AB/BC Theater Extra Credit Spring 2019 Assignment #1:

Directions: answer the following 5 FRQs, make corrections (red ink) (or highlighter) and score FRQs. Due Fri (3/29)

1) (Calculator FRQ)

t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

The temperature, in degrees Celsius (°C), of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

(a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

(b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

(c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

(Non-calculator)

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION. DURING THE TIMED PORTION FOR PART B, YOU MAY GO BACK AND CONTINUE TO WORK ON THE PROBLEMS IN PART A WITHOUT THE USE OF A CALCULATOR.

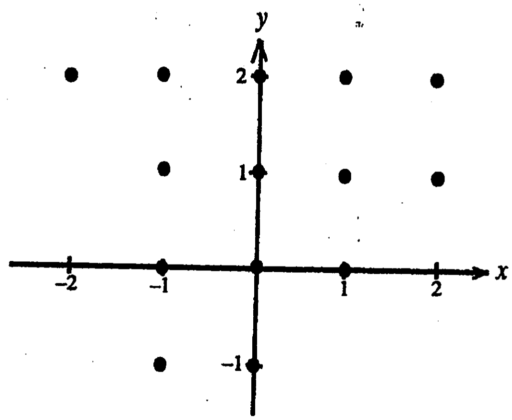
2) Consider the differential equation $\frac{dy}{dx} = \frac{xy}{(x^2+4)}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the fourteen points indicated.

(b) Sketch the solution curve that contains the point (–2, 2).

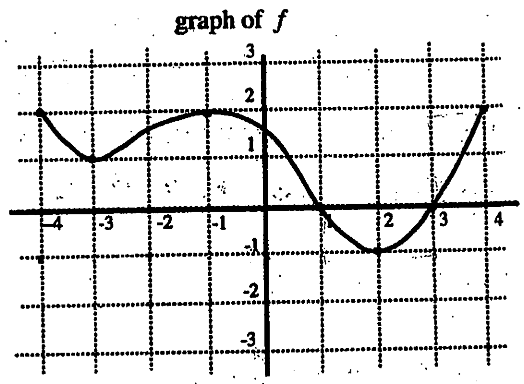
(c) Find a general solution to the differential equatiion.

(d) Find the particular solution to the differential equation that satisfies the initial condition $y(0) = 4$.



(Non-calculator)

3) The graph of a differentiable function f on the closed interval $[-4, 4]$ is shown at the right. The graph of f has horizontal tangents at $x = -3, -1$ and 2.



Let $G(x) = \int_{-4}^{x} f(t)\,dt$ for $-4 \le x \le 4$.

(a) Find $G(-4)$.

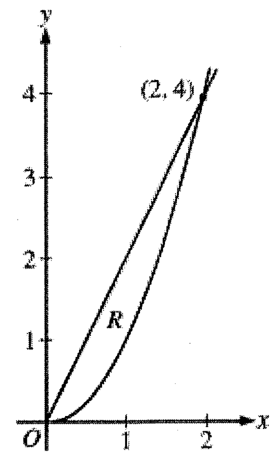
(b) Find $G'(-1)$.

(c) On which interval or intervals is the graph of G concave down. Justify your answer.

(d) Find the value of x at which G has its maximum on the closed interval $[-4, 4]$. Justify your answer.

4) Non-Calculator

Question 4



Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

(a) Find the area of R.

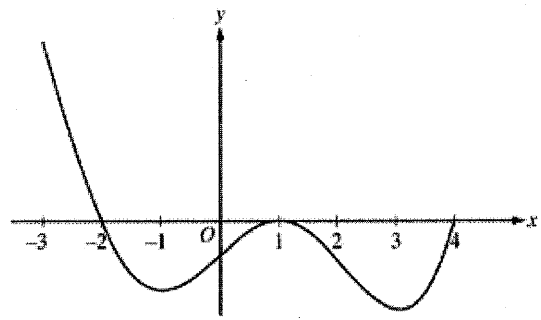
(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

5) Non-Calculator

Question 5

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



Graph of f'

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

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, make corrections (red ink) and

1) (Calculator FRQ)

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(b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

(c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

estimate rate of change by finding slope

$\boxed{2}$ a) $H'(10) \approx \dfrac{H(12)-H(8)}{12-8} = \dfrac{80-73}{12-8} = \boxed{\dfrac{7}{4}\ {}^\circ C/min}$

