AB Calculus Fall Exam Review #2: Additional Practice/MC Problems (non-calculator unless otherwise noted)

1. $Given\ g(t) = 2t\sqrt{3-2t}$, find all critical value of g(t)

2. . $Given\ f(x) = \sqrt{x-1}$ in interval [10, 50], use Mean Value Theorem to find a tangent line equation with same slope as secant line that passes through the endpoints.

3.

If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{x^2+y}{x+2y^2}$ (B) $-\frac{x^2+y}{x+y^2}$ (C) $-\frac{x^2+y}{x+2y}$ (D) $-\frac{x^2+y}{2y^2}$ (E) $\frac{-x^2}{1+2y^2}$

4.

At what value of x does the graph of $y=\frac{1}{x^2}-\frac{1}{x^3}$ have a point of inflection?

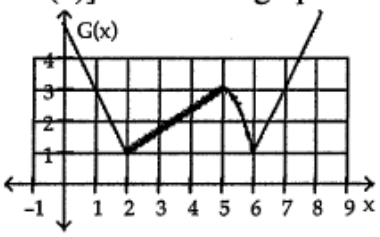
(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

5. What is the minimum **value** for $f(x) = xlnx$

$(A)\ -e$ $(B)\ -\frac{1}{e}$ $(C)\ e$ (D) $\frac{1}{e}$ D) $No\ minimum\ value$

6.

The function F is defined by $F(x) = G[x + G(x)]$ where the graph of the function G is shown.



The approximate value of $F'(1)$ is

7.

Let $f(x) = \ln x + e^{-x}$. Which of the following is TRUE at $x = 1$?

(A) f is increasing

(B) f is decreasing

(C) f is discontinuous

(D) f has a relative minimum

(E) f has a relative maximum

8. What is $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1} = ?$

9. Find the maximum value for $f(x) = 2x^3 - 9x^2 + 12x - 1$ *on* $[-1,2]$. Justify your answer.

$(A) -1$ $(B) 1$ $(C) 2$ $(D) 3$ $(E) 4$

10.

If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

11.

An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1,5)$ is

(A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$

(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

12.

If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

13. What is the average rate of change of $3t^3 - t^2$ over the interval $-1 \leq t \leq 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

14.

What is $\lim_{x \to 0} \frac{e^{2x} - 1}{\tan x}$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) The limit does not exist.

15.

If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6\sin 3x \cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x \cos 3x$

16.

If $\tan(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

17.

If $f(x) = e^{1/x}$, then $f'(x) =$

(A) $-\frac{e^{1/x}}{x^2}$ (B) $-e^{1/x}$ (C) $\frac{e^{1/x}}{x}$ (D) $\frac{e^{1/x}}{x^2}$ (E) $\frac{1}{x}e^{(1/x)-1}$

18.

If $y = \ln\left(x^2 + y^2\right)$, then the value of $\frac{dy}{dx}$ at the point $(1, 0)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) undefined

19.

If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $\frac{1}{x}$ (B) $\frac{1}{\ln x}$ (C) $\frac{\ln x}{x}$ (D) x (E) $\frac{1}{x\ln x}$

20.

If $y = x^{\ln x}$, then y' is

(A) $\dfrac{x^{\ln x}\ln x}{x^2}$ (B) $x^{1/x}\ln x$ (C) $\dfrac{2x^{\ln x}\ln x}{x}$

(D) $\dfrac{x^{\ln x}\ln x}{x}$ (E) None of the above

21.

If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$

22.

If $y = \arctan(\cos x)$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\dfrac{1}{(\arccos x)^2 + 1}$ (E) $\dfrac{1}{1+\cos^2 x}$

23.

$\dfrac{d}{dx}\left(\dfrac{1}{x^3} - \dfrac{1}{x} + x^2\right)$ at $x = -1$ is

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6

24.

If $f(x)=e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h\to 0}\frac{e^{x+h}}{h}$ (B) $\lim_{h\to 0}\frac{e^{x+h}-e^e}{h}$ (C) $\lim_{h\to 0}\frac{e^{e+h}-e}{h}$

(D) $\lim_{h\to 0}\frac{e^{x+h}-1}{h}$ (E) $\lim_{h\to 0}\frac{e^{e+h}-e^e}{h}$

25.

