AP Calculus AB **Trig Quiz #2 (Chapter 2)** **REVIEW #2** **Name:** _____

No Calculators. Leave all answers reasonably simplified

1. If $y = 4x^3 \tan\left(4x^3\right)$ find $\frac{dy}{dx}$.

2. If $y = \frac{\csc(4x)}{\cot(3x)}$ find $\frac{dy}{dx}$.

3. If $\cot(y) = 3x\sin(5x)$ find $\frac{dy}{dx}$.

4. $y = \frac{4}{3}\tan(4\theta)$ find $\frac{dy}{d\theta}$. And find $\frac{d^2y}{d\theta^2}$

5. Find the equation of the tangent line to the curve $y = \csc x$ at $x = \frac{2\pi}{3}$

6. The position of a particle moving along the x-axis is given by $x(t) = \cot 2t$ find the acceleration of the particle at $t = \frac{3\pi}{4}$

AP Calculus AB **Trig Quiz #2 (Chapter 2)** **REVIEW #2** **Name:** Key

No Calculators. Leave all answers reasonably simplified

1. If $y = 4x^3 \tan(4x^3)$ find $\frac{dy}{dx}$. *product rule

$y' = \underbrace{12x^2}_{f'} \cdot \underbrace{\tan(4x^3)}_{g} + \underbrace{4x^3}_{f} \cdot \underbrace{\sec^2(4x^3) \cdot 12x^2}_{g'}$ $\quad f'g + fg'$

$$\boxed{y' = 12x^2\tan(4x^3) + 48x^5\sec^2(4x^3)}$$

OR

$$y' = 12x^2\left[\tan(4x^3) + 4x^3\sec^2(4x^3)\right]$$

2. If $y = \frac{\csc(4x)}{\cot(3x)}$ find $\frac{dy}{dx}$. *quotient rule $\frac{f'g - fg'}{g^2}$

$$y' = \frac{\overbrace{-\csc(4x)\cot(4x)\cdot 4}^{f'}\overbrace{(\cot(3x))}^{g} - \overbrace{\csc(4x)}^{f}\cdot\overbrace{-\csc^2(3x)\cdot 3}^{g'}}{[\cot(3x)]^2}$$

$$y' = \frac{-4\csc(4x)\cot(4x)\cot(3x) + 3\csc(4x)\csc^2(3x)}{[\cot(3x)]^2}$$

3. If $\cot(y) = 3x\sin(5x)$ find $\frac{dy}{dx}$. *implicit *product rule

$-\csc^2 y\left(\frac{dy}{dx}\right) = \overbrace{3}^{f'}\overbrace{\sin(5x)}^{g} + \overbrace{3x}^{f}\cdot\overbrace{\cos(5x)\cdot 5}^{g'}$

$$\boxed{\frac{dy}{dx} = \frac{3\sin(5x) + 15x\cos(5x)}{-\csc^2 y}}$$

4. $y = \frac{4}{3}\tan(4\theta)$ find $\frac{dy}{d\theta}$.

$y' = \frac{4}{3}\sec^2(4\theta)\cdot 4$

$$\boxed{y' = \frac{16}{3}\sec^2(4\theta)}$$

Find $y''(\theta)$

$y' = \frac{16}{3}\left[\sec(4\theta)\right]^2$

$y'' = \frac{16}{3}\cdot 2\left[\sec 4\theta\right]^1 \cdot \sec 4\theta\tan 4\theta \cdot 4$

$$\boxed{y'' = \frac{128}{3}\sec^2 4\theta\tan 4\theta}$$

5. Find the equation of the tangent line to the curve $y = \csc x$ at $x = \frac{2\pi}{3}$

*find point
*find slope: $y'\left(\frac{2\pi}{3}\right)$
*equation: $y - y_1 = m(x - x_1)$

point: $y\left(\frac{2\pi}{3}\right) = \csc\left(\frac{2\pi}{3}\right) = \frac{2\sqrt{3}}{3}$ $\quad = \frac{1}{\sin\frac{2\pi}{3}} = \frac{1}{\sqrt{3}/2}$

point: $\left(\frac{2\pi}{3}, \frac{2\sqrt{3}}{3}\right)$

$y' = -\csc x\cot x \cdot 1$

$y'\left(\frac{2\pi}{3}\right) = -\csc\left(\frac{2\pi}{3}\right)\cot\left(\frac{2\pi}{3}\right)$

$= -\left(\frac{2\sqrt{3}}{3}\right)\left(-\frac{\sqrt{3}}{3}\right) = \frac{2}{3}$

slope: $m = \frac{2}{3}$

$$\boxed{y - \frac{2\sqrt{3}}{3} = \frac{2}{3}\left(x - \frac{2\pi}{3}\right)}$$

6. The position of a particle moving along the x-axis is given by $x(t) = \cot 2t$ find the acceleration of the particle at $t = \frac{3\pi}{4}$

$v(t) = -\csc^2(2t)\cdot 2 = -2\csc^2(2t)$

$= -2\left[\csc(2t)\right]^2$ chain rule

$a(t) = -2\cdot 2\left[\csc(2t)\right]\cdot -\csc(2t)\cot(2t)\cdot 2$

$a(t) = 8\csc^2(2t)\cot(2t)$

$a\left(\frac{3\pi}{4}\right) = 8\csc^2\left(2\left(\frac{3\pi}{4}\right)\right)\cot\left(2\left(\frac{3\pi}{4}\right)\right)$

$= 8(-1)^2(0) = \boxed{0}$