

AB Calculus Fall Exam Review Packet #3

Key

Non-Calculators

Ch. 1 Limits

1) $\lim_{x \rightarrow 2^-} \frac{(x+2)^2}{x^2-4} = \frac{(2+2)^2}{4-4} = \frac{16}{0}$ → one-sided limit at an asymptote * test using decimals +∞ -∞ * use 1.9 $\frac{(1.9+2)^2}{1.9^2-4} = \frac{+}{-} = \boxed{-\infty}$

A) 4 B) 1 C) ∞ D) $-\infty$ E) Does Not Exist

2) $\lim_{x \rightarrow -11^-} \frac{\sqrt{x+19}-\sqrt{8}}{x+11} = \frac{\sqrt{-11+19}-\sqrt{8}}{-11+11} = \frac{0}{0}$

- A) $\frac{1}{2\sqrt{8}}$ B) $-\frac{1}{2\sqrt{8}}$ C) ∞ D) $-\infty$ E) Does Not Exist

$$\begin{aligned} \lim_{x \rightarrow -11^-} \frac{\sqrt{x+19}-\sqrt{8}}{x+11} \cdot \frac{\sqrt{x+19}+\sqrt{8}}{\sqrt{x+19}+\sqrt{8}} &= \lim_{x \rightarrow -11^-} \frac{x+19-8}{(x+11)(\sqrt{x+19}+\sqrt{8})} = \lim_{x \rightarrow -11^+} \frac{(x+11)}{(x+11)(\sqrt{x+19}+\sqrt{8})} \\ &= \frac{1}{\sqrt{-11+19}+\sqrt{8}} = \frac{1}{\sqrt{8}+\sqrt{8}} = \boxed{\frac{1}{2\sqrt{8}}} \end{aligned}$$

3) $\lim_{x \rightarrow -\infty} \frac{5x^4-5x-5}{(2x^2-3)^2} = \frac{\text{* compare degrees}}{\text{* compare degrees}} = \lim_{x \rightarrow -\infty} \frac{5x^4-5x-5}{(2x^2-3)(2x^2-3)}$

- A) $\frac{5}{2}$ B) $\frac{5}{4}$ C) ∞ D) $-\infty$ E) Does Not Exist

$$\lim_{x \rightarrow -\infty} \frac{5x^4-5x-5}{4x^4-12x^2+9} = \boxed{\frac{5}{4}}$$

4) $\lim_{x \rightarrow 1} \frac{(x+1)^2}{x^2-1} = \frac{2^2}{1-1} = \frac{4}{0}$ Limit DNE

- A) 1 B) 2 C) ∞ D) $-\infty$ E) Does Not Exist

5) $\lim_{x \rightarrow -\infty} \frac{4-x^3}{2-x^2} = \frac{\text{* compare degrees}}{\text{* compare degrees}} \lim_{x \rightarrow -\infty} \frac{4-x^3}{2-x^2} \begin{cases} \nearrow +\infty \\ \searrow -\infty \end{cases}$

- A) 4 B) 2 C) ∞ D) $-\infty$ E) -2

$$\frac{4-(-100)^3}{2-(-100)^2} = \frac{+}{-} = \boxed{-\infty}$$

6) What is $\lim_{x \rightarrow 0} \left(\frac{\frac{1}{x-1} + 1}{x} \right)$? $\frac{\frac{1}{0-1} + 1}{0} = \frac{0}{0}$

(A) -1 (B) 0 (C) 1 (D) 2 (E) the limit does not exist

$$\lim_{x \rightarrow 0} \frac{\frac{1}{x-1} + 1}{x} = \lim_{x \rightarrow 0} \frac{1 + (x-1)}{x(x-1)} = \lim_{x \rightarrow 0} \frac{1+x-1}{x(x-1)} = \lim_{x \rightarrow 0} \frac{x}{x(x-1)} = \lim_{x \rightarrow 0} \frac{1}{x-1} = \frac{1}{-1} = -1$$

7) What is $\lim_{x \rightarrow \infty} \frac{3x^2 + 1}{(3-x)(3+x)}$? * compare degrees

- (A) -9 (B) -3 (C) 1 (D) 3 (E) The limit does not exist.

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 1}{9 - x^2} = \frac{3}{-1} = -3$$

Which of the following functions is both continuous and differentiable at all x in the interval $-2 \leq x \leq 2$?
 (no breaks) (smooth curve, no sharp turns)

- ✗ (A) $f(x) = |x^2 - 1|$ (not differentiable at $x=1, -1$)
- ✗ (B) $f(x) = \sqrt{x^2 - 1}$ $y = (x^2 - 1)^{1/2}$ $y' = \frac{x}{\sqrt{x^2 - 1}}$ not differentiable at $x=1, -1$
- ✓ (C) $f(x) = \sqrt{x^2 + 1}$ $y = (x^2 + 1)^{1/2}$ $y' = \frac{1}{2}(x^2 + 1)^{-1/2}(2x)$
- ✗ (D) $f(x) = \frac{1}{x^2 - 1}$ V.A. at $x=1, x=-1$
- ✗ (E) none of these

9)

$$\lim_{x \rightarrow 1} \left(\frac{\sqrt{x+3} - 2}{1-x} \right) = \frac{\sqrt{1+3} - 2}{1-1} = \frac{0}{0}$$

(A) 0.5

(B) 0.25

(C) 0

(D) -0.25

(E) -0.5

$$\lim_{x \rightarrow 1} \frac{(\sqrt{x+3} - 2)(\sqrt{x+3} + 2)}{(1-x)(\sqrt{x+3} + 2)}$$

$$\lim_{x \rightarrow 1} \frac{x+3-4}{(1-x)(\sqrt{x+3} + 2)}$$

$$\lim_{x \rightarrow 1} \frac{x-1}{(1-x)(\sqrt{x+3} + 2)}$$

$$\lim_{x \rightarrow 1} \frac{-1(1-x)}{(1-x)(\sqrt{x+3} + 2)}$$

$$= \frac{-1}{\sqrt{1+3} + 2} = \frac{-1}{\sqrt{4} + 2}$$

$$= \frac{-1}{2+2} = \boxed{\frac{-1}{4}} = -0.25$$

10)

Let f be defined by $f(x) = \begin{cases} \frac{x^2 - 2x + 1}{x - 1} & \text{for } x \neq 1 \\ k & \text{for } x = 1. \end{cases}$

Determine the value of k for which f is continuous for all real x .

