1) **AB–3 / BC–3** **1999**

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24–hour period.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

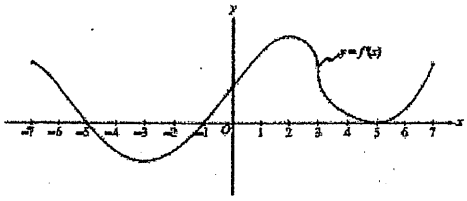
(c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24–hour time period. Indicate units of measure.

c) Approximate $R'(18)$

3)

AP Calculus AB–3 **2000**

The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.



(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

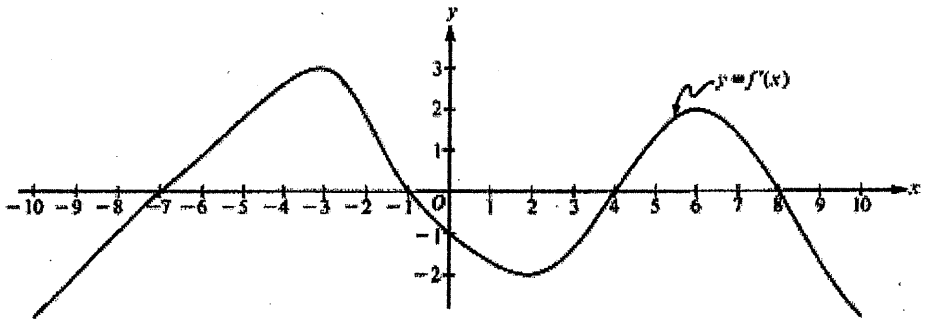
(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f have point(s) of inflection? Justify.

5)

Mario is standing 25 feet from the launch pad of a hot air balloon. If the balloon is rising at the rate of 20 ft/min, how will the angle between Mario's eye and the balloon be changing after 5 minutes?

1989 AB5



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \le x \le 10$.

(a) For what values of x does the graph of f have a horizontal tangent?

(b) For what values of x in the interval $(-10,10)$ does f have a relative maximum? Justify your answer.

(c) For value of x is the graph of f concave downward?

d) For what value of x does the graph of f have point(s) of inflection?

Key

1)

AB–3 / BC–3 — 1999

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c) Approximate $R'(18)$

c) $R'(18) \approx \frac{R(21)-R(18)}{21-18} = \frac{10.2-10.7}{21-18}$

$= \boxed{\frac{-1}{6} \text{ gallons}^3/\text{hr}^2}$

or $\frac{R(24)-R(21)}{24-21}$

b) Rolle's theorem applies since

i) $R(t)$ is continuous on $[0, 24]$

ii) $R(t)$ is differentiable on $(0, 24)$

iii) $R(0) = R(24) = 9.6$

Therefore $r'(t) = 0$

Question 3

3)

AP Calculus AB–3 — 2000

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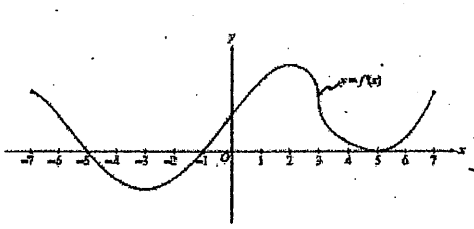

pos slope

m = 0

neg. slope

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(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

d) At what values of x, for $-7 \le x \le 7$, is there POI, and justify

$f'(x)$: + (−7, −5), − (−5, −1), + (−1, 5), + (5, 7)

$f''(x)$: ∩ (−7, −3), ∪ (−3, 2), ∩ (2, 5), ∪ (5, 7)

b) Relative max at $x = -5$ b/c f' changes from + to −

a) Relative min at $x = -1$ b/c f' changes from − to +

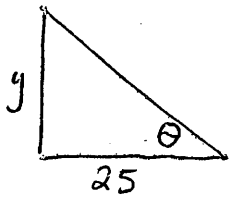
c) Concave down $(-7, -3) \cup (2, 5)$ b/c $f''(x) < 0$

d) POI at $x = -3, 2, 5$ since $f''(x)$ change signs

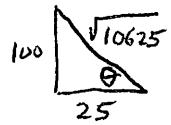
4)

5)

Mario is standing 25 feet from the launch pad of a hot air balloon. If the balloon is rising at the rate of 20 ft/min, how will the angle between Mario's eye and the balloon be changing after 5 minutes?



