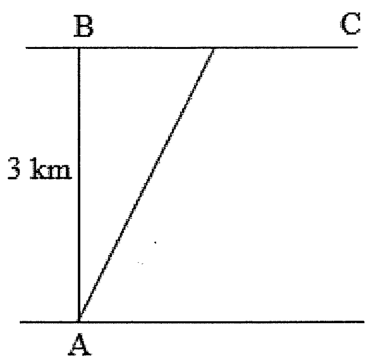
Optimization Problems WS #2

1. Points A and B are opposite each other on shores of a straight river 3 km wide. Point C is on the same shore as B, but 5 km down the river from B. A telephone company wishes to lay cable from A to C. If the cost per kilometer of the cable is 25% more under the water than it is on land, what line of cable would be least expensive for the company?



2. You are standing at the edge of a slow-moving river which is one mile wide and wish to return to your campground on the opposite side of the river. You can swim at 2 mph and walk at 3 mph. You must first swim across the river to any point on the opposite bank. From there walk to the campground, which is one mile from the point directly across the river from where you start your swim. What route will take the least amount of time? Recall that (time = distance/velocity)

3. A cylindrical can is to hold 20π m^3 .The material for the top and bottom costs \$10/ m^2 and material for the side costs \$8/m^2 . Find the radius r and height h of the most economical can.

Surface Area = $2\pi r^2 + 2\pi rh$ Volume: $\pi r^2 h$

4. There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

Key

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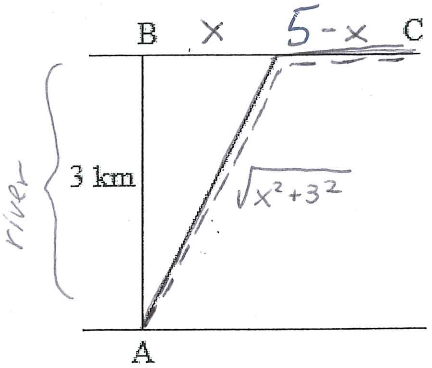
1. Points A and B are opposite each other on shores of a straight river 3 km wide. Point C is on the same shore as B, but 5 km down the river from B. A telephone company wishes to lay cable from A to C. If the cost per kilometer of the cable is 25% more under the water than it is on land, what line of cable would be least expensive for the company?



$$C(x) = (1)(5-x) + (1.25)(x^2+9)^{1/2}$$

$$C'(x) = -1 + (1.25)\left(\tfrac{1}{2}\right)(x^2+9)^{-1/2}(2x)$$

$$C'(x) = -1 + \frac{1.25x}{\sqrt{x^2+9}}$$

$$0 = -1 + \frac{1.25x}{\sqrt{x^2+9}}$$

$$1 = \frac{1.25x}{\sqrt{x^2+9}}$$

$$\left(\sqrt{x^2+9}\right)^2 = (1.25x)^2$$

$$x^2+9 = 1.5625x^2$$

$$0.5625x^2 = 9$$

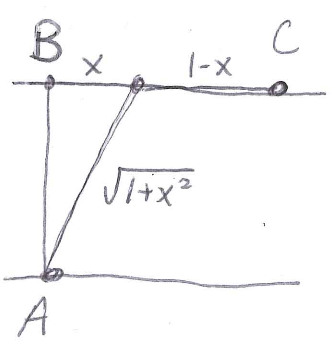
$$x^2 = 16$$

$$x = \pm 4$$

$$\boxed{x = 4}$$

Optimal line of cable would be underwater line from Point A to a point 4 miles down from point B

2. You are standing at the edge of a slow-moving river which is one mile wide and wish to return to your campground on the opposite side of the river. You can swim at 2 mph and walk at 3 mph. You must first swim across the river to any point on the opposite bank. From there walk to the campground, which is one mile from the point directly across the river from where you start your swim. What route will take the least amount of time? Recall that (time = distance/velocity)



$$T(x) = \frac{\sqrt{1+x^2}}{2} + \frac{1-x}{3}$$

$$T(x) = \tfrac{1}{2}(1+x^2)^{1/2} + \tfrac{1}{3} - \tfrac{1}{3}x$$

$$T'(x) = \tfrac{1}{2}\left(\tfrac{1}{2}\right)(1+x^2)^{-1/2}(2x) - \tfrac{1}{3}$$

$$0 = \frac{x}{2\sqrt{1+x^2}} - \frac{1}{3}$$

$$\frac{1}{3} = \frac{x}{2\sqrt{1+x^2}}$$

$$3x = 2\sqrt{1+x^2}$$

$$(3x)^2 = \left(2\sqrt{1+x^2}\right)^2$$

$$9x^2 = 4(1+x^2)$$

$$9x^2 = 4 + 4x^2$$

$$5x^2 = 4$$

$$x^2 = 4/5 \qquad x = \pm\frac{2}{\sqrt{5}}$$

$$\boxed{x = \frac{2}{\sqrt{5}}}$$

3. A cylindrical can is to hold 20π m^3 .The material for the top and bottom costs \$10/ m^2 and material for the side costs \$8/m^2 . Find the radius r and height h of the most economical can.

Surface Area = $2\pi r^2 + 2\pi rh$ Volume: $\pi r^2 h$ $\longrightarrow$ $20\pi = \pi r^2 h$ $\frac{20}{r^2} = h$

$10(\pi r^2) + 10(\pi r^2) + 8(2\pi rh)$

$C = 20\pi r^2 + 16\pi r\underline{\underline{h}}$

$C = 20\pi r^2 + 16\pi r\left(\frac{20}{r^2}\right)$

$C = 20\pi r^2 + 320\pi r^{-1}$

$C'(r) = 40\pi r - 320\pi r^{-2}$

$0 = 40\pi r - \frac{320\pi}{r^2}$

$40\pi r = \frac{320\pi}{r^2}$

$40\pi r^3 = 320\pi$

$40\pi r^3 - 320\pi = 0$

$40\pi(r^3 - 8) = 0$

$r^3 = 8$

$\boxed{r = 2m}$

$20\pi = \pi r^2 h$

$20\pi = \pi(2)^2 h$

$h = \frac{20\pi}{\pi \cdot 4} = 5$

$\boxed{h = 5m}$

4. There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

$P = (\#$ of trees$)($apple output per tree$)$

$P = (50 + x)(800 - 10x)$

$P = 40{,}000 + 300x - 10x^2$

$P'(x) = 300 - 2x$

$0 = 20(15 - x)$

15 trees added

$\boxed{x = 15, \quad 65 \text{ total trees}}$