x} + \frac{12x^2}{2x^3-1}$

b. $\frac{x}{x^2+1} - \frac{1}{x} + \frac{6x^2}{2x^3-1}$

c. $\frac{1}{(x^2+1)^{1/2}(4x^2)(2x^3-1)}$

d. $\frac{x}{x^2+1} - \frac{1}{x} - \frac{12x^2}{2x^3-1}$

e. none of these

D

56. Find y' if $y = \frac{x^3}{3^x}$

$$*\frac{d}{dx} a^u = \ln a \cdot a^u \cdot u'$$

*quotient rule

$$\begin{aligned}y' &= \frac{3x^2 \cdot 3^x - x^3 \cdot \ln 3 \cdot 3^x (1)}{(3^x)^2} \\ &= \frac{3^x [3x^2 - x^3 (\ln 3)]}{3^x \cdot 3^x}\end{aligned}$$

$$\frac{x^2 [3 - x \ln 3]}{3^x}$$

a. $\frac{x}{3^{x-2}}$

b. $\frac{3x^2}{3^x(\ln 3)}$

c. $\frac{x^2(9-x^2)}{3^{x+1}}$

d. $\frac{x^2[3-x(\ln 3)]}{3^x}$

e. none of these

C

57. Find $f'(x)$ for $f(x) = \frac{2}{2x+e^{2x}}$

*quotient rule

$$*\frac{d}{dx} e^u = e^u \cdot u'$$

$$f'(x) = \frac{0(2x+e^{2x}) - 2[2+e^{2x} \cdot 2]}{(2x+e^{2x})^2}$$

$$= \frac{-4[1+e^{2x}]}{(2x+e^{2x})^2}$$

a. 0

b. $\frac{1}{1+e^{2x}}$

c. $\frac{-4(1+e^{2x})}{(2x+e^{2x})^2}$

d. $\frac{1+xe^{2x}-1}{(2x+e^{2x})^2}$

e. none of these

B

58. If $y = \log_3(x^3 - 8x)$, find $\frac{dy}{dx}$

$$*\frac{d}{dx} \log_a u = \frac{1}{\ln a} \cdot \frac{u'}{u}$$

$$y' = \frac{1}{\ln 3} \cdot \frac{3x^2 - 8}{x^3 - 8x}$$

$$\frac{3x^2 - 8}{(\ln 3)(x^3 - 8x)}$$

a. $\frac{3x^2 - 8}{x^3 - 8x}$

b. $\frac{3x^2 - 8}{(x^3 - 8x)(\ln 3)}$

c. $(3x^2 - 8)(\ln 3)$

d. $\frac{(3x^2 - 8)(\ln 3)}{(x^3 - 8x)}$

e. none of these

E

59. Find $\frac{dy}{dx}$ for $y = \arctan \frac{x}{2}$

$$*\frac{d}{dx} \arctan u = \frac{u'}{1+u^2}$$

$$y = \arctan \left(\frac{1}{2}x \right)$$

$$y' = \frac{\frac{1}{2}}{1 + \left(\frac{x}{2} \right)^2} \cdot \frac{4}{4+x^2}$$

a. $\frac{4}{4+x^2}$

b. $\frac{4}{1+x^2}$

c. $\frac{1}{\sqrt{4-x^2}}$

d. $\frac{1}{2} \sec^2 \left(\frac{x}{2} \right)$

e. $\frac{2}{4+x^2}$

B

60. If $f(x) = \frac{1}{27}(x^5 + 2x^3)$, and $g(x) = f^{-1}(x)$, find $(g')(-11)$ if $g(-11) = -3$.

a. 17

b. $\frac{1}{17}$

c. $\frac{27}{73931}$

d. $\frac{73931}{27}$

e. none of these

$$\begin{aligned}f(a) &= b & g(b) &= a \\ f'(a) &= n & g'(b) &= \frac{1}{n}\end{aligned}$$

$$f(-3) = -11 \quad g(-11) = -3$$

$$\begin{array}{|c|c|} \hline f'(-3) & - \\ \hline g'(-11) & - \\ \hline \end{array}$$

$$f'(x) = \frac{1}{27}(5x^4 + 6x^2)$$

$$f'(-3) = \frac{1}{27}(5(-3)^4 + 6(-3)^2)$$

$$\frac{1}{27}(405 + 54)$$

$$f'(-3) = \frac{459}{27} = 17$$

$$g'(-11) = \frac{1}{17}$$