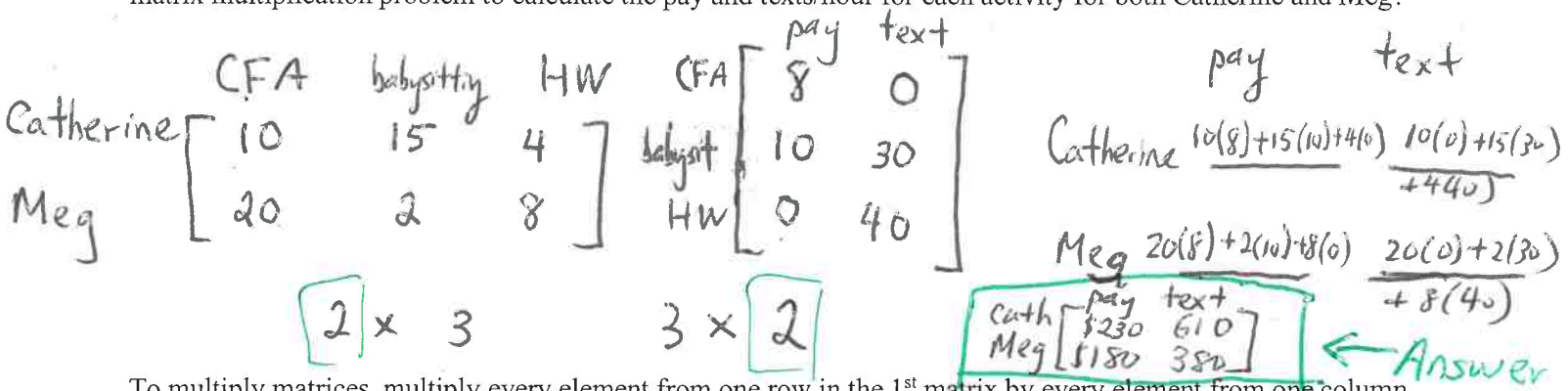
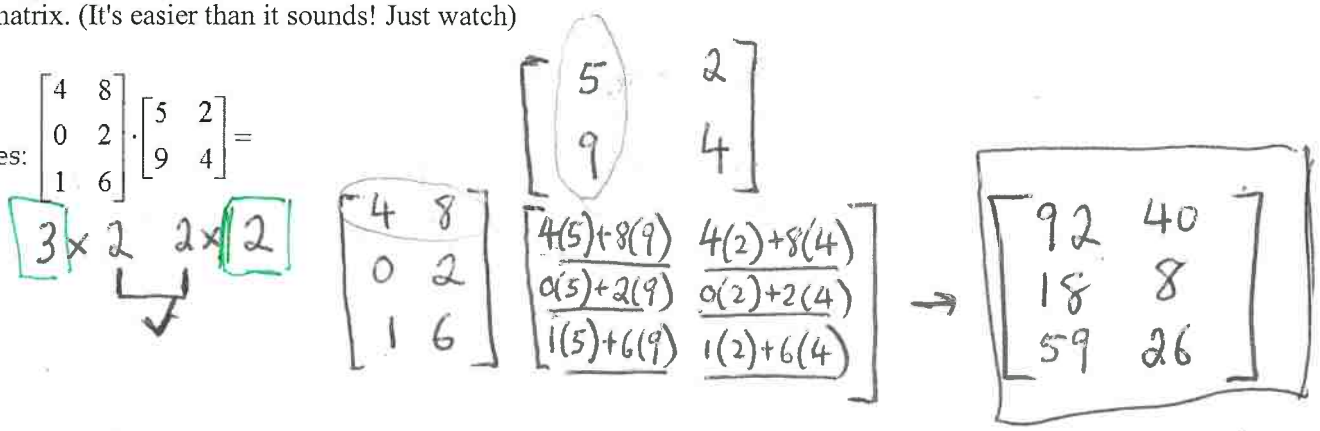
5.02 Matrix Multiplication Notes

Scenario: Catherine and Meg spend their spare time working at Chick-Fil-A, babysitting, and working on homework. Chick-Fil-A pays them each $8.00/hour, they make $10/hour babysitting, and their homework will pay them in dividends when they're older (translation: they don't get paid). When working at Chick-Fil-A, they can't use their phones at all. While babysitting, they can send roughly 30 texts/hour. While working on homework, they can send about 40 texts/hour. Create hours/schedules for each student. How can we set up a matrix multiplication problem to calculate the pay and texts/hour for each activity for both Catherine and Meg?

