AB Calculus Fall Exam Review Packet #3

Non-Calculators

Ch. 1 Limits

1) $\lim_{x\to 2^-} \frac{(x+2)^2}{x^2-4} =$

A) 4 B) 1 C) ∞ D) $-\infty$ E) Does Not Exist

2) $\lim_{x\to -11^-} \frac{\sqrt{x+19}-\sqrt{8}}{x+11} =$

A) $\frac{1}{2\sqrt{8}}$ B) $-\frac{1}{2\sqrt{8}}$ C) ∞ D) $-\infty$ E) Does Not Exist

3) $\lim_{x\to -\infty} \frac{5x^4-5x-5}{(2x^2-3)^2} =$

A) $\frac{5}{2}$ B) $\frac{5}{4}$ C) ∞ D) $-\infty$ E) Does Not Exist

4) $\lim_{x\to 1} \frac{(x+1)^2}{x^2-1} =$

A) 1 B) 2 C) ∞ D) $-\infty$ E) Does Not Exist

5) $\lim_{x\to -\infty} \frac{4-x^3}{2-x^2} =$

A) 4 B) 2 C) ∞ D) $-\infty$ E) - 2

6) What is $\lim_{x\to 0}\left(\dfrac{\frac{1}{x-1}+1}{x}\right)$?

(A) –1 (B) 0 (C) 1 (D) 2 (E) the limit does not exist

7)

What is $\lim_{x\to\infty}\dfrac{3x^2+1}{(3-x)(3+x)}$?

(A) –9 (B) –3 (C) 1 (D) 3 (E) The limit does not exist.

8)

Which of the following functions is both continuous and differentiable at all x in the interval $-2 \le x \le 2$?

(A) $f(x) = |x^2 - 1|$

(B) $f(x) = \sqrt{x^2 - 1}$

(C) $f(x) = \sqrt{x^2 + 1}$

(D) $f(x) = \dfrac{1}{x^2 - 1}$

(E) none of these

9)

$\lim_{x\to 1}\left(\dfrac{\sqrt{x+3}-2}{1-x}\right)$

(A) 0.5

(B) 0.25

(C) 0

(D) –0.25

(E) –0.5

10)

Let f be defined by $f(x) = \begin{cases} \dfrac{x^2 - 2x + 1}{x - 1} & \text{for } x \neq 1 \\ k & \text{for } x = 1. \end{cases}$

Determine the value of k for which f is continuous for all real x.

(A) 0

(B) 1

(C) 2

(D) 3

(E) none of the above

11)

The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

(A) $\lim_{x \to c} f(x)$ exists (B) $\lim_{x \to c} f(x) = f(c)$ (C) $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$

(D) $f(c)$ is defined (E) $f'(c)$ exists

Ch. 2 Derivatives

12)

Find the point on the graph of $y = \sqrt{x}$ between (1, 1) and (9, 3) at which the tangent to the graph has the same slope as the line through (1, 1) and (9, 3).

(A) (1, 1)

(B) $(2, \sqrt{2})$

(C) $(3, \sqrt{3})$

(D) (4, 2)

(E) none of the above

13)

If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{3}{2x}$

(B) $\frac{3x}{(1+x^2)^2}$

(C) $\frac{6x}{(4+x^2)^2}$

(D) $\frac{-6x}{(4+x^2)^2}$

(E) $\frac{-3}{(4+x^2)^2}$

14)

Consider the curve $x + xy + 2y^2 = 6$. The slope of the line tangent to the curve at the point (2,1) is

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $-\frac{1}{3}$

(D) $-\frac{1}{5}$

(E) $-\frac{3}{4}$

15)

If $p(x) = (x-1)(x+k)$ and if the line tangent to the graph of p at the point $(4, p(4))$ is parallel to the line $5x - y + 6 = 0$, then $k =$

(A) 2

(B) 1

(C) 0

(D) –1

(E) –2

16)

The formula $x(t) = \ln t + \frac{t^2}{18} + 1$ gives the position of an object moving along the x-axis during the time interval $1 \le t \le 5$. At the instant when the acceleration of the object is zero, the velocity is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 1 (E) undefined

17)

A particle moves along the x-axis so that its distance from the origin at time t is given by $10t - 4t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?