$\frac{d}{dx}\ln\left(\frac{1}{1-x}\right)=$

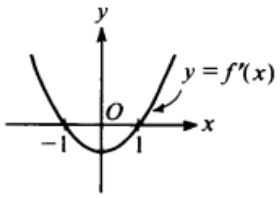
(A) $\frac{1}{1-x}$ (B) $\frac{1}{x-1}$ (C) $1-x$ (D) $x-1$ (E) $(1-x)^2$

26.

If $f(x)=x\ln\left(x^2\right)$, then $f'(x)=$

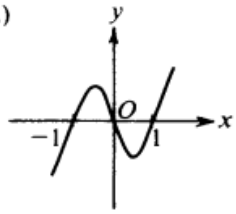
(A) $\ln\left(x^2\right)+1$ (B) $\ln\left(x^2\right)+2$ (C) $\ln\left(x^2\right)+\frac{1}{x}$ (D) $\frac{1}{x^2}$ (E) $\frac{1}{x}$

27.

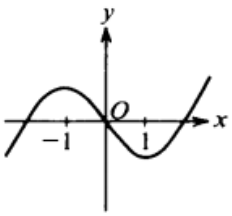


The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

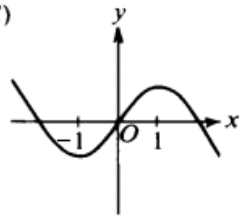
(A)



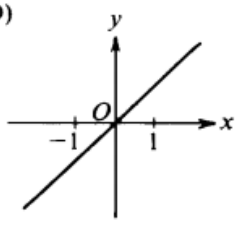
(B)



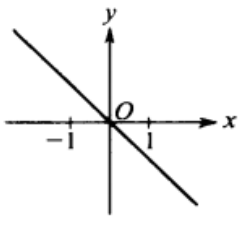
(C)



(D)



(E)



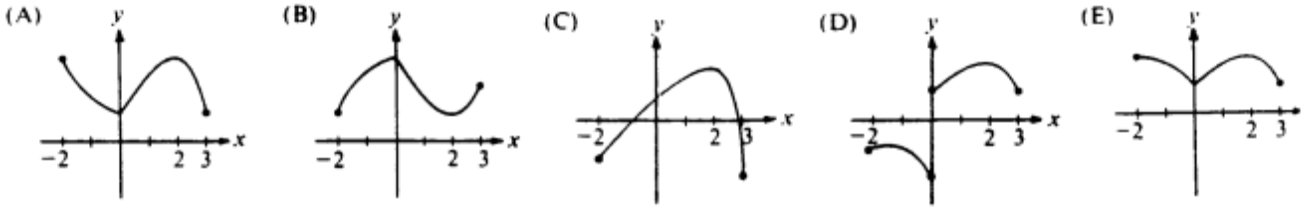
28.

$$\lim_{x\to\frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}} \text{ is}$$

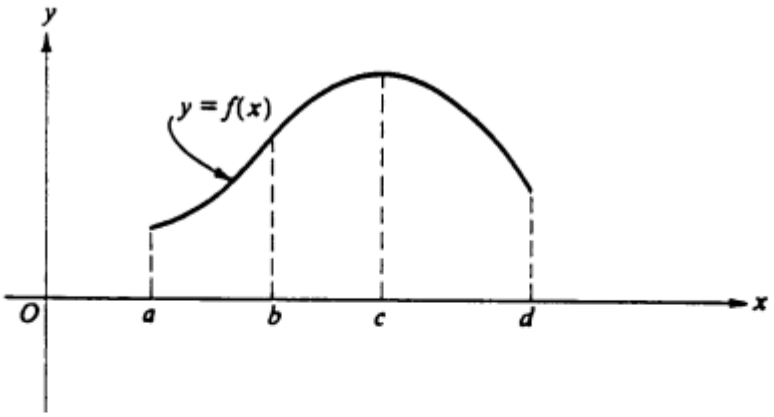
(A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\pi}{4}$ (D) 1 (E) nonexistent

29.

Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2)=0$, and $f''(x)<0$ for all x except $x=0$. Which of the following could be the graph of f?



30.



The graph of $y=f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx}>0$ and $\frac{d^2y}{dx^2}<0$?

I. $a<x<b$
II. $b<x<c$
III. $c<x<d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

31.

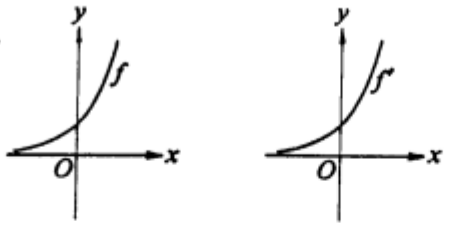
$$\lim_{x\to 5}\frac{2x^2-50}{x^2-15x+50}=$$

(A) -4 (B) -1 (C) 0 (D) 1 (E) 2

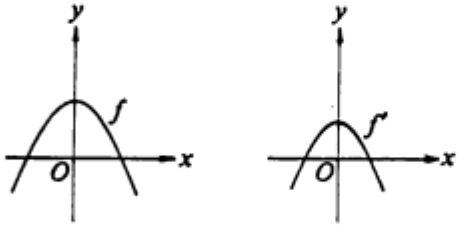
32.

Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?

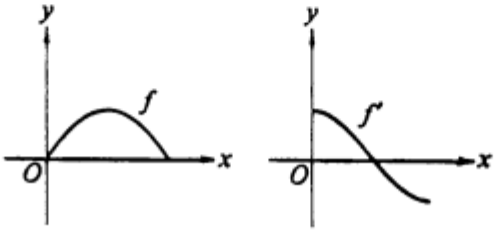
I.



II.



III.



(A) I only (B) II only (C) III only (D) I and III (E) II and III

33.

If the graph of $y=x^3+ax^2+bx-4$ has a point of inflection at $(1,-6)$, what is the value of b?

(A) -3 (B) 0 (C) 1 (D) 3

(E) It cannot be determined from the information given.

34.

The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

(A) $\frac{1}{\pi}$ (B) $\frac{1}{2}$ (C) $\frac{2}{\pi}$ (D) 1 (E) 2

35.

$$\frac{d}{dx}\cos^2(x^3) =$$

(A) $6x^2 \sin(x^3)\cos(x^3)$

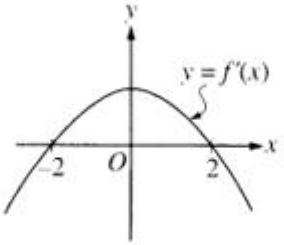
(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

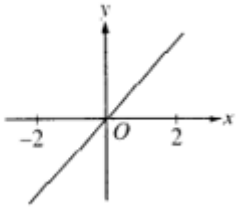
(E) $-2\sin(x^3)\cos(x^3)$

36.

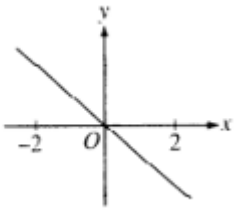


The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

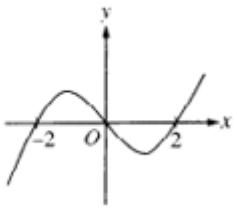
(A)



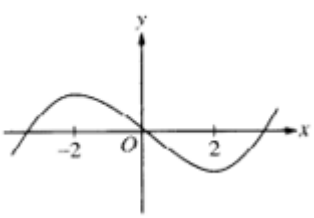
(B)



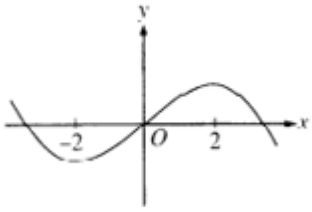
(C)



(D)



(E)

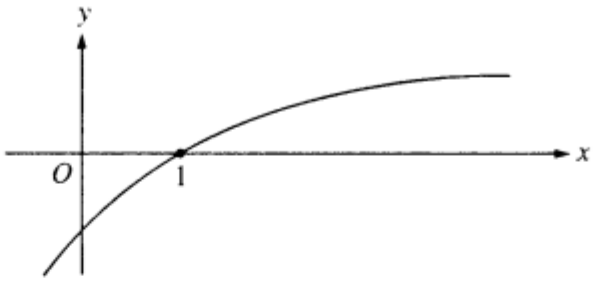


37.

