Name ________________ Period Key Aug. 2021

Accelerated Pre-Calculus

Unit 1 Packet

Introduction to Trigonometry

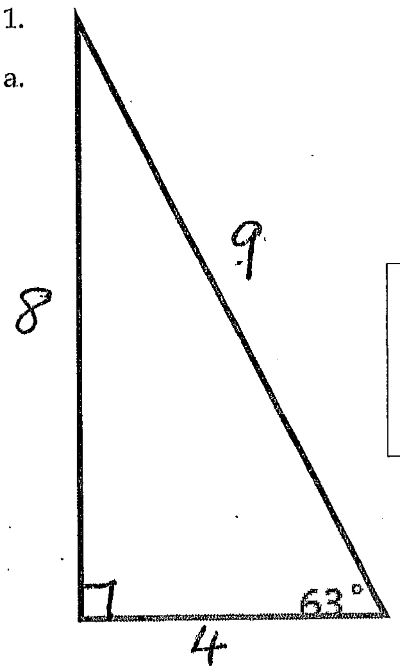
Name: ______________________

Accel Pre-Calculus Trig Ratio Investigation $S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

Working with a partner, measure the side lengths of the following triangles (in cm) then find the sine, cosine, and tangent of theta for each triangle (remember SOHCAHTOA). Discuss the results with your partner.

1.

a.

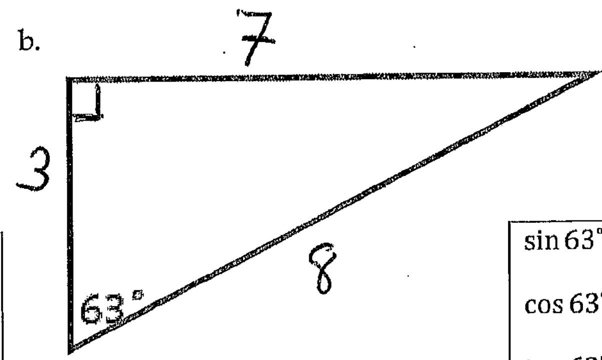


sin 63° = 8/9

cos 63° = 4/9

tan 63° = 8/4 = 2

b.

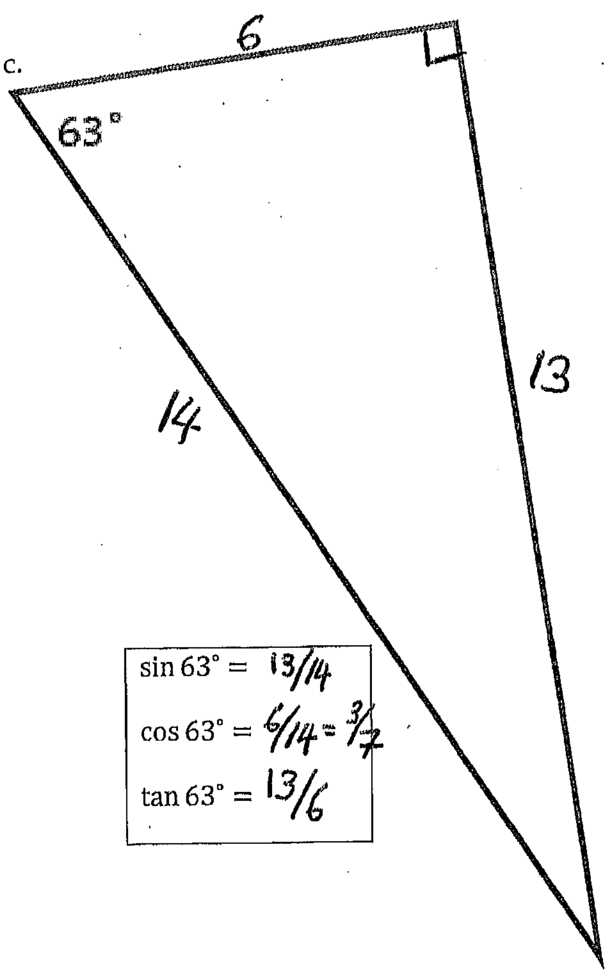


sin 63° = 7/8

cos 63° = 3/8

tan 63° = 7/3

c.