(A) 1.0

(B) 1.5

(C) 2.0

(D) 2.5

(E) 3.0

18)

If $xy + x^2 = 6$, then the value of $\frac{dy}{dx}$ at $x = -1$ is

(A) –7 (B) –2 (C) 0 (D) 1 (E) 3

19)

A particle moves along the x-axis so that its position at any time $t \ge 0$ is given by $x(t) = \frac{t}{t^2 + 4}$. The particle is at rest when $t =$

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) 2 (E) 4

20)

If y is a differentiable function of x, then the slope of the tangent to the curve $xy - 2y + 4y^2 = 6$ at the point where $y = 1$ is

(A) $\frac{1}{12}$ (B) $-\frac{1}{10}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $-\frac{5}{6}$

21)

The y-intercept of the tangent line to the curve $y = \sqrt{x+3}$ at the point $(1, 2)$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$ (E) $\frac{7}{4}$

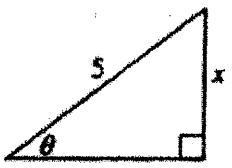
Use the table data and rules of differentiation to solve each problem:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	2	1	3
1	0	2	1	0
2	1	2	2	1
3	1	3	1	0

22) $If\ h(x) = \frac{g(x)}{f(x)}$ find $h'(3)$

23) $If\ k(x) = g(f(x))$ find $k'(1)$

24) $If\ p(x) = [(f(x)]^3$ find $p'(2)$



25) In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

26)

The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?

(A) 5 in^3/sec

(B) 10 in^3/sec

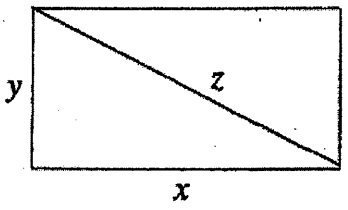
(C) 15 in^3/sec

(D) 20 in^3/sec

(E) 25 in^3/sec

27)

The diagonal z of the rectangle at the right is increasing at the rate of 2 cm/sec and $\frac{dy}{dt} = 3\frac{dx}{dt}$. At what rate is the length x increasing when $x = 3$ cm and $y = 4$ cm ?



(A) 1 cm/sec

(B) $\frac{3}{4}$ cm/sec

(C) $\frac{2}{3}$ cm/sec

(D) $\frac{1}{3}$ cm/sec

(E) $\frac{1}{15}$ cm/sec

28)

Find the maximum value of $f(x) = 2x^3 + 3x^2 - 12x + 4$ on the closed interval [0,2].

(A) –3

(B) 2

(C) 4

(D) 8

(E) 24

29)

The function $f(x) = x^4 - 18x^2$ has a relative minimum at $x =$

(A) 0 and 3 only

(B) 0 and –3 only

(C) –3 and 3 only

(D) 0 only

(E) –3, 0, 3

30)

The graph of $y = 3x^5 - 10x^4$ has an inflection point at

(A) $(0, 0)$ and $(2, -64)$

(B) $(0, 0)$ and $(3, -81)$

(C) $(0, 0)$ only

(D) $(-3, 81)$ only

(E) $(2, -64)$ only

31)

Consider the function $f(x) = \frac{x^4}{2} - \frac{x^5}{10}$. The *derivative* of f attains its maximum value at $x =$

(A) 3

(B) 4

(C) 5

(D) 0

(E) there is no maximum

32)

Let $f(x) = x^4 + ax^2 + b$. The graph of f has a relative maximum at $(0, 1)$ and an inflection point when $x = 1$. The values of a and b are

