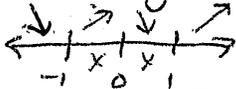


AP Practice Exam M.C. II

Solution Key (Larson)

1. C	2. B	3. D	4. A	5. C
6. B	7. E	8. D	9. D	10. A
11. E	12. A	13. C	14. C	15. E
16. A	17. E	18. B	19. A	20. E
21. B	22. A	23. E	24. A	25. C
26. C	27. B	28. D	29. A	30. B
31. D	32. D	33. D	34. B	35. D
36. C	37. E	38. B	39. E	40. A
41. D	42. E	43. C	44. A	45. D

1) $f(x) = \ln(x^2 - 1)$ Find when function is decreasing
C $f'(x) = \frac{2x}{x^2 - 1}$ critical pts: $0, -1, +1$ 
 * Recall Domain of $\ln|u|, |u| > 0$
 so $x^2 - 1 > 0$, so $|x| > 1$, so function decreasing **$x < -1$**

2) $\lim_{x \rightarrow 0} \frac{\cos x}{|x|}$ limit DNE because limit from both sides of zero approaches ∞ .
E

3) $\int 3^{-x} + \frac{1}{x} dx$ $\int a^u du = \frac{a^u}{\ln a} + C$
D $= \frac{-3^{-x}}{\ln 3} + \ln|x| + C$

4) $\cos(x+y) = x$ Find $\frac{dy}{dx}$ $\sin(x+y) \left[1 + \frac{dy}{dx} \right] = 1$
A $\frac{dy}{dx} = \frac{1 - \sin(x+y)}{\sin(x+y) - 1}$
 $\frac{dy}{dx} = -\csc(x+y) - 1$

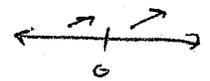
5) $g(x) = \int_0^x f(t) dt$ $g(-1) = \int_0^{-1} f(t) dt = -\int_{-1}^0 f(t) dt = -(1) = -1$
C $g'(-1) = 0$ $g''(-1) = 2$ (slope)
 $g(-1) + g'(-1) + g''(-1) = -1 + 0 + 2 = 1$

6) $\int_{-3}^5 f(t) dt = -2 + 6 + 1 + (2 - \frac{\pi(1)^2}{2}) = 7 - \frac{\pi}{2}$ **B**

7) **E** $g(x)$ is not concave down in $-2 < x < -1$ because slope is positive, therefore concave up.

8) $x(t) = 2 + 3t - t^3$ Find speed at $t = 4$.
D $v(t) = 3 - 3t^2$ $v(4) = 3 - 3(4)^2 = |-45| = 45$

9) $x = y^2 - 1$ and y -axis $y^2 - 1 = 0$ $y = \pm 1$  $R(y) = 0 - (y^2 - 1)$
D $V = \pi \int_{-1}^1 [-y^2 + 1]^2 dy = \pi \int_{-1}^1 [y^4 - 2y^2 + 1] dy = \pi \left[\frac{y^5}{5} - \frac{2y^3}{3} + y \right]_{-1}^1$
 $= \frac{1}{5} - \frac{2}{3} + 1 - \left[-\frac{1}{5} + \frac{2}{3} - 1 \right] = \frac{16}{15}\pi$

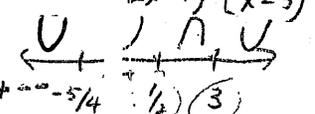
10) $f(x) = \sqrt[3]{x+1} = x^{1/3} + 1$ $f'(x) = \frac{1}{3}x^{-2/3}$ $f'(x) = \frac{1}{3\sqrt[3]{x^2}}$
A  $f(x)$ not differentiable at $x = 0$ b/c $f'(x)$ DNE at $x = 0$
 Tangent to curve at $x = 0$ is not horizontal, since $f(x)$ is not differentiable, so MVT does not apply in interval.