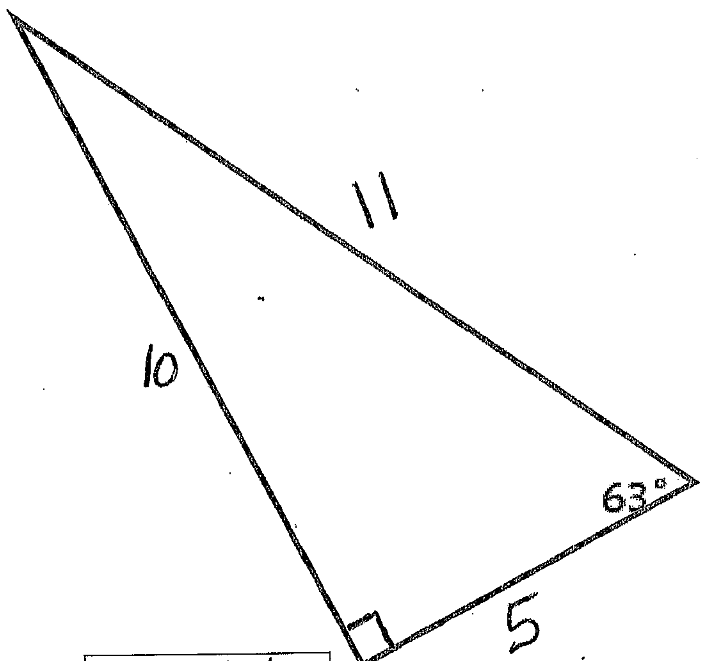


sin 63° = 13/14

cos 63° = 6/14 = 3/7

tan 63° = 13/6

d.



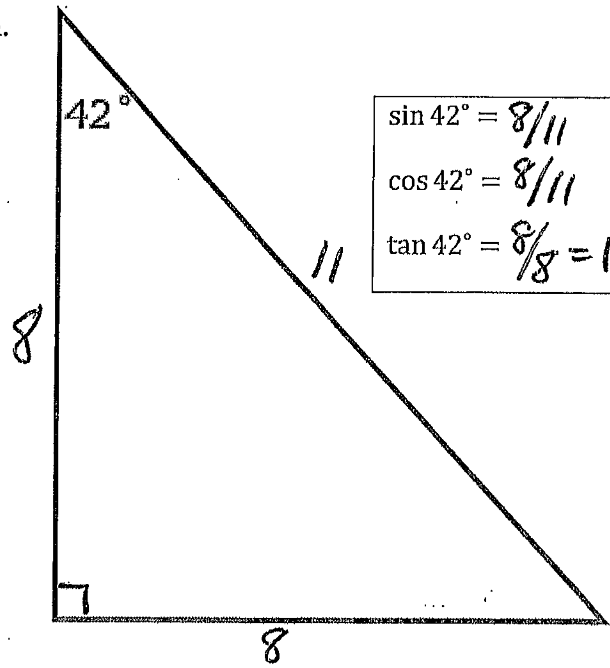
sin 63° = 10/11

cos 63° = 5/11

tan 63° = $\frac{10}{5}$ = 2

2.

a.

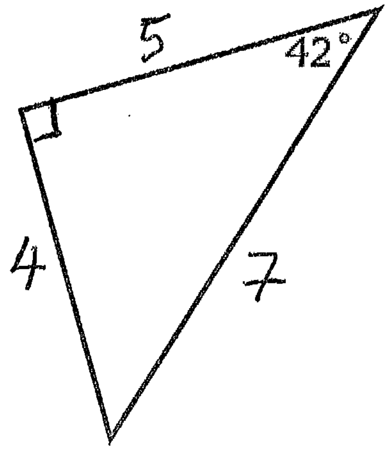


$\sin 42° = 8/11$

$\cos 42° = 8/11$

$\tan 42° = 8/8 = 1$

b.