(A) $a = 1, \quad b = -6$

(B) $a = 1, \quad b = 6$

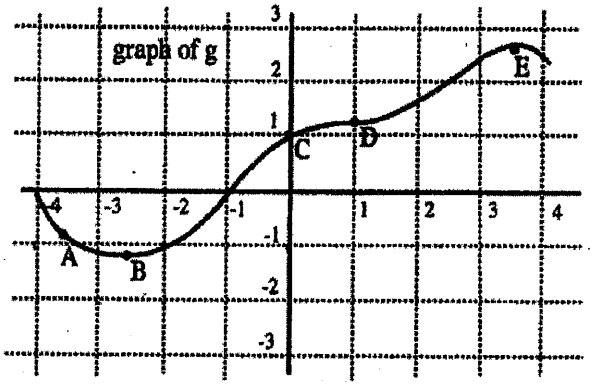
(C) $a = -6, \quad b = 5$

(D) $a = -6, \quad b = 1$

(E) $a = 6, \quad b = 1$

33)

At which point on the graph of $y = g(x)$ below is $g'(x) = 0$ and $g''(x) = 0$?



(A) A (B) B (C) C (D) D (E) E

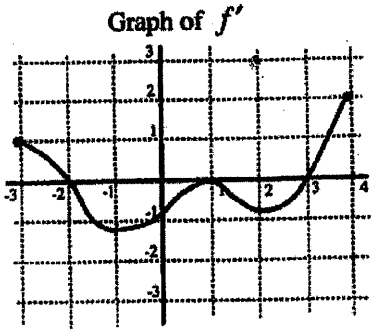
34)

If $f(x) = x^3 - 5x^2 + 3x$, then the graph of f is decreasing and concave down on the interval

(A) $\left(0, \frac{1}{3}\right)$ (B) $\left(\frac{1}{3}, \frac{2}{3}\right)$ (C) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\left(\frac{5}{3}, 3\right)$ (E) $(3, \infty)$

35)

The figure shows the graph of f', the derivative of a function f. The domain of f is the closed interval $[-3,4]$. Which of the following is true?



I. f is increasing on the interval $(2,4)$.

II. f has a relative minimum at $x = -2$.

III. The f-graph has an inflection point at $x = 1$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, III

Ch. 5 Logs/Exponentials

36)

If $f(x) = \frac{x^2+1}{e^x}$, then the graph of f is decreasing and concave down on the interval

(A) $(-\infty, 0)$ (B) $(0, 1)$ (C) $(1, 3)$ (D) $(3, 4)$ (E) $(4, \infty)$

37)

The slope of the line tangent to the graph of $y = \ln \sqrt{x}$ at $(e^2, 1)$ is

(A) $\frac{e^2}{2}$ (B) $\frac{2}{e^2}$ (C) $\frac{1}{2e^2}$ (D) $\frac{1}{2e}$ (E) $\frac{1}{e}$

38)

If $y = e^{kx}$, then $\frac{d^5y}{dx^5} =$

(A) k^5e^x

(B) k^5e^{kx}

(C) $5!\,e^{kx}$

(D) $5!\,e^x$

(E) $5e^{kx}$

39)

What is the x-coordinate of the point of inflection on the graph of $y = xe^x$?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

40)

If $f(x)=\ln(\cos 2x)$, then $f'(x)=$

(A) $-2\tan 2x$ (B) $\cot 2x$ (C) $\tan 2x$ (D) $-2\cot 2x$ (E) $2\tan 2x$

Trig Unit

41)

If $y=\cos^2(2x)$, then $\frac{dy}{dx}=$

(A) $2\cos 2x\sin 2x$

(B) $-4\sin 2x\cos 2x$

(C) $2\cos 2x$

(D) $-2\cos 2x$

(E) $4\cos 2x$

42)

If $g(x)=\text{Arcsin } 2x$, then $g'(x)=$

(A) $2\text{Arccos } 2x$ (B) $2\csc 2x\cot 2x$ (C) $\frac{2}{1+4x^2}$

(D) $\frac{2}{\sqrt{4x^2-1}}$ (E) $\frac{2}{\sqrt{1-4x^2}}$

43)

Suppose the function f is defined so that $f(0)=1$ and its derivative, f', is given by $f'(x)=e^{\sin x}$. Which of the following statement are TRUE?

I $f''(0)=1$

II The line $y=x+1$ is tangent to the graph of f at $x=0$.

III If $h(x)=f(x^3-1)$, then h is increasing for all real numbers x.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

44)

What is the instantaneous rate of change at $x = 0$ of the function f given by $f(x) = e^{2x} - 3\sin x$?