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) none of the above

*step through continuity conditions:

a) $f(c)$ exists

b) $\lim_{x \rightarrow c} f(x)$ exists ($\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x)$)

c) $f(c) = \lim_{x \rightarrow c} f(x)$

i) $f(1) = k$

ii) $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x - 1} = \frac{0}{0}$ $\lim_{x \rightarrow 1} \frac{(x-1)(x-1)}{(x-1)}$

$1-1=0$

$\lim_{x \rightarrow 1} f(x) = 0$

iii) $f(1) = \lim_{x \rightarrow 1} f(x)$

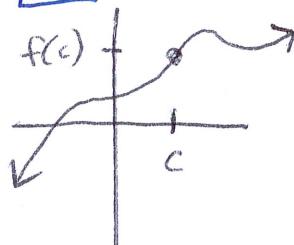
$k = 0$

11)

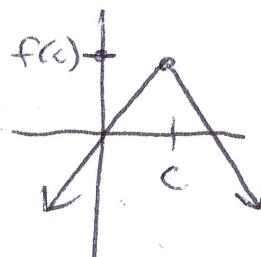
The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

- ✓ (A) $\lim_{x \rightarrow c} f(x)$ exists
- ✓ (B) $\lim_{x \rightarrow c} f(x) = f(c)$
- ✓ (C) $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x)$
- ✓ (D) $f(c)$ is defined
- (E) $f'(c)$ exists \rightarrow graph does not have to have point where slope exists

Ch. 2 Derivatives



... could be



12)

Find the point on the graph of $y = \sqrt{x}$ between $(1, 1)$ and $(9, 3)$ at which the tangent to the graph has the same slope as the line through $(1, 1)$ and $(9, 3)$.

- (A) $(1, 1)$
- (B) $(2, \sqrt{2})$
- (C) $(3, \sqrt{3})$
- (D) $(4, 2)$
- (E) none of the above



*Apply MVT:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\begin{aligned} \frac{1}{2\sqrt{x}} &= \frac{f(9) - f(1)}{9 - 1} \\ \frac{1}{2\sqrt{x}} &= \frac{3 - 1}{9 - 1} \\ \frac{1}{2\sqrt{x}} &= \frac{2}{8} \end{aligned}$$

$$4\sqrt{x} = 8$$

$$\sqrt{x} = 2$$

$$(\sqrt{x})^2 = (2)^2$$

$$x = 4$$

$$y = \sqrt{x}$$

$$y(4) = \sqrt{4} = 2$$

point: $(4, 2)$

13)

If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{3}{2x}$

(B) $\frac{3x}{(1+x^2)^2}$

(C) $\frac{6x}{(4+x^2)^2}$

(D) $\frac{-6x}{(4+x^2)^2}$

(E) $\frac{-3}{(4+x^2)^2}$

$$y = 3(4+x^2)^{-1}$$

$$y' = -3(4+x^2)^{-2}(2x)$$

$$\boxed{y' = \frac{-6x}{(4+x^2)^2}}$$

*chain rule

14)

Consider the curve $x + xy + 2y^2 = 6$. The slope of the line tangent to the curve at the point $(2,1)$ is

(A) $\frac{2}{3}$

* implicit differentiation

(B) $\frac{1}{3}$

* product rule

(C) $-\frac{1}{3}$

$$x + \overbrace{xy}^f + 2y^2 = 6$$

(D) $-\frac{1}{5}$

$$1 + \overbrace{1 \cdot y}^f + x \cdot \overbrace{\frac{dy}{dx}}^g + 4y \left(\frac{dy}{dx}\right) = 0$$

(E) $-\frac{3}{4}$

$$1 + y + x\left(\frac{dy}{dx}\right) + 4y\left(\frac{dy}{dx}\right) = 0$$

$$x\left(\frac{dy}{dx}\right) + 4y\left(\frac{dy}{dx}\right) = -1 - y$$

$$\frac{dy}{dx}(x+4y) = -1 - y$$

$$\frac{dy}{dx} = \frac{-1-y}{x+4y}$$

$$\left. \frac{dy}{dx} \right|_{(2,1)} = \frac{-1-1}{2+4(1)} = \frac{-2}{6} = \boxed{-\frac{1}{3}}$$

15)

If $p(x) = (x-1)(x+k)$ and if the line tangent to the graph of p at the point $(4, p(4))$ is parallel to the line $5x - y + 6 = 0$, then $k =$

(A) 2 *set $p'(x)$ equal to the slope of the line $5x - y + 6 = 0$

(B) 1 $5x - y + 6 = 0$

$$p(x) = (x-1)(x+k)$$

$$2x-1+k=5 \leftarrow \text{*at point } (4, p(4))$$

(C) 0 $5x + 6 = y$

$$p(x) = x^2 - x + kx - k$$

$$2(4)-1+k=5$$

(D) -1

\uparrow
slope of line
is 5

$$p'(x) = 2x-1+k-0$$

$$8-1+k=5$$

(E) -2

*set $p'(x) = 5$.

$$7+k=5$$

$$k=-2$$

16)

$x(t) = \ln t + \frac{t^2}{18} + 1$

The formula $x(t) = \ln t + \frac{t^2}{18} + 1$ gives the position of an object moving along the x -axis during the time interval $1 \leq t \leq 5$. At the instant when the acceleration of the object is zero, the velocity is

- (A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 1 (E) undefined

*set $a(t) = 0$ $x'(t) = \frac{1}{t} + \frac{2}{18}t$ $V(t) = t^{-1} + \frac{1}{9}t$ $\frac{1}{t^2} = \frac{1}{9}$ $V(t) = \frac{1}{t} + \frac{1}{9}t$

*find velocity $v(t) = \frac{1}{t} + \frac{t}{9}$ $a(t) = -t^{-2} + \frac{1}{9}$ $t^2 = 9$ $v(3) = \frac{1}{3} + \frac{1}{9}(3)$

17)

$O = -\frac{1}{t^2} + \frac{1}{9}$ $t = 3$ $v(3) = \frac{1}{3} + \frac{1}{3} = \boxed{\frac{2}{3}}$

A particle moves along the x -axis so that its distance from the origin at time t is given by

$10t - 4t^2$. What is the total distance covered by the point between $t = 1$ and $t = 2$?

- (A) 1.0

*create sign line from $v(t) = 0$



- (B) 1.5

$$x(t) = 10t - 4t^2$$

- (C) 2.0

$$x'(t) = 10 - 8t$$

- (D) 2.5

$$O = 10 - 8t$$

- (E) 3.0

$$8t = 10$$

18)

$$t = \frac{10}{8} = \frac{5}{4}$$

If $xy + x^2 = 6$, then the value of $\frac{dy}{dx}$ at $x = -1$ is

$x(1) = 10(1) - 4(1)^2 = 10 - 4 = 6$
 $x(\frac{5}{4}) = 10(\frac{5}{4}) - 4(\frac{5}{4})^2 = \frac{25}{2} - \frac{25}{4} = \frac{50}{4} - \frac{25}{4} = \frac{25}{4} = 6.25$
 $x(2) = 10(2) - 4(2)^2 = 4$

*product rule
*implicit differentiation

$$\begin{aligned} x(1) &= 6 &> 0.25 \\ x(\frac{5}{4}) &= 6.25 &> 2.25 \\ x(2) &= 4 &+ \end{aligned}$$

$\boxed{2.50}$

- (A) -7

- (B) -2

- (C) 0

- (D) 1

- (E) 3

$$\frac{f'g}{f} + \frac{f}{f'} \cdot g' + 2x = 0$$

$$1 \cdot y + x \cdot \frac{dy}{dx} + 2x = 0$$

$$y + x \left(\frac{dy}{dx} \right) + 2x = 0$$

$$\frac{dy}{dx} = \frac{-2x-y}{x}$$

*find y -value

$$xy + x^2 = 6$$

$$-ly + (-1)^2 = 6$$

$$-y + 1 = 6$$

$$-y = 5$$

$$y = 5$$

$$\frac{dy}{dx} \Big|_{(-1, 5)} = \frac{-2(-1)-5}{-1} = \frac{2-5}{-1} = \frac{-3}{-1} = 3$$