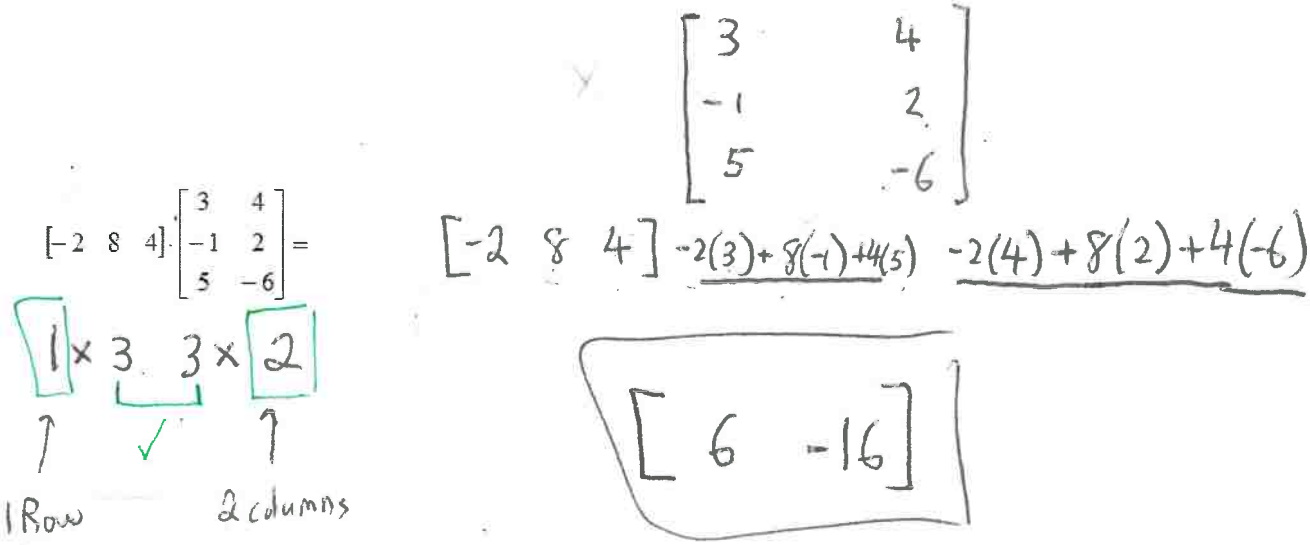


To multiply matrices, multiply every element from one row in the 1st matrix by every element from one column in the 2nd matrix and find the sum of the products. This gives you the element in that row and column in the answer matrix. (It's easier than it sounds! Just watch)

Examples: $\begin{bmatrix}4 & 8\\ 0 & 2\\ 1 & 6\end{bmatrix}\cdot\begin{bmatrix}5 & 2\\ 9 & 4\end{bmatrix}=$



$[-2\ \ 8\ \ 4]\cdot\begin{bmatrix}3 & 4\\ -1 & 2\\ 5 & -6\end{bmatrix}=$



5.02 Notes continued

$$\begin{bmatrix}6&3&0\\2&5&1\\9&8&6\end{bmatrix}\cdot\begin{bmatrix}7&4\\6&7\\5&0\end{bmatrix}=$$

$3\times 3 \quad 3\times 2$ ✓

$$\begin{bmatrix}6&3&0\\2&5&1\\9&8&6\end{bmatrix}\begin{bmatrix}7&4\\6&7\\5&0\end{bmatrix}$$

$$\begin{bmatrix}6(7)+3(6)+0(5) & 6(4)+3(7)+0(0)\\ 2(7)+5(6)+1(5) & 2(4)+5(7)+1(0)\\ 9(7)+6(8)+6(5) & 9(4)+8(7)+6(0)\end{bmatrix}$$

$$\begin{bmatrix}42+18+0 & 24+21+0\\ 14+30+5 & 8+35+0\\ 63+48+30 & 36+56+0\end{bmatrix}=\begin{bmatrix}60&45\\49&43\\141&92\end{bmatrix}$$

Matrix Multiplication and Dimensions:

What happens if we switch the order of the matrices?