(A) –2 (B) –1 (C) 0 (D) 4 (E) 5

45)

If $f(x) = \sin(2x) + \ln(x+1)$, then $f'(0) =$

(A) –1 (B) 0 (C) 1 (D) 2 (E) 3

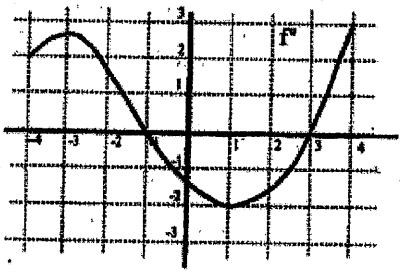
Calculator Active Practice Problems

46)

The graph of the **second derivative** of a function f is shown at the right. Which of the following is true?

I. The graph of f has an inflection point at $x = -1$.
II. The graph of f is concave down on the interval $(-1, 3)$.
III. The graph of the derivative function f' is increasing at $x = 1$.

the graph of f''



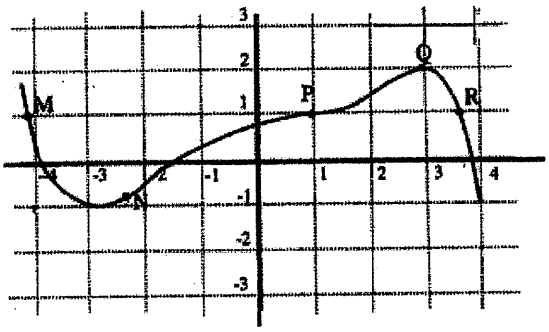
(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

47)

The graph of the function f is shown at the right. At which point on the graph of f are all the following true?

$f(x) > 0$, and $f'(x) < 0$ and $f''(x) < 0$

the graph of f



(A) M (B) N (C) P (D) Q (E) R

48)

Functions f and g are defined by $f(x) = \frac{1}{x^2}$ and $g(x) = \arctan x$. What is the approximate value of x for which $f'(x) = g'(x)$?

(A) –3.36 (B) –2.86 (C) –2.36 (D) 1.36 (E) 2.36

49)

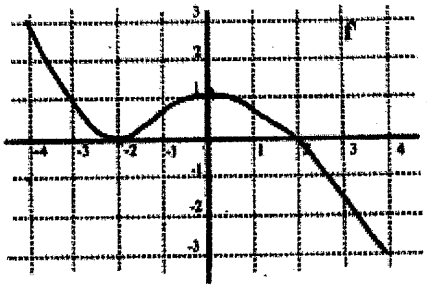
The graph of the *derivative* of f is shown at the right. If the graph of f' has horizontal tangents at $x = -2$ and 0, which of the following is true about the function f?

I. f is decreasing at $x = 0$.

II. f has a local maximum at $x = 2$.

III. The graph of f is concave up at $x = -1$.

graph of the derivative of f



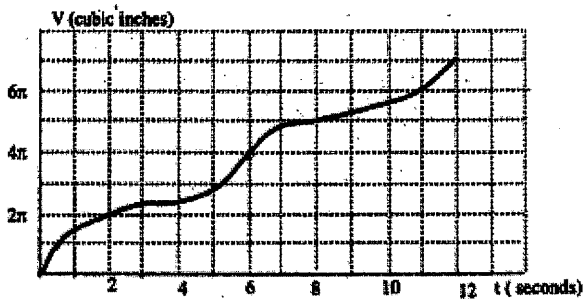
(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

50)

The function V whose graph is sketched below gives the volume of air, $V(t)$, (measured in cubic inches) that a man has blown into a balloon after t seconds.

$$\left(V = \frac{4}{3}\pi r^3\right)$$

The rate at which the radius is changing after 6 seconds is nearest to



(A) 0.05 in/sec (B) 0.12 in/sec (C) 0.21 in/sec (D) 0.29 in/sec (E) 0.37 in/sec

51)

At how many points on the inteval $-2\pi \le x \le 2\pi$ does the tangent to the graph of the curve $y = x\cos x$ have slope $\frac{\pi}{2}$?