$\boxed{3}$ b) Avg. temperature $= \dfrac{1}{16}\displaystyle\int_0^{16} H(t)\,dt$ *use Avg. Value theorem $\dfrac{1}{b-a}\displaystyle\int_a^b f(t)\,dt$

Approximate using left Riemann Sum: (4 subintervals) $\displaystyle\int_0^{16} H(t)\,dt \approx 4(65)+4(68)+4(73)+4(80) = 1144$

$\dfrac{1}{16}\displaystyle\int_0^{16} H(t)\,dt \approx \dfrac{1}{16}(1144) = \boxed{71.5\,{}^\circ C}$

$\boxed{1}$ c) Since the graph of H is continually increasing (positive slope), the left Riemann Sum will be an underapproximation of the average temperature

$\boxed{3}$ d) Since the slopes of the values in the subinterval are increasing, this supports the claim that tempeature is increasing at an incrasing rate

slopes: $\dfrac{3}{4} < \dfrac{5}{4} < \dfrac{7}{4} < \dfrac{10}{4}$

(Non-calculator)

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2) **Consider the differential equation $\frac{dy}{dx} = \frac{xy}{(x^2+4)}$.**

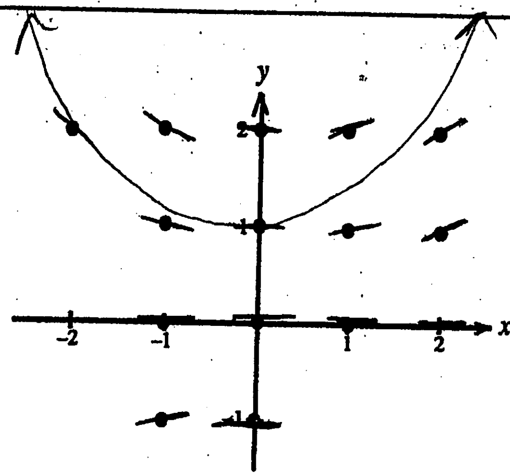
points:

$\boxed{1}$ **(a) On the axes provided, sketch a slope field for the given differential equation at the fourteen points indicated.**

$\boxed{1}$ **(b) Sketch the solution curve that contains the point (–2, 2).**

$\boxed{4}$ **(c) Find a general solution to the differential equatiion.**

$\boxed{3}$ **(d) Find the particular solution to the differential equation that satisfies the initial condition $y(0) = 4$.**



2) $\frac{dy}{dx} = \frac{xy}{x^2+4}$

c) $(x^2+4)\,dy = xy\,dx$

$\frac{dy}{y} = \frac{x}{x^2+4}\,dx$

$\int \frac{1}{y}\,dy = \int \frac{x}{x^2+4}\,dx$ $\quad u = x^2+4$, $\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$

$\ln|y| = \int \frac{x}{u}\cdot\frac{du}{2x} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u|$

$\ln|y| = \frac{1}{2}\ln|x^2+4| + c$

$e^{\ln|y|} = e^{\ln|x^2+4|^{1/2} + c}$

$e^{\ln|y|} = e^{\ln|x^2+4|^{1/2}}\cdot e^{c}$

$|y| = |x^2+4|^{1/2}\cdot C$

$\boxed{y = C(x^2+4)^{1/2}}$

plug in (0,4)

d) Find particular solution

$4 = C(0^2+4)^{1/2}$

$4 = C(2)$

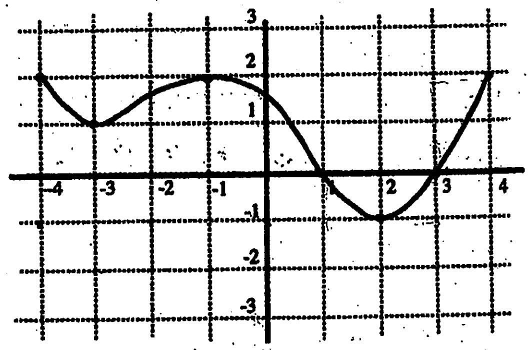
$2 = C$

$\boxed{y = 2(x^2+4)^{1/2}}$

(Non-calculator)

3) The graph of a differentiable function f on the closed interval $[-4, 4]$ is shown at the right. The graph of f has horizontal tangents at $x = -3, -1$ and 2.

graph of f



Let $G(x) = \int_{-4}^{x} f(t)\,dt$ for $-4 \le x \le 4$.

points:

$\boxed{1}$ (a) Find $G(-4)$.