If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

38.



The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

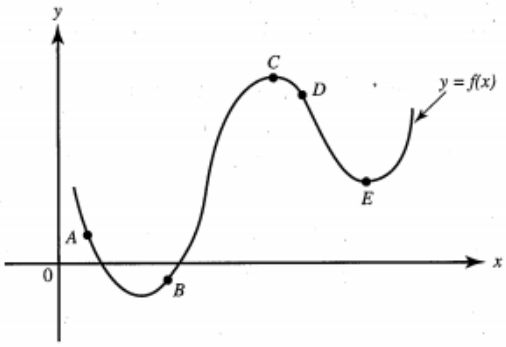
(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

39.

If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only



40. At which point on the graph of $y = f(x)$ shown above is $f'(x) < 0$ and $f''(x) > 0$?

(A) A **(B)** B **(C)** C **(D)** D **(E)** E

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41. Let $f(x) = x^5 + 1$, and let g be the inverse function of f. What is the value of $g'(0)$?

(A) -1 **(B)** $\frac{1}{5}$ **(C)** 1 **(D)** $g'(0)$ does not exist.

(E) $g'(0)$ cannot be determined from the given information.

42.

The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

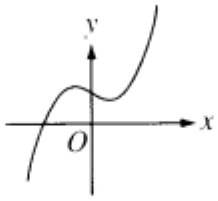
(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

43 . (Calculator)

A missile rises vertically from a point on the ground75,000 feet from a radar station. If the missile is rising at the rate of 40,000 feet per minute at the instant when it is 100,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?

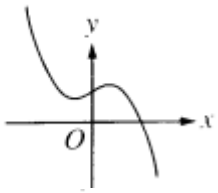
(A) $\frac{18}{25}$ (B) $\frac{8}{15}$ (C) $\frac{24}{125}$ (D) $\frac{18}{125}$ (E) $\frac{8}{25}$

44.

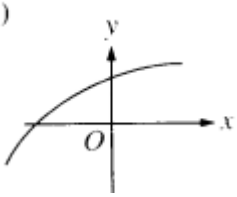


The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

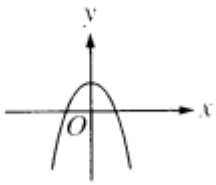
(A)



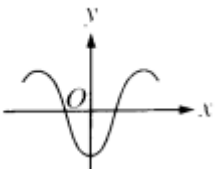
(B)



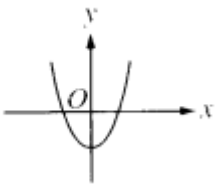
(C)



(D)



(E)



45.

The function $f(x) = x^5 + 3x - 2$ passes through the point (1,2). Let f^{-1} denote the inverse of f. Then $(f^{-1})'(2)$ equals

(A) $\frac{1}{83}$ **(B)** $\frac{1}{8}$ **(C)** 1 **(D)** 8 **(E)** 83

46.

The graph of f'', the second derivative of f, is shown in Figure 3T-3. The graph of f'' has horizontal tangents at $x = -2$ and $x = 2$. For what values of x does the graph of the function f have a point of inflection?

(A) -4, 0, and 4
(B) -2, 0, and 2
(C) -4 and 4 only
(D) -2 and 2 only
(E) 0 only

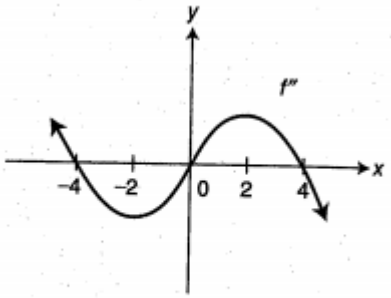


Figure 3T-3

47.