(A) 5

(B) 4

(C) 3

(D) 2

(E) 1

52)

If $f(x) = 2x + \sin x$ and the function g is the inverse of f, then $g'(2) =$

(A) 0.32

(B) 0.34

(C) 0.36

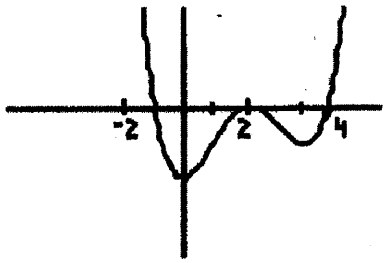
(D) 0.38

(E) 0.40

53)

Let f be a function that has domain $[-2, 5]$. The graph of f' is shown at the right. Which of the following statements are TRUE?

The graph of f'



I. f has a relative maximum at $x = -1$.

II. f has an absolute minimum at $x = 0$.

III. The graph of f is concave down for $-2 < x < 0$.

IV. The graph of f has inflection points at $x = 0$ and $x = 2$ and $x = 3$.

(A) I, II, IV (B) I, III, IV (C) II, III, IV (D) I, II, III (E) I, II, III, IV

54)

The function f is defined on all the reals such that $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \le 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

(A) $k = -1, b = -3$

(B) $k = 1, b = 3$

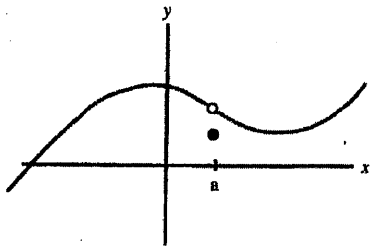
(C) $k = 1, b = 4$

(D) $k = 1, b = -4$

(E) $k = -1, b = 6$

55)

The graph of a function f is shown to the right.
Which of the following statements about f is false?



(A) f has a relative minimum at $x = a$.

(B) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

(C) $\lim_{x \to a} f(x) \neq f(a)$

(D) $f(a) > 0$

(E) $f'(a) < 0$

56)

The function f defined by $f(x) = e^{3x} + 6x^2 + 1$ has a horizontal tangent at $x =$

(A) -0.144 (B) -0.150 (C) -0.156 (D) -0.162 (E) -0.168

57)

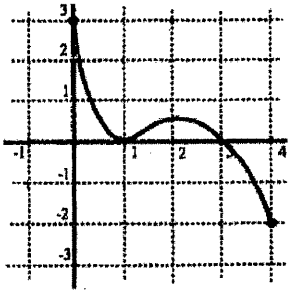
The graph of the second derivative of a function g is shown in the figure. Use the graph to determine which of the following are true.

I. The g-graph has points of inflection at $x = 1$ and $x = 3$.

II. The g-graph is concave down on the interval $(3, 4)$.

III. If $g'(0) = 0$, g is increasing at $x = 2$.

Graph of g''



(A) I only

(B) II only

(C) II and III only

(D) I and II only

(E) I, II, III

58)

If $k \neq 0$, then $\lim_{x \to k} \dfrac{x^2 - k^2}{x^2 - kx} =$

(A) 0

(B) 2

(C) $2k$

(D) $4k$

(E) nonexistent

59)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	0	−1	−2	5
2	4	3	5	1
3	2	3	−1	0

The table shows some of the values of differentiable functions f and g and their derivatives. If $h(x) = f(g(x))$, then $h'(2)$ equals

(A) −2
(B) −1
(C) 0
(D) 1
(E) 2

60)

At what value(s) of x do the graphs of $y = e^x$ and $y = x^2 + 5x$ have parallel tangent lines?

(A) −2.5
(B) 0
(C) 0 and 5
(D) −5 and 0.24
(E) −2.45 and 2.25

61)

How many points of inflection does the graph of $y = \cos x + \frac{1}{3}\cos 3x - \frac{1}{5}\cos 5x$ have on the interval $0 \le x \le \pi$?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Implicit Differentiation (2nd derivative)

62) Find the $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the following: $x^2 - y^2 = 1$