A particle moves along the x -axis so that its position at any time $t \geq 0$ is given by

$x(t) = \frac{t}{t^2 + 4}$. The particle is at rest when $t =$ *particle at rest when $V(t) = 0$

- (A) 0

- (B) $\frac{1}{4}$

- (C) 1

- (D) 2

- (E) 4

$$x'(t) = \frac{(1)(t^2+4) - t(2t)}{(t^2+4)^2}$$

$$x'(t) = \frac{t^2+4-2t^2}{(t^2+4)^2}$$

$$x'(t) = \frac{4-t^2}{(t^2+4)^2}$$

$$O = \frac{4-t^2}{(t^2+4)^2}$$

*set just numerator = 0

$$4-t^2 = 0$$

$$-t^2 = -4$$

$$t^2 = 4$$

$$t = \pm 2$$

$$\boxed{t=2}, t \neq 2$$

20)

* implicit differentiation
* product rule

If y is a differentiable function of x , then the slope of the tangent to the curve $xy - 2y + 4y^2 = 6$ at the point where $y = 1$ is

(A) $\frac{1}{12}$

(B) $-\frac{1}{10}$

(C) $-\frac{1}{6}$

(D) $\frac{1}{4}$

(E) $-\frac{5}{6}$

point: $xy - 2y + 4y^2 = 6$
 $(at y=1) x(1) - 2(1) + 4(1)^2 = 6$
 $x - 2 + 4 = 6$
 $x + 2 = 6$
 $x = 4$
21) point: $(4, 1)$

$\frac{f \cdot g}{x} - 2y + 4y^2 = 6$
 $f' \cdot g + f \cdot g' - 2\left(\frac{dy}{dx}\right) + 8y\left(\frac{dy}{dx}\right) = 0$
 $y + x\left(\frac{dy}{dx}\right) - 2\left(\frac{dy}{dx}\right) + 8y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}(x - 2 + 8y) = -y$
 $\frac{dy}{dx} = \frac{-y}{x - 2 + 8y}$
 $\frac{dy}{dx}|_{(4,1)} = \frac{-1}{4 - 2 + 8(1)} = \boxed{-\frac{1}{10}}$

The y -intercept of the tangent line to the curve $y = \sqrt{x+3}$ at the point $(1, 2)$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{5}{4}$

(E) $\frac{7}{4}$

*Find tangent line

*then replace $x=0$ to find y -int:

$y = (x+3)^{1/2}$

$$\begin{cases} y'(x) = \frac{1}{2}(x+3)^{-1/2}(1) \\ y'(x) = \frac{1}{2\sqrt{x+3}} \end{cases}$$

$$\begin{cases} y'(1) = \frac{1}{2\sqrt{1+3}} = \frac{1}{2\sqrt{2}} = \frac{1}{4} \\ \text{point: } (1, 2) \\ \text{slope: } m = \frac{1}{4} \end{cases}$$

$y = \frac{1}{4}(x-1) + 2$

$$\begin{cases} y = \frac{1}{4}(0-1) + 2 \\ y = -\frac{1}{4} + 2 = \frac{7}{4} \\ y = \boxed{\frac{7}{4}} \end{cases}$$

Use the table data and rules of differentiation to solve each problem:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	2	1	3
1	0	2	1	0
2	1	2	2	1
3	1	3	1	0

22) If $h(x) = \frac{g(x)}{f(x)}$ find $h'(3)$

*quotient rule:

$$h'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{f(x)^2}$$

$$h'(3) = \frac{g'(3)f(3) - g(3)f'(3)}{[f(3)]^2}$$

$$\begin{cases} h'(3) = (0)(1) - 3(1) \\ = -\frac{3}{1} = \boxed{-3} \end{cases}$$

23) If $k(x) = g(f(x))$ find $k'(1)$

*chain rule: $\frac{d}{dx} f[g(x)] = f'[g(x)] \cdot g'(x)$

$$k'(x) = g'[f(x)] \cdot f'(x)$$

$$k'(1) = g'[f(1)] \cdot f'(1)$$

$$k'(1) = g'[0] \cdot f'(1)$$

$$k'(1) = 3 \cdot 1 = \boxed{3}$$

24) If $p(x) = [f(x)]^3$ find $p'(2)$

$$p'(x) = 3[f(x)]^2 \cdot f'(x)$$

$$p'(2) = 3[f(2)]^2 \cdot f'(2)$$

$$= 3[1]^2 \cdot f'(2)$$

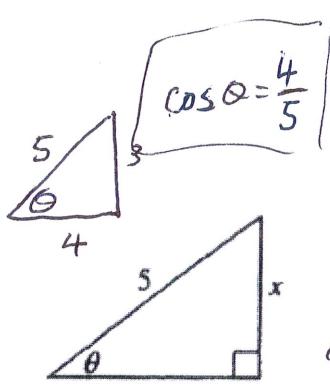
$$= 3 \cdot 1^2 \cdot 2 = 6$$

$$\boxed{p'(2) = 6}$$

Ch. 2.6 Related Rates

$$\frac{d\theta}{dt} = 3 \text{ rad/min}$$

$$\frac{dx}{dt} = \quad x = 3$$



$$\sin \theta = \frac{x}{5}$$

$$\sin \theta = \frac{1}{5}x$$

$$\cos \theta \left(\frac{d\theta}{dt} \right) = \frac{1}{5} \left(\frac{dx}{dt} \right)$$

$$5 \cos \theta \left(\frac{d\theta}{dt} \right) = \frac{dx}{dt}$$

$$5 \left(\frac{4}{5} \right) (3) = \frac{dx}{dt}$$

$$12 = \frac{dx}{dt}$$

25)

In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3

(B) $\frac{15}{4}$

(C) 4

(D) 9

(E) 12

26) $\frac{dx}{dt} = 0.2 \text{ in/sec}$

$S = 150$

$\frac{dV}{dt} = \quad ?$

The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?

(A) $5 \text{ in}^3/\text{sec}$

(B) $10 \text{ in}^3/\text{sec}$

(C) $15 \text{ in}^3/\text{sec}$

(D) $20 \text{ in}^3/\text{sec}$

(E) $25 \text{ in}^3/\text{sec}$

$S = 150 \quad 6x^2 = 150$

$S = 6x^2 \quad x^2 = 25$

$x = 5$

27)

$S = 6x^2$

$\frac{dS}{dt} = 12x \left(\frac{dx}{dt} \right)$

$V = x^3$

$\frac{dV}{dt} = 3x^2 \left(\frac{dx}{dt} \right)$

$\frac{dV}{dt} = 3(5)^2(0.2)$

$\frac{dV}{dt} = 3 \cdot 25 \cdot \frac{1}{5} = 15 \text{ in}^3/\text{sec}$

$\frac{dz}{dt} = 2$

$x = 3$

$y = 4$

$z = 5$

$\frac{dx}{dt} = ?$

(B) $\frac{3}{4} \text{ cm/sec}$

(C) $\frac{2}{3} \text{ cm/sec}$

(D) $\frac{1}{3} \text{ cm/sec}$

(E) $\frac{1}{15} \text{ cm/sec}$

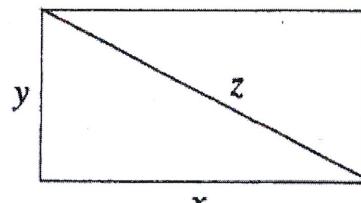
$x^2 + y^2 = z^2$

$2x \left(\frac{dx}{dt} \right) + 2y \left(\frac{dy}{dt} \right) = 2z \left(\frac{dz}{dt} \right)$

