5.01 Intro to Matrices Notes

Introduction to Matrices

Matrix: A rectangular array of numbers

Uses: to solve systems of equations (next week) and to record data (think Excel)

$$\text{Example: } A = \begin{array}{cl} \begin{bmatrix} 5 & 0 & -1 \\ -3 & 1 & 9 \end{bmatrix} & \begin{array}{l} \text{Row 1} \\ \text{Row 2} \end{array} \\ \begin{array}{ccc} \text{Column 1} & \text{Column 2} & \text{Column 3} \end{array} & \end{array}$$

Dimensions size of a matrix : measured as rows × columns

$\boxed{\text{Ex: } 2 \times 3 \text{ matrix}}$

Elements Values in a matrix, locations are described as rows, columns

$[A]$ $\quad a_{13} \rightarrow \boxed{-1}$

Row 1 Column 3

More examples:

$$B = \begin{bmatrix} -2 \\ 4 \\ 0 \\ 7 \end{bmatrix}$$

B is known as a "column matrix".
What are the dimensions of matrix B?

4×1 matrix

4 Rows 1 column

How about a "row matrix"?
Create a 1x3 matrix C.

$$C = \begin{bmatrix} 4 & 6 & 9 \end{bmatrix}$$

How about a "square matrix"?
Create a 4x4 matrix D.

$$D = \begin{bmatrix} 3 & 4 & -1 & 5 \\ 0 & 4 & 6 & 9 \\ 10 & -3 & 0 & 6 \\ -1 & 5 & 7 & 8 \end{bmatrix}$$

Example $2T = 2\begin{bmatrix} -5 & -4 & 1 \\ 7 & -8 & 4 \end{bmatrix} = \begin{bmatrix} -10 & -8 & 2 \\ 14 & -16 & 8 \end{bmatrix}$ 4

Operations

Addition and subtraction can only be done with matrices of the same size (dimensions).

Scalar multiplication can be done with a matrix of any size.

$$S = \begin{bmatrix} 2 & -1 & 3 \\ -4 & -2 & -8 \end{bmatrix} \qquad T = \begin{bmatrix} -5 & -4 & 1 \\ 7 & -8 & 4 \end{bmatrix} \qquad U = \begin{bmatrix} 9 & -2 \\ -3 & 0 \end{bmatrix}$$

Evaluate each:

S + U = not possible

S - T = $\begin{bmatrix} 2 & -1 & 3 \\ -4 & -2 & -8 \end{bmatrix} - \begin{bmatrix} -5 & -4 & 1 \\ 7 & -8 & 4 \end{bmatrix}$ $\boxed{\begin{bmatrix} 7 & 3 & 2 \\ -11 & 6 & -12 \end{bmatrix}}$

2T = $2\begin{bmatrix} -5 & -4 & 1 \\ 7 & -8 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} -10 & -8 & 2 \\ 14 & -16 & 8 \end{bmatrix}}$

½ U = $\frac{1}{2}\begin{bmatrix} 9 & -2 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 9/2 & -2/2 \\ -3/2 & 0 \end{bmatrix}$ or $\boxed{\begin{bmatrix} 9/2 & -1 \\ -3/2 & 0 \end{bmatrix}}$

Use matrices A, B, and C to solve for X.

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} -4 \\ 12 \end{bmatrix} \qquad C = \begin{bmatrix} -2 & 8 \\ 5 & -6 \end{bmatrix}$$

X = 2B

$X = 2\begin{bmatrix} -4 \\ 12 \end{bmatrix}$

$\boxed{X = \begin{bmatrix} -8 \\ 24 \end{bmatrix}}$

A + X = C

$X = C - A$

$X = \begin{bmatrix} -2 & 8 \\ 5 & -6 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ -1 & 7 \end{bmatrix}$

$\boxed{X = \begin{bmatrix} -5 & 8 \\ 6 & -13 \end{bmatrix}}$

5.01 Practice

State the dimensions of each matrix.

1. $\begin{bmatrix} 1 & -8 \\ 6 & -2 \end{bmatrix}$

2. $\begin{bmatrix} -9 & -8 \\ 2 & 17 \\ 11 & -6 \end{bmatrix}$

1) 2×2 2) 3 × 2

Find the value of each element in $A = \begin{bmatrix} -3 & 45 & 28 \\ 24 & 36 & -22 \\ -15 & 4 & 29 \end{bmatrix}$

3) $a_{22} = \boxed{36}$

Row 2 Column 2

4. $a_{31} = \boxed{-15}$

Row 3 Column 1

Find each of the following for $Q = \begin{bmatrix} 13 & -6 \\ 2 & -10 \\ -4 & 8 \end{bmatrix}$, $R = \begin{bmatrix} 1 & -3 \\ -5 & 9 \\ 12 & 7 \end{bmatrix}$, $S = \begin{bmatrix} 5 & -2 & 1 \\ -6 & 14 & 8 \end{bmatrix}$, and $T = \begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix}$.

If the matrix does not exist, write *impossible*.

5. Q + R

$\begin{bmatrix} 14 & -9 \\ -3 & -1 \\ 8 & 15 \end{bmatrix}$

6. $T - R$

not possible

7. $T - S$

$\begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -2 & 1 \\ -6 & 14 & 8 \end{bmatrix}$

$\boxed{\begin{bmatrix} -16 & 5 & 6 \\ 10 & -23 & 8 \end{bmatrix}}$

8. $2R + Q$

$\begin{bmatrix} 2 & -6 \\ -10 & 18 \\ 24 & 14 \end{bmatrix} + \begin{bmatrix} 13 & -6 \\ 2 & -10 \\ -4 & 8 \end{bmatrix} = \boxed{\begin{bmatrix} 15 & -12 \\ -8 & 8 \\ 20 & 22 \end{bmatrix}}$

9. $\frac{1}{2}(T + S)$

$\frac{1}{2}\begin{bmatrix} -6 & 1 & 8 \\ -2 & 5 & 24 \end{bmatrix} = \begin{bmatrix} -3 & 1/2 & 4 \\ -1 & 5/2 & 12 \end{bmatrix}$

Use matrices Q, R, S, and T to solve for X. If the matrix does not exist, write *impossible*.

10. $\left(Q - R = \frac{1}{3}X\right)3$

$3(Q-R) = X$

$3\begin{bmatrix} 12 & -3 \\ 7 & -19 \\ -16 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 36 & -9 \\ 21 & -57 \\ -48 & 3 \end{bmatrix}}$

11. $3S - X = T$

$3S - T = X$

$3\begin{bmatrix} 5 & -2 & 1 \\ -6 & 14 & 8 \end{bmatrix} - \begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix}$

$\begin{bmatrix} 15 & -6 & 3 \\ -18 & 42 & 24 \end{bmatrix} - \begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix}$

$\boxed{\begin{bmatrix} 26 & -9 & -4 \\ -22 & 51 & 8 \end{bmatrix}}$

12. $\left(2(Q - X) = -T\right)\frac{1}{2}$

$Q - X = -\frac{1}{2}T$

$Q + \frac{1}{2}T = X$

not possible (dimensions are not the same)

13. Jessica took her two children to the community swimming pool once a week for six weeks. The daily admission fees are \$4.50 for a child and \$6.75 for an adult. Write a 1 x 3 matrix with a scalar multiple that represents the total cost of admission. What is the total cost?

$6\begin{bmatrix} 4.50 & 4.50 & 6.75 \end{bmatrix} \rightarrow \begin{bmatrix} 27 & 27 & 40.50 \end{bmatrix}$

Total cost is \$94.50