11) $a(t) = \frac{t^2 + t}{t^2 + 1}$ for $0 \leq t < 1$ $v(0) = 1$ cm/sec.
E

0	2	4	6	8	10	12
12	18	10	15	13	16	8

12) $R(t)$ = rate of water dripping

Avg. rate = $\frac{1}{12-0} \int_0^{12} R(t) dt \approx \frac{1}{12} (4) [f(4) + f(8) + f(12)]$
 $\approx \frac{1}{3} [10 + 12 + 8] = \frac{30}{3} = 10 \text{ cm}^3/\text{min}$

13) $h''(x) = e^{-x} (2x-1)^2 (x-3)^3 (4x+5)$ critical pts: $x = 1/2, 3, -5/4$
 C  2 POI's

14) $\int_1^2 f'(x) dx = k$ $f(2) - f(-1) = \int_{-1}^2 f'(x) dx = \int_{-1}^0 f'(x) dx + \int_0^1 f'(x) dx + \int_1^2 f'(x) dx$
 C $= -1 + 6 + k = k - 1/2$

15) $\int \frac{1}{16+x^2} dx = \int \frac{1}{(4)^2 + (x)^2} dx$ $a=4, u=x, \frac{du}{dx}=1$
 E $= \frac{1}{4} \arctan \frac{x}{4} + C$

16) $v(t) = -t + \frac{9}{4}t^4$ $a(t) = -3t^2 + 9t^3$
 Acceleration at minimum where $a'(t) = 0$ $a'(t) = -6t + 27t^2$
 $0 = 3t(-2 + 9t)$ $t = 0, 2/9$
 A $t = 2/9$

17) $\lim_{h \rightarrow 0} \frac{\sin^{-1}(1+h) - \pi/2}{h}$ $f(x) = \sin^{-1}(x)$ $f'(x) = \frac{1}{\sqrt{1-x^2}}$
 E $f'(1) = \frac{1}{\sqrt{1-1}} = \frac{1}{0}$ DNE

18) $f(x) = e^{5x} + x$ at $x=0$

B $f(x) = e^{5x} (\cos x) + 1$ $y-1 = 2(x-0)$
 $f(0) = 1$ $f'(0) = 2$ $y = 2x + 1$

19) A

20) $F(x) = \int_2^2 f(2t) dt = - \int_2^{3x-2} f(2t) dt$

E $\frac{d}{dx} \int_2^{3x-2} f(2t) dt = (-) f[2(3x-2)] \cdot (3) = -3f(6x-4)$

21) slope of line normal (perpendicular) at $x=1$
 B Since slope is $h'(1)$ then slope \perp to tangent is $-\frac{1}{h'(1)}$
 $h(x) = \sqrt{5x^3 - 2x + 1}$ $h'(x) = \frac{1}{2} (5x^3 - 2x + 1)^{-1/2} (15x^2 - 2)$
 $h'(1) = \frac{1}{2} (5 - 2 + 1)^{-1/2} (15 - 2) = \frac{13}{4}$ so $-\frac{1}{h'(1)} = \frac{-4}{13}$

22) $y = 2x(\sin 2x + x \cos 2x)$ in $0 \leq x \leq \pi/2$. Find Avg. R.O.C.
 A $y(\pi/2) = 2(\pi/2) [\sin \pi + \pi/2 \cos \pi] = \pi [0 - \pi/2] = -\frac{\pi^2}{2}$
 $y(0) = 0$ $\frac{y(\pi/2) - y(0)}{\pi/2 - 0} = \frac{-\pi^2/2}{\pi/2} = -\pi$

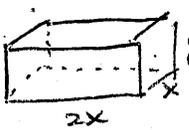
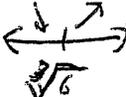
23) $f(x) = x^3 - \ln(k+x)$ $f'(x) = 3x^2 - \frac{1}{k+x}$
 E Rel. min occurs at $x=1$
 $3x^2 - \frac{1}{k+x} = 0$ $3 - \frac{1}{k+1} = 0$
 $3 = \frac{1}{k+1}$ $3k+3=1$ $3k=-2$ $k = -2/3$

24) $C(t) = \frac{2t}{8100+t^2}, t \geq 0$ Find when concentration is greatest.
 A $C'(t) = \frac{2(8100+t^2) - (2t)(2t)}{(8100+t^2)^2}$ $C'(t) = \frac{16200 + 2t^2 - 4t^2}{(8100+t^2)^2} = \frac{16200 - 2t^2}{(8100+t^2)^2}$
 (A) 90 minutes $t=90$