$2(3) \left(\frac{dx}{dt} \right) + 2(4) \left(3 \frac{dx}{dt} \right) = 2(5)(2)$

$6 \left(\frac{dx}{dt} \right) + 24 \left(\frac{dx}{dt} \right) = 20$

$30 \left(\frac{dx}{dt} \right) = 20$



$\frac{dx}{dt} = \frac{20}{30} = \frac{2}{3} \text{ cm/sec}$

Ch. 3 Curve Sketching

28)

→ max y-value

Find the maximum value of $f(x) = 2x^3 + 3x^2 - 12x + 4$ on the closed interval $[0, 2]$.

(A) -3

*Apply EVT

(B) 2

a) Test endpts.

(C) 4

b) Test critical pts.

(D) 8

$f(x)$ continuous on $[0, 2]$

(E) 24

$$f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2)$$

$$0 = 6(x+2)(x-1)$$

$$x = 1, -2 \quad \begin{matrix} \text{outside} \\ \text{interval} \end{matrix}$$

Test $x = 0, 1, 2$

$$f(0) = 4$$

$$f(1) = 2 + 3 - 12 + 4 = -3$$

$$f(2) = 2(2)^3 + 3(2)^2 - 12(2) + 4 = 8 \quad *(\text{Abs max})$$

29)

The function $f(x) = x^4 - 18x^2$ has a relative minimum at $x =$

(A) 0 and 3 only

* 1st derivative test:

(B) 0 and -3 only

$$f'(x) = 4x^3 - 36x$$

(C) -3 and 3 only

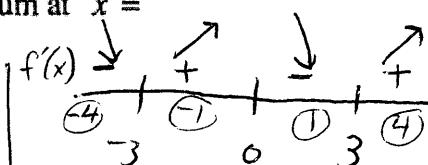
$$0 = 4x(x^2 - 9)$$

(D) 0 only

$$0 = 4x(x+3)(x-3)$$

(E) -3, 0, 3

$$x = 0, 3, -3$$



Rel. min at $x = -3, x = 3$

30)

The graph of $y = 3x^5 - 10x^4$ has an inflection point at

(A) (0, 0) and (2, -64)

$$y' = 15x^4 - 40x^3$$

(B) (0, 0) and (3, -81)

$$y'' = 60x^3 - 120x^2$$

(C) (0, 0) only

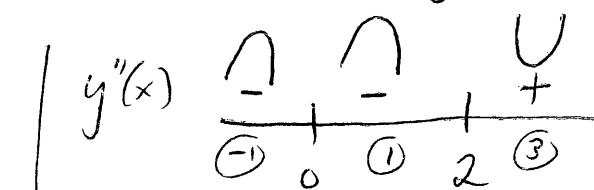
$$0 = 60x^2(x-2)$$

(D) (-3, 81) only

$$x = 0, 2$$

(E) (2, -64) only

* create 2nd derivative sign line



POI at $x = 2$

$$y(2) = -64$$

31)

*when $f''(x)$ changes from + to - ✓

Consider the function $f(x) = \frac{x^4}{2} - \frac{x^5}{10}$. The derivative of f attains its maximum value at $x =$

(A) 3

(B) 4

(C) 5

(D) 0

(E) there is no maximum

*In other words, find location where maximum steepness (POI) occurs

$$f(x) = \frac{1}{2}x^4 - \frac{1}{10}x^5$$

$$f'(x) = \frac{4}{2}x^3 - \frac{5}{10}x^4 = 2x^3 - \frac{1}{2}x^4$$

$$f''(x) = 6x^2 - 2x^3$$

$$0 = 2x^2(3-x)$$

$$\begin{array}{c} f''(x) \\ \hline + & + & - \\ (-1) & 0 & (1) & 3 & (4) \end{array}$$

POI at $x = 3$

32)

$$x=0, 3$$

Let $f(x) = x^4 + ax^2 + b$. The graph of f has a relative maximum at $(0, 1)$ and an inflection point when $x = 1$. The values of a and b are

(A) $a = 1, b = -6$ (B) $a = 1, b = 6$ (C) $a = -6, b = 5$ (D) $a = -6, b = 1$ (E) $a = 6, b = 1$ *Rd. max at $(0, 1)$ when $f'(x) = 0$

$$f'(x) = 4x^3 + 2ax \quad (\text{at } (0, 1))$$

$$0 = 4x^3 + 2ax$$

$$0 = 4(0)^3 + 2a(0)$$

$$f(x) = x^4 + -6x^2 + b \quad \text{pt. } f(0, 1)$$

$$1 = 0^4 - 6(0)^2 + b$$

$$1 = b \quad (\text{sl. pt. } = 0)$$

*POI at $x = 1$ when

$$f''(x) = 0$$

$$f''(x) = 12x^2 + 2a$$

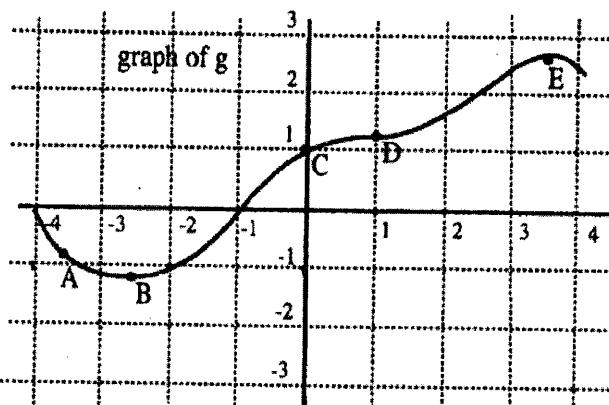
$$0 = 12(1)^2 + 2a$$

$$-12 = 2a$$

$$-6 = a$$

33)

At which point on the graph of $y = g(x)$ below is $\underline{g'(x) = 0}$ and $\underline{g''(x) = 0}$? (possible POI)



(A) A

(B) B

(C) C

(D) D

(E) E

concavity = 0

34)

If $f(x) = x^3 - 5x^2 + 3x$, then the graph of f is decreasing and concave down on the interval

(A) $\left(0, \frac{1}{3}\right)$

(B) $\left(\frac{1}{3}, \frac{2}{3}\right)$

(C) $\left(\frac{1}{3}, \frac{5}{3}\right)$

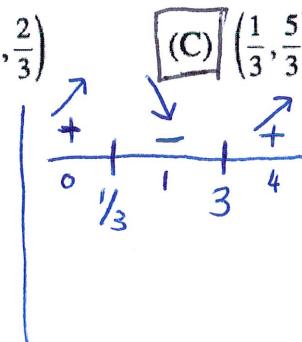
(D) $\left(\frac{5}{3}, 3\right)$

(E) $(3, \infty)$

$f'(x) = 3x^2 - 10x + 3$

$O = (3x-1)(x-3)$

$x = \frac{1}{3}, x = 3$



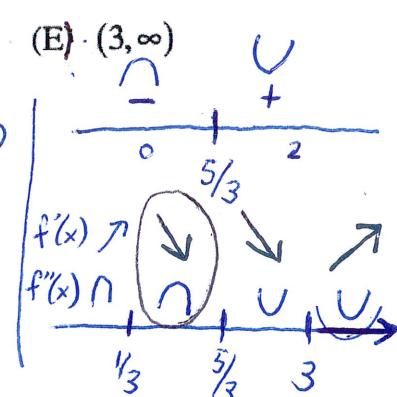
$f''(x) = 6x - 10$

$O = 2(3x-5)$

$x = \frac{5}{3}$

$f'(x) > 0$

$f''(x) < 0$



35)

The figure shows the graph of f' , the derivative of a function f . The domain of f is the closed interval $[-3, 4]$. Which of the following is true?