$\boxed{2}$ (b) Find $G'(-1)$.

$\boxed{3}$ (c) On which interval or intervals is the graph of G concave down. Justify your answer.

$\boxed{3}$ (d) Find the value of x at which G has its maximum on the closed interval $[-4, 4]$. Justify your answer.

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3) $\boxed{G(x) = \int_{-4}^{x} f(t)\,dt}$

$G'(x) = \frac{d}{dx}\int_{-4}^{x} f(t)\,dt = f(x)\cdot 1$ $\boxed{G'(x) = f(x)}$

$\boxed{G''(x) = f'(x)}$

$g'(x)$ slope sign line: -4 ... $+$ ... 1 ... $-$ ... 3 ... $+$ ... 4

$g''(x)$ concavity sign line: -4 ... $-$ ... -3 ... $+$ ... -1 ... $-$ ... 2 ... $+$ ... 4

a) $G(-4) = \int_{-4}^{-4} f(t)\,dt = \boxed{0}$

b) $G'(-1) = f(-1) = \boxed{2}$

c) G is concave down when slope of f is negative $(-4,-3) \cup (-1, 2)$

d) * Apply EVT to locate absolute maximum i) test endpoints ii) test relative max, min

$G(-4) = \int_{-4}^{-4} f(t)\,dt = 0$ $G(1) = \int_{-4}^{1} f(t)\,dt \approx 6.5$

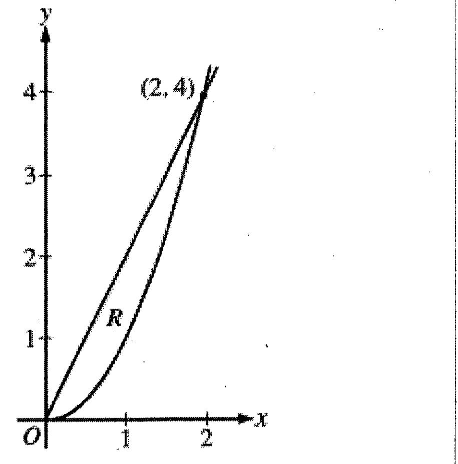
$G(4) = \int_{-4}^{4} f(t)\,dt \approx 7$

Absolute maximum is occuring at $x = 1$ in interval $[-4, 4]$

4) Non-Calculator

Question 4

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

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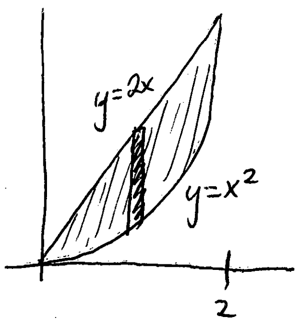
a)



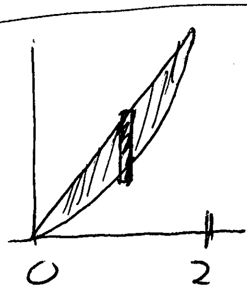
Top/bottom

$$\text{Area} = \int_0^2 2x - x^2\,dx \rightarrow \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\Bigg]_0^2$$

$$= 2^2 - \frac{2^3}{3} - \left(0^2 - \frac{0^3}{3}\right)$$

$$= 4 - \frac{8}{3} = \frac{12}{3} - \frac{8}{3} = \boxed{\frac{4}{3}}$$

b)



$$\text{Area} = \sin\left(\frac{\pi}{2}x\right)$$

$$V = \int [\text{Area}]\,dx$$

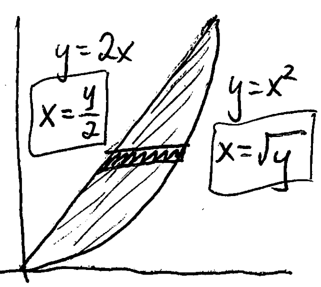
$$V = \int_0^2 \sin\left(\frac{\pi}{2}x\right)dx \xrightarrow{u\text{-sub}} \quad u = \frac{\pi}{2}x \quad \frac{du}{dx} = \frac{\pi}{2} \quad \pi dx = 2du \quad dx = \frac{2}{\pi}du$$

$$\int \sin u \cdot \frac{2}{\pi}du = \frac{2}{\pi}\int \sin u\,du = \frac{2}{\pi}(-\cos u)$$

$$-\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Bigg]_0^2 = -\frac{2}{\pi}\cos(\pi) - \left(-\frac{2}{\pi}\cos(0)\right)$$

$$= -\frac{2}{\pi}(-1) + \frac{2}{\pi} = \boxed{\frac{4}{\pi}}$$

c)