25) $\lim_{x \rightarrow \infty} \frac{ae^x}{xte^x} = 3$ $a=3$

C

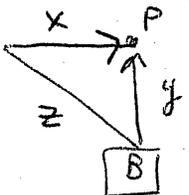
26) $\frac{d}{dx} [f(g(x^2))] = f'(g(x^2)) \cdot g'(x^2) \cdot 2x$ when $x=2$
C $f'(g(2^2)) \cdot g'(2^2) \cdot 2(2) = f'(g(4)) \cdot g'(4) \cdot 4$
 $f'(1) \cdot g'(4) \cdot 4 = \left(\frac{2}{3}\right) \left(-\frac{1}{2}\right) (4) = \boxed{-\frac{4}{3}}$

27) **B**  $V = 8 \text{ cm}^3$ $V = (2x)(x)(y) = 2x^2y$
 $S = 2x^2 + 2xy + 4xy$ $8 = 2x^2y$
 $S = 2x^2 + 6xy$ $y = \frac{4}{x^2}$
 $S = 2x^2 + \frac{24}{x}$ $S'(x) = 4x - 24x^{-2}$
 $0 = 4x - \frac{24}{x^2}$ $4x = \frac{24}{x^2}$ $4x^3 = 24$ $x = \sqrt[3]{6}$ 

28) **D** $y = \sqrt{x+1}$ cross section \perp to x -axis
 $\text{base} = \sqrt{x+1} - 0$
 $\text{Squares} = \text{base}^2 = (\sqrt{x+1})^2$
 $V = \int_{-1}^1 (\sqrt{x+1})^2 dx = \int_{-1}^1 (x+1) dx = \left[\frac{x^2}{2} + x\right]_{-1}^1 = \frac{1}{2} + 1 - \left(\frac{1}{2} - 1\right) = \frac{1}{2} + 1 - \frac{1}{2} + 1 = \boxed{2}$

29) **A** $g(0) = 4$ $g'(0) = 8$ $g''(0) = -12$ $h(x) = \sqrt{g(x)}$
 Find $h''(0)$.
 product rule $h'(x) = \frac{1}{2}(g(x))^{-1/2} (g'(x)) = \frac{1}{2\sqrt{g(x)}} g'(x)$
 $h''(x) = \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) (g(x))^{-3/2} (g'(x))(g'(x)) + \frac{1}{2}(g(x))^{-1/2} g''(x)$
 $= -\frac{1}{4}(4)^{-3/2} (8)(8) + \frac{1}{2}(4)^{-1/2} (-12)$
 $= -\frac{1}{4} \left(\frac{1}{8}\right) (8)(8) + \left(\frac{1}{2}\right) \left(\frac{1}{2}\right) (-12) = -2 - 3 = \boxed{-5}$

30) $g''(x) = \frac{x}{2+e^x}$ and $g'(0) = -1$ Find $g'(3)$
B $g'(3) = g'(0) + \int_0^3 g''(x) dx = -1 + 0.5274 = \boxed{-0.473}$

31) **A**  $\frac{dx}{dt} = -60 \text{ mph}$ $\frac{dy}{dt} = -50 \text{ mph}$
 $x = 12$ $y = 10$ $z = 15.62$ $12^2 + 10^2 = z^2$ $z = 15.62$
 $x^2 + y^2 = z^2$ $2x \left(\frac{dx}{dt}\right) + 2y \left(\frac{dy}{dt}\right) = 2z \left(\frac{dz}{dt}\right)$
 $2(12)(-60) + 2(10)(-50) = 2(15.62) \left(\frac{dz}{dt}\right)$
 $-2440 = 31.24 \frac{dz}{dt}$ $\frac{dz}{dt} = -78.102 \text{ mph}$
 Find $\frac{dz}{dt}$

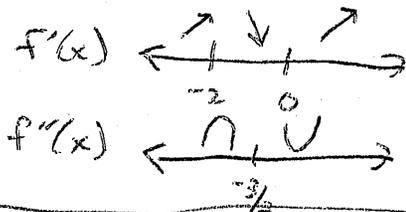
32) line tangent to $f(x)$ has slope $= -1$
 $f(x) = \frac{1}{2^{2x}}$ $f(x) = 2^{-2x}$
 $f'(x) = \ln 2 \cdot 2^{-2x} \cdot -2$ $\frac{d}{dx} a^u = \ln a \cdot a^u \cdot u'$
 $-1 = -2 \ln 2 \cdot 2^{-2x}$ $x \approx 0.236$ point $(0.236, 0.72)$ $m = -1$
 $y - 0.72 = -1(x - 0.236)$ $y = -x + 0.956$
 $0 = -x + 0.956$ $x = \boxed{0.956}$