I. f is increasing on the interval $(2, 4)$. \times II. f has a relative minimum at $x = -2$. \times III. The f -graph has an inflection point at $x = 1$. ✓

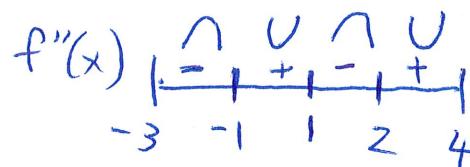
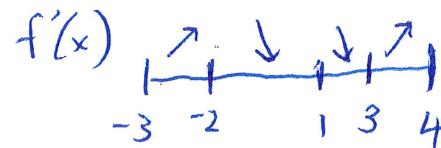
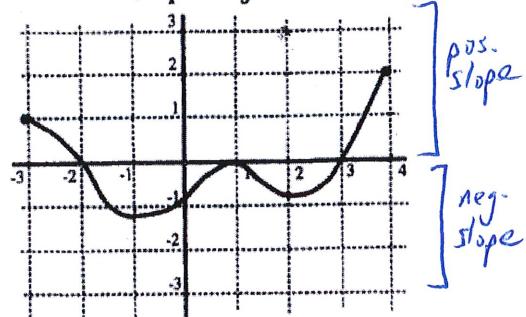
(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, III

Graph of f' Ch. 5 Logs/Exponentials

36)

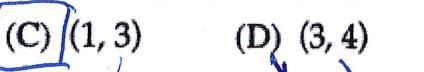
If $f(x) = \frac{x^2+1}{e^x}$, then the graph of f is decreasing and concave down on the interval

(A) $(-\infty, 0)$ (B) $(0, 1)$ (C) $(1, 3)$ (D) $(3, 4)$ (E) $(4, \infty)$

$f'(x) = \frac{(2x)(e^x) - (x^2+1)(e^x)}{(e^x)^2}$

$f'(x) = \frac{2xe^x - x^2e^x - e^x}{e^{2x}} = \frac{2x-x^2-1}{e^x}$

$f'(x) = \frac{-e^x(x^2-2x+1)}{e^{2x}} = \frac{-(x-1)(x-1)}{e^x}$

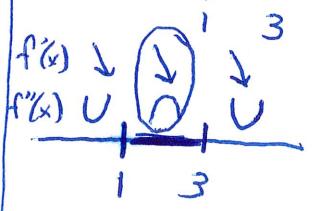


$f''(x) = \frac{(2-2x)e^x - (2x-x^2-1)e^x}{(e^x)^2} = \frac{2-4x+x^2}{e^x}$

$0 = e^x[2-2x-2x+x^2+1] = e^x[x^2-4x+3]$

$f''(x) = \frac{x^2-4x+3}{e^x}$

$$\begin{aligned} x^2-4x+3 &= 0 \\ (x-3)(x-1) &= 0 \\ x &= 1, 3 \end{aligned}$$



37)

The slope of the line tangent to the graph of $y = \ln \sqrt{x}$ at $(e^2, 1)$ is

- (A) $\frac{e^2}{2}$ (B) $\frac{2}{e^2}$ (C) $\frac{1}{2e^2}$ (D) $\frac{1}{2e}$ (E) $\frac{1}{e}$

38)

$$\begin{aligned} y &= \ln x^{1/2} \\ y &= \frac{1}{2} \ln x \end{aligned} \quad \left| \begin{array}{l} y'(x) = \frac{1}{2} \left(\frac{1}{x} \right) = \frac{1}{2x} \\ y'(e^2) = \boxed{\frac{1}{2e^2}} \end{array} \right.$$

If $y = e^{kx}$, then $\frac{d^5y}{dx^5} = \frac{d}{dx} e^u = e^u \cdot u'$

(A) $k^5 e^x$

$$y' = e^{kx} \cdot k = k e^{kx}$$

(B) $k^5 e^{kx}$

$$y'' = k e^{kx} \cdot k = k^2 e^{kx}$$

(C) $5! e^{kx}$

$$y'''(x) = k^2 e^{kx} \cdot k = k^3 e^{kx}$$

(D) $5! e^x$

$$y''''(x) = k^3 e^{kx} \cdot k = k^4 e^{kx}$$

(E) $5e^{kx}$

$$y''''(x) = k^4 e^{kx} \cdot k = k^5 e^{kx}$$

$$y''''(x) = k^5 e^{kx} \cdot k = k^5 e^{kx}$$

39)

What is the x -coordinate of the point of inflection on the graph of $y = xe^x$?

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

*find $y''(x)$, test intervals

$$y'(x) = 1 \cdot e^x + x \cdot e^x$$

$$y''(x) = e^x + 1 \cdot e^x + x \cdot e^x$$

$$y''(x) = 2e^x + xe^x$$

$$y''(x) = e^x(2+x)$$

$$0 = e^x(2+x)$$

$$\begin{array}{l} e^x = 0 \\ \text{none} \end{array} \quad \left| \begin{array}{l} x+2=0 \\ x=-2 \end{array} \right.$$

$$\begin{array}{c} f''(x) \\ \hline -3 & -2 & 0 \end{array}$$

POI at $x = -2$

POI at $x = -2$

$$y(-2) = -2e^{-2}$$

$$y(-2) = \frac{-2}{e^2}$$

40)

If $f(x) = \ln(\cos 2x)$, then $f'(x) =$

$$\frac{d}{dx} \ln u = \frac{u'}{u} \quad \frac{d}{dx} \cos u = -\sin u \cdot u'$$

- (A) $-2 \tan 2x$ (B) $\cot 2x$ (C) $\tan 2x$ (D) $-2 \cot 2x$ (E) $2 \tan 2x$

$$f'(x) = \frac{-\sin(2x) \cdot 2}{\cos(2x)} = \frac{-2 \sin(2x)}{\cos(2x)} = -2 \cdot \tan(2x) = \boxed{-2 \tan(2x)}$$

Trig Unit

41)

If $y = \cos^2(2x)$, then $\frac{dy}{dx} =$ * rewrite equation:

(A) $2 \cos 2x \sin 2x$

$y = [\cos(2x)]^2$

(B) $-4 \sin 2x \cos 2x$

$y' = 2[\cos(2x)] \cdot -\sin(2x) \cdot 2$

(C) $2 \cos 2x$

$$\boxed{y' = -4 \sin(2x) \cos(2x)}$$

(D) $-2 \cos 2x$

(E) $4 \cos 2x$

42)

If $g(x) = \arcsin 2x$, then $g'(x) =$

* $\frac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}}$

$$g'(x) = \frac{2}{\sqrt{1-(2x)^2}}$$

(A) $2 \arccos 2x$

(B) $2 \csc 2x \cot 2x$

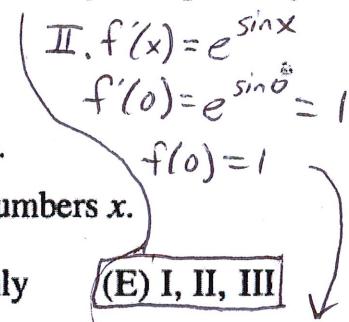
(C) $\frac{2}{1+4x^2}$

(D) $\frac{2}{\sqrt{4x^2-1}}$

(E) $\frac{2}{\sqrt{1-4x^2}}$

$$\boxed{g'(x) = \frac{2}{\sqrt{1-4x^2}}}$$

43)

Suppose the function f is defined so that $f(0) = 1$ and its derivative, f' , is given by $f'(x) = e^{\sin x}$. Which of the following statement are TRUE?

✓ I $f'''(0) = 1$

✓ II The line $y = x + 1$ is tangent to the graph of f at $x = 0$.

✓ III If $h(x) = f(x^3 - 1)$, then h is increasing for all real numbers x .

- (A) I only (B) II only (C) III only (D) I and II only

- (E) I, II, III

I. $f''(x) = e^{\sin x} \cdot \cos x$

$f''(0) = e^{\sin 0} \cdot \cos 0 = e^0 \cdot 1 = \boxed{1}$

III: $h(x) = f(x^3 - 1)$
 $h'(x) = \underbrace{f'(x^3 - 1)}_{\text{positive}} \cdot \underbrace{3x^2}_{\text{positive}}$
 so $f'(x) > 0$ for all x -values

Tangent Line:

point: $(0, 1)$ slope: $m = 1$

$y - 1 = 1(x - 0)$

$$\boxed{y = x + 1}$$

44)

$$*\frac{d}{dx} e^u = e^u \cdot u' \quad \frac{d}{dx} \sin u = \cos u \cdot u'$$

What is the instantaneous rate of change at $x = 0$ of the function f given by
 $f(x) = e^{2x} - 3 \sin x$?

- (A) -2 (B) -1 (C) 0 (D) 4 (E) 5

*find $f'(x)$

$$f'(x) = e^{2x} \cdot 2 - 3 \cos x = 2e^{2x} - 3 \cos x$$

$$f'(0) = 2e^0 - 3 \cos 0 = 2 - 3 = \boxed{-1}$$

45)

If $f(x) = \sin(2x) + \ln(x+1)$, then $f'(0) =$

$$*\frac{d}{dx} \ln u = \frac{u'}{u}$$

- (A) -1 (B) 0 (C) 1 (D) 2

$$f'(x) = \cos(2x) \cdot 2 + \frac{1}{x+1}$$

$$f'(x) = 2\cos 2x + \frac{1}{x+1}$$

$$\left| \begin{array}{l} f'(0) = 2\cos(0) + \frac{1}{0+1} \\ = 2(1) + 1 = \boxed{3} \end{array} \right.$$

(E) 3

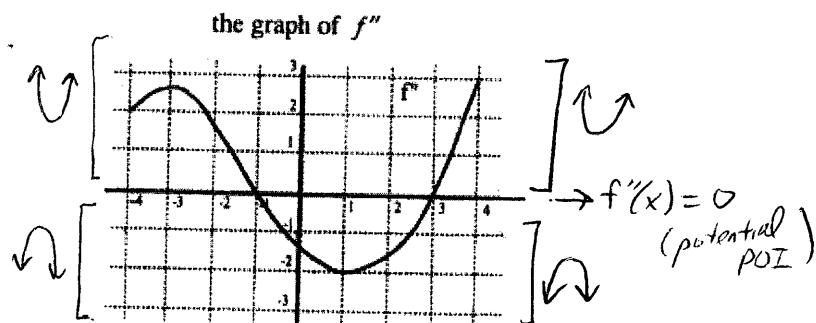
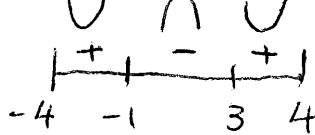
Calculator Active Practice Problems

46)

The graph of the second derivative of a function f is shown at the right. Which of the following is true?

- ✓ I. The graph of f has an inflection point at $x = -1$.
- ✓ II. The graph of f is concave down on the interval $(-1, 3)$.
- ✗ III. The graph of the derivative function f' is increasing at $x = 1$. False, since $f'(x)$ is concave down at $x = 1$.

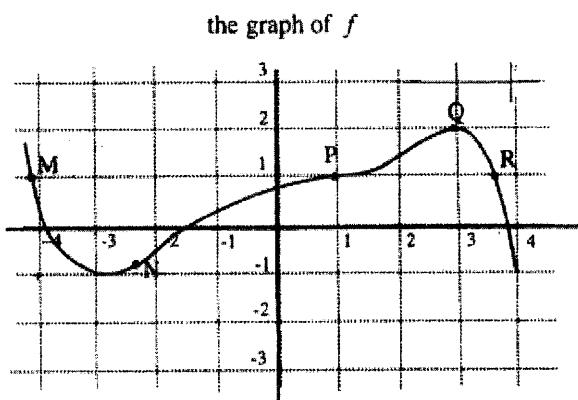
- (A) I only (B) II only (C) III only (D) I and II only (E) I, II, III



47)

The graph of the function f is shown at the right. At which point on the graph of f are all the following true?

$f(x) > 0$, and $f'(x) < 0$ and $f''(x) < 0$
 above the x -axis slope is negative graph is concave down

(A) M (B) N (C) P (D) Q (E) R

48)

Functions f and g are defined by $f(x) = \frac{1}{x^2}$ and $g(x) = \arctan x$. What is the approximate value of x for which $f'(x) = g'(x)$?

