Spring 2014 Calculus AB Topics Summary / Formula Sheet

Summation Formulas:

$\sum_{i=1}^{n} 1 = n$ $\qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$ $\qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$

Riemann Sum: Estimate Area under curve using rectangles

a) Right-handed sum
b) Left-handed sum
c) Mid-Point Rule

Trapezoid Rule: ½(width)(h_1 +2 h_2+ 2 h_3 + ... h_n)

width = $\frac{b-a}{n}$

Area using Limit Definition

$$\lim_{n\to\infty} \sum_{i=1}^{n} (width) * f(left\ endpoint + width * i)$$

Integral Formulas:

Power Rule:

$\int u^n du = \frac{u^{n+1}}{n+1} + C$

Log Rule:

$\int \frac{1}{u} du = \ln|u| + C$

Exponential Rule: (Base e)

$\int e^u du = e^u + C$

Exponential Rule (base other than e)

$\int a^u du = \frac{a^u}{\ln a} + C$

Note: ln a is a constant

Integrals of Even/Odd Functions Rules:

Even: $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ $\qquad$ Odd: $\int_{-a}^{a} f(x)dx = 0$

CheckList Order for determining Method for Finding Integrals

(Check for correct method in the following order)
1.**Expand/Simplify/PowerRule** 2. Trig/Exponential/Log Rule
3. **U-Substitution** 4. U-Sub(Change of Variable) 5. Long Division/ Synthetic divison 6. ArcTrig (Complete the square) 7. IBP/Tab method

Trig Integrals:

$\int \sin u\, du = -\cos u + C$ $\qquad \int \cos u\, du = \sin u + C$

$\int \sec^2 u\, du = \tan u + C$ $\qquad \int \sec u \tan u\, du = \sec u + C$

$\int \csc^2 u\, du = -\cot u + C$ $\qquad \int \csc u \cot u\, du = -\csc u + C$

More Trig Integrals:

$\int \tan u\, du = -\ln|\cos u| + C$ $\qquad \int \cot u\, du = \ln|\sin u| + C$

$\int \sec u\, du = \ln|\sec u + \tan u| + C$

$\int \csc u\, du = -\ln|\csc u + \cot u| + C$

Trig properties:

$\frac{\sin x}{\cos x} = \tan x$ $\qquad \frac{\cos x}{\sin x} = \cot x$

Integral properties

$\int_{a}^{a} f(x)dx = 0$ $\qquad \int_a^b f(x)dx = -\int_b^a f(x)dx$

1st Fundamental Theorem of Calculus (FFTC)

$\int_a^b f(x)dx$ = F(b) – F(a) where F is the antiderivative of f.

* If a function is continuous on a closed interval, then it is integrable on that interval.

distance vs displacement

displacement = integral of velocity $\quad \int_a^b v(t)dt$

distance = integral of absolute value of velocity $\quad \int_a^b |v(t)|dt$

Mean Value Theorem for Integrals (Average Value Theorem)

If f is integrable on the closed interval [a, b], then the average value of f on the interval is $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$.

*This is derived from the Area of a rectangle:

Since height * width = Area, $f(c) * (b-a) = \int_a^b f(x)dx$

Therefore, $f(c) = \frac{\int_a^b f(x)dx}{b-a}$ or $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$

2nd Fundamental Theorem of Calculus: (SFTC)

$$\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right] = f(p(x)) \cdot p'(x)$$

"a" is constant

*Highlights the inverse relationship between derivatives and integrals

PVA integral problem: Suppose f''(x) = 6x + 4
f'(0) = 3 and f(1) = 5. Find f(x)

$f''(x) = 6x + 4$ so $f'(x) = 3x^2 + 4x + C$.
Plug in (0, 3) to get $f'(x) = 3x^2 + 4x + 3$.
So $f(x) = x^3 + 2x^2 + 3x + K$.
Plug in (1, 5) to get that $f(x) = x^3 + 2x^2 + 3x - 1$

SFTC (Alternate)

$$\frac{d}{dx}\left[\int_{q(x)}^{p(x)} f(t)dt\right] = f(p(x)) \cdot p'(x) - f(q(x)) \cdot q'(x)$$

Example of (SFTC)

$$\frac{d}{dx}\left[\int_{10}^{x^2} \sqrt{t-1}\,dt\right] = \sqrt{x^2-1} \cdot 2x = 2x\sqrt{x^2-1}$$

Differential Equations: Equations involving derivatives
(Separation of Variables)
1)Rewrite y' as $\frac{dy}{dx}$ 2)Separate variables (y's on left, x's on right)
3)Take integral of both sides 4)Exponentiate both sides
5)Move constant of integration as leading coefficient
(ex. e^{kt+C} becomes $e^{kt}e^{C}$, then is written as Ce^{kt})
6. Solve for C (initial condition) 7. Solve for k
Varies directly or is directly proportional: use y = kx
Varies inversely or is inversely proportional: use $y=\frac{k}{x}$

Slope Fields: graphical approach to looking at solutions of a differential equation:
*Use the differential equation to match the individual slope segments which creates the slope fields (ex: $\frac{dy}{dx}=x-1$)
*Use the solution (integral) of the differential equation to match the shape and pattern of the slope field (ex: $y=\frac{1}{2}x^2-x+C$) (this solution will show graph patterms that resemble a parabola)

Exponential Growth/Decay
Be able to rewrite word problem in the form of differential equation: "The rate of increase of population is proportional to the population": Differential equation is $\frac{dP}{dt}=kp$