33) How many times are tangents to $g(x)$ parallel to $y = 2x - 1$
D set $g'(x) = 2$
 $g(x) = \tan(x+2)$ $g'(x) = \sec^2(x+2)$ $\frac{1}{\cos^2(x+2)} = 2$
 $-4 < x < 5$ $\boxed{D: 5}$ 

34) **B** $\int_1^{2k} x - \frac{k}{x^2} dx = \left[\frac{x^2}{2} + \frac{k}{x}\right]_1^{2k}$ $\frac{4k^2}{2} + \frac{k}{2k} - \left(\frac{1}{2} + k\right)$
 $2k^2 + \frac{1}{2} - \frac{1}{2} - k = 15$
 $2k^2 - k - 15 = 0$ $k = -\frac{5}{2}, 3$ $-\frac{5}{2} + 3 = \boxed{\frac{1}{2}}$
 $(2k+5)(k-3) = 0$

35) Draw sign lines

D



36) Find Avg. rate of growth using Trapezoid Rule:

C $\frac{1}{4} \left[\frac{1}{2}(H(1) + 2H(1) + 2H(2) + 2H(3) + H(4)) \right]$

37) $v(t) = (-1)e^{t+2t}$ Find critical pts: set $v'(t) = 0$

E Distance = $\int_0^1 |v(t)| dt + \int_1^{1.5} v(t) dt$

= 2.411 + 2.257 = **31.669 cm**

38) I. False: $\Delta V T$ can only be guaranteed in closed interval

B II. True: area is decreasing to the right $[0, 2]$

III. False: since graph is concave down and decreasing, the tangent approximation will overestimate function value.

39) $f(x) = x^2$

E $\int_{-2}^x g(t) dt$ $f(-2) = (-2)^2 + \int_{-2}^{-2} g(t) dt = 4$

40) $f'(-2) = 2x$

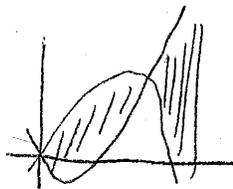
A $\frac{d}{dx} \int_{-2}^x g(t) dt = 2x + g(x) \cdot 1 = 2(-2) + (-2) = -6$

41) $f''(2) = 2$

D $g'(x) = 2 + g'(2) = 2 + 0 = 2$

42)

E



$A = \int_0^{1.18} (2x \sin(2x) - (-2x \cos 2x)) dx$
 $+ \int_{1.18}^{\pi/2} (-2x \cos 2x - 2x \sin 2x) dx$

= 1.166 + 0.595 = **1.761**

43)

C Skip

44) $R =$ radioactive isotope (time, weight)

$(0, 25)$ $(276, 12.5)$

A

$\frac{dR}{dt} = kR$

$(120, -)$

$\int \frac{dR}{R} = \int k dt$ $e^{\ln|R|} = e^{kt+C}$ $R = Ce^{kt}$ $R = 25e^{kt}$

$12.5 = 25e^{k(276)}$ $0.5 = e^{276k}$ $\ln 0.5 = \ln e^{276k}$
 $-0.693 = 276k$ $k = -0.0025$

$R = 25e^{-0.0025t}$

$R = 25e^{-0.0025(120)}$

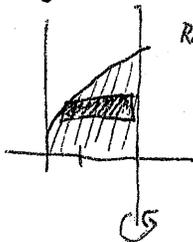
$R = 18.52$ on the 120th day

$\frac{dR}{dt} = k(R) = (-0.0025)(18.52) = -0.0463 \text{ grams/day}$

45)

$y = \sin^{-1}(x-1) + \pi/2$

D



Radius = Right - Left

$y - \pi/2 = \sin^{-1}(x-1)$

$\sin(y - \pi/2) + 1 = x$

$2 - [\sin(y - \pi/2) + 1] = R$

$R = 1 - \sin(y - \pi/2)$