The graph of f is shown in Figure 3T-5 and f is twice differentiable. Which of the following has the largest value?

I. $f(0)$
II. $f'(0)$
III. $f''(0)$

(A) I
(B) II
(C) III
(D) I and II
(E) II and III

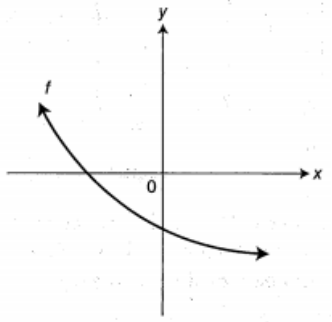


Figure 3T-5

48.

Let f and g be differentiable functions such that $g(x) = f^{-1}(x)$. If $f(2) = 4$, $f(3) = 9$, $g'(4) = \frac{1}{4}$, and $g'(9) = \frac{1}{6}$, what is the value of $f'(3)$?

(A) 0
(B) 3
(C) 4
(D) 6
(E) 9

49.

The graph of f' is shown in Figure 3T-7. Which of the following statements is/are true?

I. The function f is decreasing on the interval $(-\infty, -1)$.
II. The function f has an absolute maximum at $x = 2$.
III. The function f has a point of inflection at $x = -1$.

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

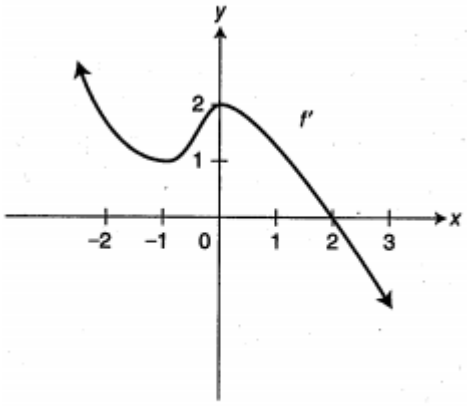


Figure 3T-7

50. The function $G(x) = \frac{(x-2)(x-3)}{(x-1)}$ does not satisfy the hypothesis of Rolle's Theorem on the interval $[-3, 2]$ because

A) $G(-3) = G(2) = 0$

B) $G(x)$ is not differentiable on $[-3, 2]$

C) $G(x)$ is not continuous on $[-3, 2]$

D) $G(0) \neq 0$

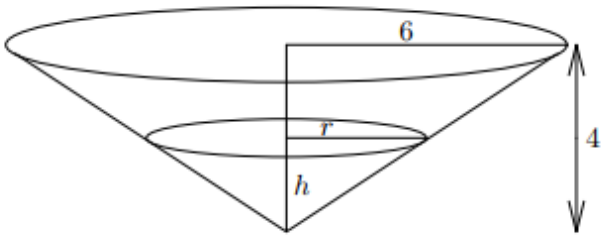
E) None of these

51.

If c is the number defined by Rolle's Theorem, then for $R(x) = 2x^3 - 6x$ on the interval $0 \le x \le \sqrt{3}$, c must be

A) 1 B) -1 C) ± 1 D) 0 E) $\sqrt{3}$

52. (Calculator)



The conical reservoir shown above has diameter 12 feet and height 4 feet. Water is flowing into the reservoir at the constant rate of 10 cubic feet per minute. At the instant when the surface of the water is 2 feet above the vertex, the water level is rising at the rate of

A) 0.177 ft per min

B) 0.354 ft per min

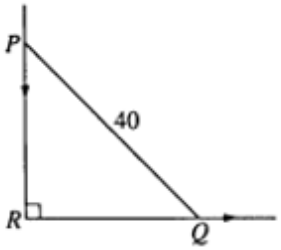
C) 0.531 ft per min

D) 0.708 ft per min

E) 0.885 ft per min

53. (calculator)

In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?



(A) $\frac{6}{5}\sqrt{10}$ (B) $\frac{8}{5}\sqrt{10}$ (C) $\frac{80}{\sqrt{7}}$ (D) 24 (E) 32

54.

If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

(A) -45 (B) -30 (C) -15 (D) -10 (E) -5