$\cos\theta = \frac{25}{\sqrt{10625}}$



$\frac{dy}{dt} = 20\text{ ft/min}$

$y = 100 \rightarrow 5\times 20 = 100$

$z = \sqrt{100^2+25^2}$
$= \sqrt{10625}$

$\tan\theta = \frac{y}{25}$

$\tan\theta = \frac{1}{25}y$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{25}\left(\frac{dy}{dt}\right)$

$\frac{d\theta}{dt} = \frac{1}{\sec^2\theta}\cdot\frac{1}{25}\cdot\frac{dy}{dt}$

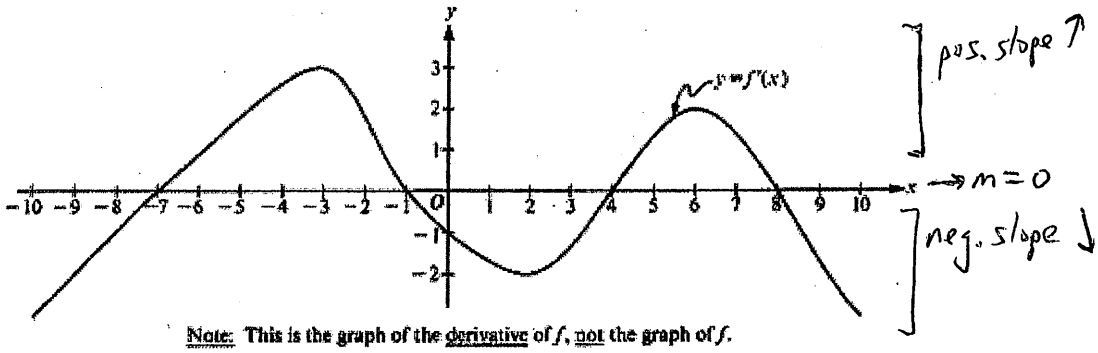
$\frac{d\theta}{dt} = (\cos\theta)^2\cdot\frac{1}{25}\cdot\frac{dy}{dt}$

$= \left[\frac{25}{\sqrt{10625}}\right]^2\cdot\frac{1}{25}\cdot 20$

$\frac{d\theta}{dt} = \frac{25^2}{10625}\cdot\frac{20}{25} = \frac{4}{85}$

$\boxed{\frac{d\theta}{dt} = \frac{4}{85}\text{ rad/min}}$

6) **1989 AB5**



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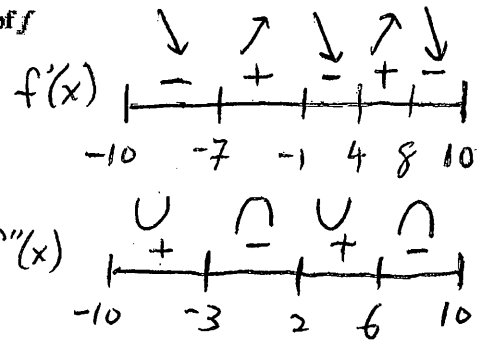

d) For what value of x does the graph of f have point(s) of inflection?

a) horizontal tangent where $f'(x)=0$ at $x=-7,-1,4,8$

b) Rel. max at $x=-1$, $x=8$ b/c f' changes from + to −

c) f is concave down $(-3,2)\cup(6,10)$ b/c $f''<0$
(or f' is decreasing)

d) POI at $x=-3,2,6$ b/c $f''(x)$ change signs
(or f' changes from inc to decrease or from dec. to increase)