(A) -3.36

(B) -2.86

(C) -2.36

(D) 1.36

(E) 2.36

$$\begin{aligned}f(x) &= \frac{1}{x^2} = x^{-2} \\f'(x) &= -2x^{-3} \\f'(x) &= -\frac{2}{x^3}\end{aligned}$$

$$\begin{aligned}g'(x) &= \frac{1}{1+x^2} \\-\frac{2}{x^3} &= \frac{1}{1+x^2} \\-2(1+x^2) &= x^3\end{aligned}$$

$$\begin{aligned}&\text{set } f'(x) = g'(x) \\&\frac{-2}{x^3} = \frac{1}{1+x^2} \\&-2(1+x^2) = x^3\end{aligned}$$

$$\frac{d}{dx} \arctan u = \frac{u'}{1+u^2}$$

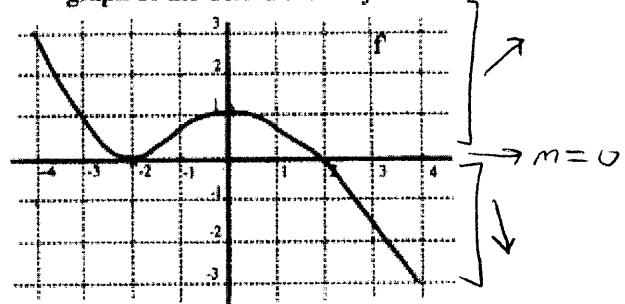
*graph
 $y = x^3 + 2x^2 + 2$
 and find
 x-intercept.

$$x \approx -2.359$$

49)

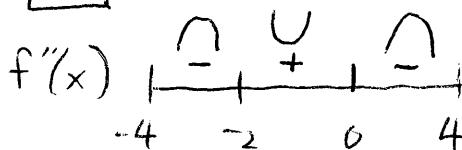
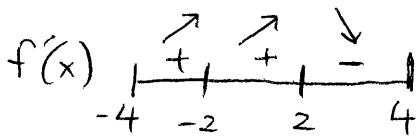
The graph of the derivative of f is shown at the right. If the graph of f' has horizontal tangents at $x = -2$ and 0 , which of the following is true about the function f ?

- I. f is decreasing at $x = 0$.
- II. f has a local maximum at $x = 2$.
- III. The graph of f is concave up at $x = -1$.

graph of the derivative of f 

(A) I only (B) II only (C) I and II only

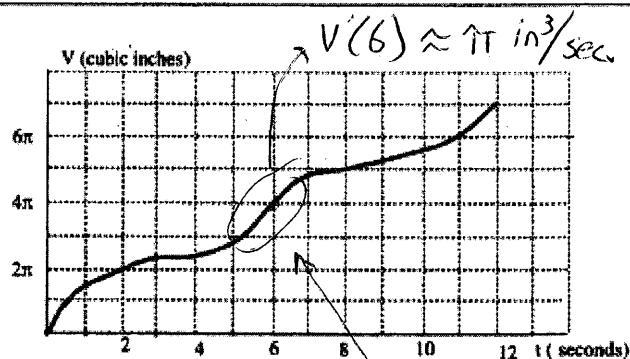
(D) II and III only (E) I, II, III



The function V whose graph is sketched below gives the volume of air, $V(t)$, (measured in cubic inches) that a man has blown into a balloon after t seconds.

$$\left(V = \frac{4}{3} \pi r^3 \right)$$

The rate at which the radius is changing after 6 seconds is nearest to



- (A) 0.05 in/sec (B) 0.12 in/sec (C) 0.21 in/sec (D) 0.29 in/sec (E) 0.37 in/sec

$$t=6 \quad \begin{cases} V = \frac{4}{3} \pi r^3 \\ \frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt} \right) \end{cases}$$

$\frac{dr}{dt} = ?$

$\frac{dV}{dt} = \frac{4}{3} \cdot 3\pi r^2 \left(\frac{dr}{dt} \right)$

* $\frac{dV}{dt}$ or $V'(6)$ is the rate of change of volume w/ respect to time. the slope is nearest to π

51)

$$(D) 0.29 \text{ in/sec}$$

$$(E) 0.37 \text{ in/sec}$$

$$\begin{aligned} V(6) &= 4\pi \\ V &= \frac{4}{3}\pi r^3 \\ 4\pi &= \frac{4}{3}\pi(r)^3 \\ 4\pi \cdot \frac{3}{4\pi} &= r^3 \\ 3 &= r^3 \quad r = \sqrt[3]{3} \end{aligned}$$

$\frac{dr}{dt} \approx 0.120 \text{ in/sec}$

At how many points on the interval $-2\pi \leq x \leq 2\pi$ does the tangent to the graph of the curve

$y = x \cos x$ have slope $\frac{\pi}{2}$?

Step 5:

1) find $y'(x)$ using product rule

(A) 5 (B) 4 2) set $f'(x) = \frac{\pi}{2}$

(C) 3 3) set equation = 0

(D) 2 4) graph equation

(E) 1 5) count the number of solutions
(x -intercepts)

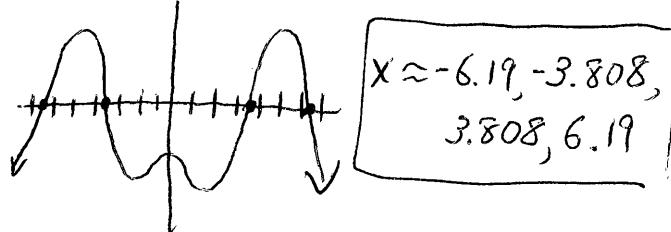
52)

$$y'(x) = 1 \cdot \cos x + x \cdot (-\sin x)$$

$$y' = \cos x - x \sin x$$

$$\frac{\pi}{2} = \cos x - x \sin x$$

$$0 = \cos x - x \sin x - \frac{\pi}{2}$$



$$x \approx -6.19, -3.808, 3.808, 6.19$$

If $f(x) = 2x + \sin x$ and the function g is the inverse of f , then $g'(2) =$

(A) 0.32 $2 = 2x + \sin x$

(B) 0.34 $0 = 2x + \sin x - 2$

(C) 0.36 $x \approx 0.684$

(D) 0.38

(E) 0.40

$$f(\underline{\hspace{0.684}}) = 2 \quad g(2) = 0.684$$

$$f'(0.684) = \underline{\hspace{0.684}} \quad g'(2) = \underline{\hspace{0.684}}$$

$$f'(x) = 2 + \cos x$$

$$f'(0.684) = 2 + \cos(0.684) = 2.775$$

* since $f'(0.684) = 2.775$,

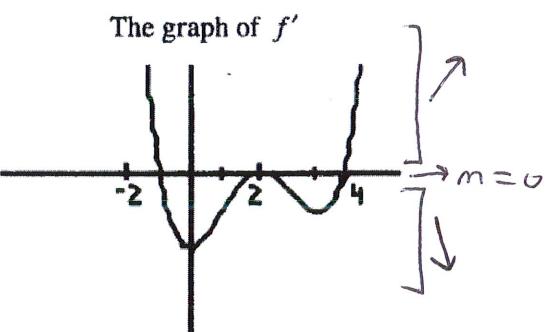
$$g'(2) \approx \frac{1}{2.775}$$

$$\approx \boxed{0.36}$$

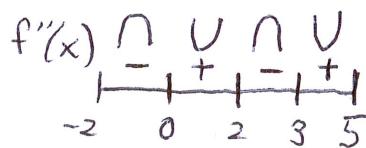
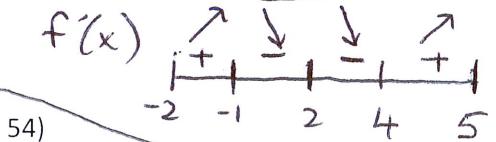
53)

Let f be a function that has domain $[-2, 5]$. The graph of f' is shown at the right. Which of the following statements are TRUE?

- I. f has a relative maximum at $x = -1$.
- II. f has an absolute minimum at $x = 0$.
- III. The graph of f is concave down for $-2 < x < 0$.
- IV. The graph of f has inflection points at $x = 0$ and $x = 2$ and $x = 3$.