Right/Left

$x = \sqrt{y}$

$x = y/2$

$\text{base} = \sqrt{y} - \frac{y}{2}$

$\text{Area} = [\text{base}]^2$

$\text{Area} = \left[\sqrt{y} - \frac{y}{2}\right]^2$

*intersection:

$\sqrt{y} = \frac{y}{2}$

$(\sqrt{y})^2 = \left(\frac{y}{2}\right)^2$

$y = \frac{y^2}{4}$

$4y = y^2$

$4y - y^2 = 0$

$y(4-y) = 0$

$y = 0, 4$

$$\boxed{V = \int_0^4 \left[\sqrt{y} - \frac{y}{2}\right]^2 dy}$$

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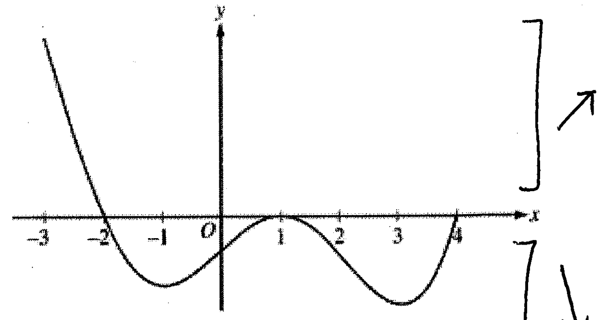

Graph of f'

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

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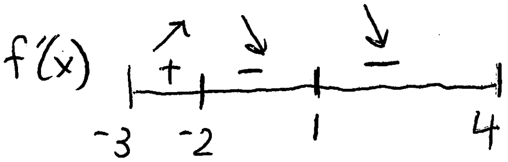
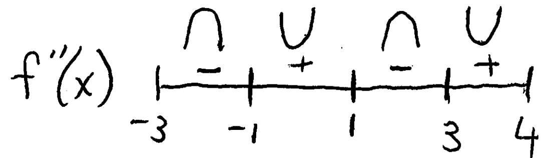
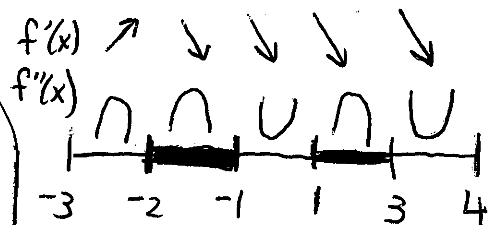
(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.





a) Relative max at $x = -2$ because $f'(x)$ changes from + to −

b) graph of f is concave down and decreasing on interval $-2 < x < -1$ and $1 < x < 3$ b/c $f'(x)$ is decreasing and negative on these intervals.



c) POI at $x = -1, 1, 3$ b/c $f''(x)$ change signs

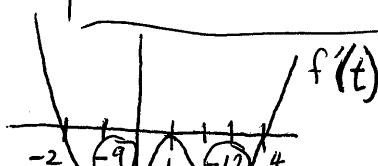
d) * final position = initial position + displacement

$$x(b) = x(a) + \int_a^b x'(t)\,dt$$

$$f(b) = f(a) + \int_a^b f'(t)\,dt$$

$$f(x) = f(1) + \int_1^x f'(t)\,dt$$

$$\boxed{f(x) = 3 + \int_1^x f'(t)\,dt}$$



$$f(4) = f(1) + \int_1^4 f'(t)\,dt$$

$$f(4) = 3 + (-12) = -9 \qquad \boxed{f(4) = -9}$$

$$f(-2) = f(1) + \int_1^{-2} f'(t)\,dt$$

$$f(-2) = f(1) - \int_{-2}^{1} f'(t)\,dt$$

$$= 3 - (-9) = 12 \qquad \boxed{f(-2) = 12}$$