3×3 and 2×3

* may not be possible if the inner dimensions do not match.

What happens if we switch the dimensions of one matrix?

* may become not possible
* may change answer (dimensions may change)

So what has to be true for matrix multiplication to work?

$3\times 3 \quad 3\times 4$ ✓

# of columns from the first matrix must equal the # of rows from the 2nd matrix

(inner dimensions of the 2 matrices must be the same)

Does matrix multiplication result in the same product when the order is reversed?

Not the same product if order is switched

$$\begin{bmatrix}2&-3\\-5&1\end{bmatrix}\begin{bmatrix}6&-2\\1&-1\end{bmatrix}=$$

$$\begin{bmatrix}2&-3\\-5&1\end{bmatrix}\begin{bmatrix}6&-2\\1&-1\end{bmatrix}$$

$$\begin{bmatrix}2(6)+(-3)(1) & 2(-2)+(-3)(-1)\\ -5(6)+1(1) & -5(-2)+1(-1)\end{bmatrix}$$

$$\begin{bmatrix}9&-1\\-29&9\end{bmatrix}$$

$$\begin{bmatrix}6&-2\\1&-1\end{bmatrix}\begin{bmatrix}2&-3\\-5&1\end{bmatrix}=$$

$$\begin{bmatrix}6&-2\\1&-1\end{bmatrix}\begin{bmatrix}2&-3\\-5&1\end{bmatrix}$$

$$\begin{bmatrix}6(2)+(-2)(-5) & 6(-3)+(-2)(1)\\ 1(2)+(-5)(-1) & 1(-3)+(-1)(1)\end{bmatrix}$$

$$\begin{bmatrix}22&-20\\7&-4\end{bmatrix}$$

$$\begin{bmatrix}9&-1\\-29&9\end{bmatrix}\neq\begin{bmatrix}22&-20\\7&-4\end{bmatrix}$$

5.02 Practice

Find AB and BA, if possible.

1. $A = [8 \quad 1]; B = \begin{bmatrix} 3 & -7 \\ -5 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & 9 \\ -7 & 3 \end{bmatrix}; B = \begin{bmatrix} 6 & -4 \\ 0 & 3 \end{bmatrix}$

3. $A = [3 \quad -5]; B = \begin{bmatrix} 4 & 0 & -2 \\ 1 & -3 & 2 \end{bmatrix}$

4. $A = \begin{bmatrix} 4 \\ 5 \end{bmatrix}; B = [6 \quad 1 \quad -10 \quad 9]$

5. $A = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix}; B = \begin{bmatrix} 6 & 0 & -1 \\ -4 & 9 & 8 \end{bmatrix}$

6. $A = \begin{bmatrix} 2 & 0 \\ -4 & -3 \\ 1 & -2 \end{bmatrix}; B = \begin{bmatrix} 0 & 6 & -5 \\ 2 & -7 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 12 & -10 \\ -6 & -3 & 17 \\ -4 & 20 & -7 \end{bmatrix}$$

7. Different point values are awarded for different shots in basketball. Use the information to determine the total points scored by each player. 3 × 3 and 3 × 1

Player	Free Throw	2-pointer	3-pointer
Rey	44	32	25
Chris	37	24	31
Jerry	35	39	29

Shots	Points
Free Throw	1
2-pointer	2
3-pointer	3

Points

Rey $44(1) + 32(2) + 25(3)$

Chris $37(1) + 24(2) + 31(3)$

Jerry $35(1) + 39(2) + 29(3)$

$\rightarrow$ Points

$$\begin{matrix} \text{Ray} \\ \text{Chris} \\ \text{Jerry} \end{matrix} \begin{bmatrix} 183 \\ 178 \\ 200 \end{bmatrix}$$