- (A) I, II, IV **(B)** I, III, IV (C) II, III, IV (D) I, II, III (E) I, II, III, IV



54)

The function f is defined on all the reals such that $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \leq 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

- (A) $k = -1, b = -3$
 (B) $k = 1, b = 3$
 (C) $k = 1, b = 4$
 (D) $k = 1, b = -4$
 (E) $k = -1, b = 6$

55)

*set equations equal (at $x=1$) b/c $f(x)$ is continuous, and piecewise will share same y-value at $x=1$

*set derivatives of piecewise equal b/c $f(x)$ is differentiable, and piecewise will share same slopes at $x=1$

at $x=1$ at $x=1$

$$\begin{aligned} x^2 + kx - 3 &= 3x + b \\ (1)^2 + k(1) - 3 &= 3(1) + b \\ 1 + k - 3 &= 3 + b \\ 1 + k - 3 &= 3 + b \end{aligned}$$

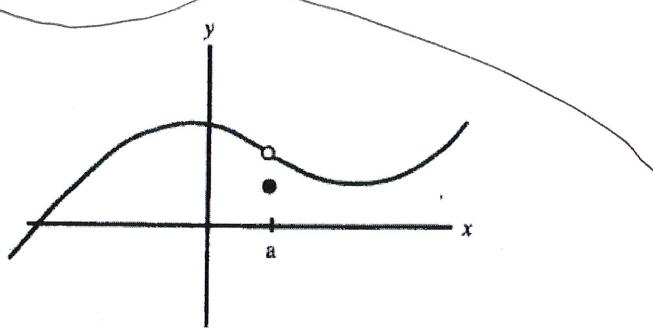
$$\begin{aligned} 2x + k &= 3 \\ 2(1) + k &= 3 \\ k &= 1 \end{aligned}$$

$$b = -4$$

The graph of a function f is shown to the right.

Which of the following statements about f is false?

- (A) f has a relative minimum at $x = a$.
- (B) $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$
- (C) $\lim_{x \rightarrow a} f(x) \neq f(a)$
- (D) $f(a) > 0$
- (E) $f'(a) < 0$



56)

The function f defined by $f(x) = e^{3x} + 6x^2 + 1$ has a horizontal tangent at $x =$

- (A) -0.144 (B) -0.150

(C) -0.156

(D) -0.162

(E) -0.168

*set $f'(x) = 0$

$$f'(x) = e^{3x} \cdot 3 + 12x$$

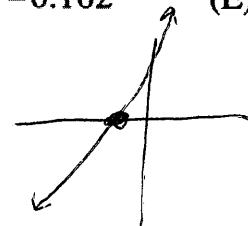
$$0 = 3e^{3x} + 12x$$

57)

*graph $y = 3e^{3x} + 12x$

*look for x-int.

x ≈ -0.156



The graph of the second derivative of a function g is shown in the figure. Use the graph to determine which of the following are true.

X I. The g -graph has points of inflection at $x = 1$ and $x = 3$.

✓ II. The g -graph is concave down on the interval $(3, 4)$.

✓ III. If $g'(0) = 0$, g is increasing at $x = 2$.

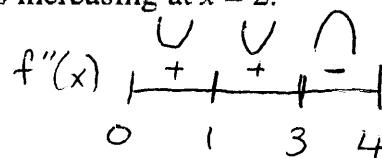
(A) I only

(B) II only

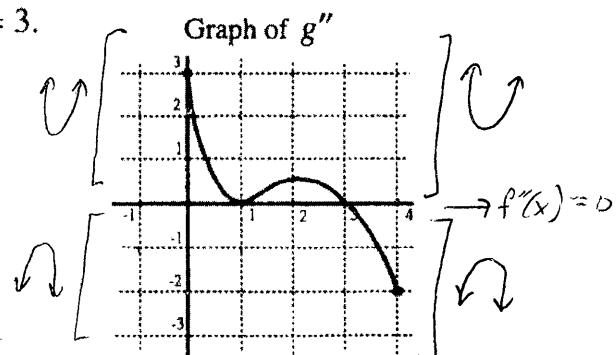
(C) II and III only

(D) I and II only

(E) I, II, III



POT: $x = 3$



58)

If $k \neq 0$, then $\lim_{x \rightarrow k} \frac{x^2 - k^2}{x^2 - kx} = \frac{0}{0} \rightarrow \lim_{x \rightarrow k} \frac{(x-k)(x+k)}{x(x-k)}$

(A) 0

(B) 2

(C) $2k$

(D) $4k$

(E) nonexistent

$$\lim_{x \rightarrow k} \frac{(x+k)}{x} = \frac{k+k}{k} = \frac{2k}{k} = \boxed{2}$$

59)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	0	-1	-2	5
2	4	3	5	1
3	2	3	-1	0

$$*\text{chain rule: } \frac{d}{dx} f[g(x)] = f'[g(x)] \cdot g'(x)$$

$$h(x) = f(g(x))$$

The table shows some of the values of differentiable functions f and g and their derivatives. If $h(x) = f(g(x))$, then $h'(2)$ equals

- (A) -2
 (B) -1
 (C) 0
 (D) 1
 (E) 2

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(2) = f'[g(2)] \cdot g'(2)$$

$$T \circ f \circ g(\alpha)$$

$$= (-1) \circ (1) = \boxed{-1}$$

60)

At what value(s) of x do the graphs of $y = e^x$ and $y = x^2 + 5x$ have parallel tangent lines?

* set derivatives of equations equal

$$y_1 = e^x$$

$$y_2 = x^2 + 5x$$

$$y_i = e^x$$

$$y_2 = 2x + 5$$

$$2x+5 = e^x$$

$$0 = e^x - 2x - 5$$

*graph y=e^x-2x-5,

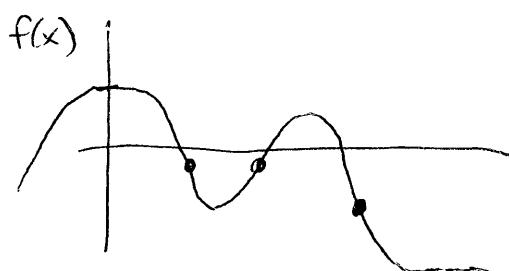
find x-int:

$$x \approx -2.457, 2.251$$

61)

How many points of inflection does the graph of $y = \cos x + \frac{1}{3} \cos 3x - \frac{1}{5} \cos 5x$ have on the interval $0 \leq x \leq \pi$?

- (A) 1
 (B) 2
 (C) 3
 (D) 4
 (E) 5



Implicit Differentiation (2nd derivative)

62) Find the $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the following: $x^2 - y^2 = 1$

$$x^2 - y^2 = 1$$

$$2x - 2y\left(\frac{dy}{dx}\right) = 0$$

$$-2y\left(\frac{dy}{dx}\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{f'}{g} - f\frac{g'}{g^2}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y - x\left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y - x\left(\frac{x}{y}\right)}{y^2} = \frac{y - \frac{x^2}{y}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y - \frac{x^2}{y}}{y^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{y^2 - x^2}{y^3}}$$

$$= \boxed{\frac{-1}{y^3}}$$

$$x^2 - y^2 = 1$$

$$-1 = y^2 - x^2$$