Exponential Growth example problem: The rate of increase of the population of a city is proportional to the population. If the population increases from 40,000 to 60,000 in 40 yrs, when will the population be 80,000?
(t, Population), (0, 40,000), (40, 60,000) (__ , 80,000)

$\frac{dP}{dt}=kp$ $\int\frac{dP}{dt}=\int kp$ $\int\frac{dP}{P}=\int kdt$ $\ln P=kt+C$ $e^{\ln P}=e^{kt+C}$

$P=Ce^{kt}$ C=40,000 $P=40{,}000e^{kt}$ $60{,}000=40{,}000e^{kt}$

$1.5=e^{kt}$ $\ln(1.5)=\ln e^{kt}$ $\ln(1.5)=k(40)$ $k=(\ln 1.5)/40$

$P=40{,}000e^{(\ln 1.5/40)t}$ $80{,}000=40{,}000e^{(\ln 1.5/40)t}$

$2=e^{(\ln 1.5/40)t}$ $\ln 2=\ln e^{(\ln 1.5/40)t}$ $\ln 2=\frac{\ln 1.5}{40}t$ Ans: t = 68.380yrs

Domain of solution to Differential Equation: the largest open interval containing the initial value for which the differential equation is defined

Arc Trig Integrals:

$$\int\frac{du}{\sqrt{a^2-u^2}}=\arcsin\frac{u}{a}+C \qquad \int\frac{du}{a^2+u^2}=\frac{1}{a}\arctan\frac{u}{a}+C$$

$$\int\frac{du}{u\sqrt{u^2-a^2}}=\frac{1}{a}\text{arcsec}\frac{|u|}{a}+C$$

Completing the Square:
1) Group x^2+bx
2) Add $\left(\frac{b}{2}\right)^2$
3) Factor

Example:
$x^2-6x+13$
$x^2-6x+\underline{9}+13-\underline{9}$
$(x-3)^2+4$

Area = $\int_a^b[f(x)-g(x)]dx$ If top – bottom, then integrate with respect to x

Area = $\int_a^b[f(y)-g(y)]dy$ If right – left, then integrate with respect to y

Volumes of Revolution: Disc Method

$$V=\pi\int_a^b R^2(dx\ or\ dy)$$

R = axis of revolution and function.
top – bottom if dx or
right – left if dy

Horizontal axis: integrate with respect to x
Vertical axis: integrate with respect to y

Volumes of Revolution: Washer Method

$$V=\pi\int_a^b(R^2-r^2)(dx\ or\ dy)$$

R = axis of revolution and outer function
(top – bottom) or (right – left)

r = axis of revolution and inner function
(top – bottom) or (right – left)

Volumes with known cross sections

$$V=\int_a^b(Area\text{ of cross section})(dx\ or\ dy)$$

*If Cross-section is perpendicular to x-axis, then Top – bottom dx
*If Cross-section is perpendicular to y-axis, then Right – Left dy

Area formula for cross sections:
1) Square: A = $(\text{base})^2$ 2) Rectangle: A = (base)(height)
3) Isosceles Right Triangle (hypotenuse on base) : $\frac{1}{4}(hypotenuse)^2$
4) Isosceles Right Triangle : A = $\frac{1}{2}(base)*(base)$
5) Equilateral Triangle : A = $\frac{\sqrt{3}}{4}(base)^2$
6) Semicircles: $A=\frac{\pi}{2}[radius]^2$ or $A=\frac{\pi}{8}[diameter]^2$

Integration by Parts: useful whenever

$$\int udv=uv-\int vdu$$

Steps: 1) Let u = L.I.P.E.T. (Preference order: Log/InverseTrig/Polynomial/Exponential/Trig)
2) find u, du, v, and dv
3) plug in and integrate

Tab Method: use whenever you have a polynomial (of degree higher than 1) multiplied by a function that you can antidifferentiate

Steps: 1) Create 3 columns: Signs | u | dv 2) Let u value be the **polynomial** 3) Let dv be the other portion 4) Find the derivative of polynomial (u) until reaching zero. 5) Find integral of dv the same number of times. 6) Assign alternating signs (+/ –). 7) Add the product of diagonal terms

Calculator Steps:
* *Remember – Calculator **ALWAYS** in **RADIAN** mode**
1. Evaluating derivatives
Math 8 → nDeriv (y_1, x, value)
[to enter y_1 go to VARS / Y-VARS / Function / 1 }
2. Evaluating Definite Integrals
Math 9 → fnInt (y_1, x, lower bound, upper bound)
3. Evaluating Total Distance:
Math 9 → fnInt (Abs (y_1), x, lower, upper)

Curve Sketching:
Evaluating f '(x) graph (velocity graph)
Portions of graph above x-axis indicates where f(x) is increasing (+ slope)
Portions of graph below x-axis indicates where f(x) is decreasing (– slope)
x-intercepts indicates critical points for potential max/min of f(x) graph
Area between graph and x-axis indicates accumulation of distance
Hills and valleys indicate POI 's
Increasing slope indicates where graph is concave up
Decreasing slope indicates where graph is concave down
Evaluating f ''(x) graph (acceleration graph)
Portions of graph above x-axis indicates concave up
Portions of graph below x-axis indicates concave down
x-intercepts indicates critical points for potential POI ' s
Area between graph and x-axis indicates accumulation of velocity