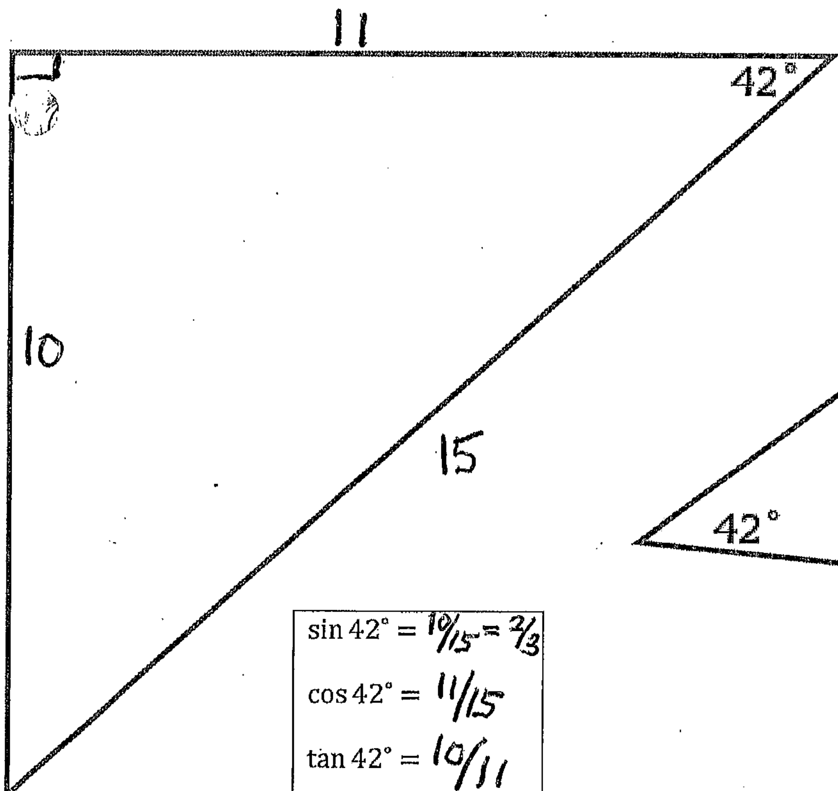


$\sin 42° = 4/7$

$\cos 42° = 5/7$

$\tan 42° = 4/5$

c.

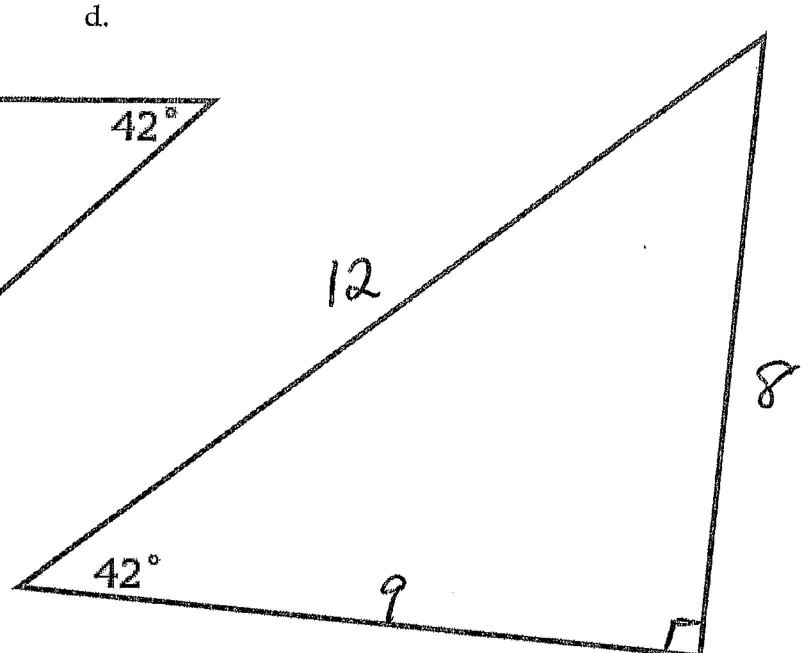


$\sin 42° = 10/15 = 2/3$

$\cos 42° = 11/15$

$\tan 42° = 10/11$

d.



$\sin 42° = 8/12 = 2/3$

$\cos 42° = 9/12 = 3/4$

$\tan 42° = 8/9$

Accel Pre-Calculus
1.02 Right Triangle Trig Notes

Name: ____________________

Warm-up:

1. Rationalize $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

2. Solve for x.

$x=36$ $\quad \frac{12}{1} = \frac{x}{3}$ $\quad \boxed{x=36}$

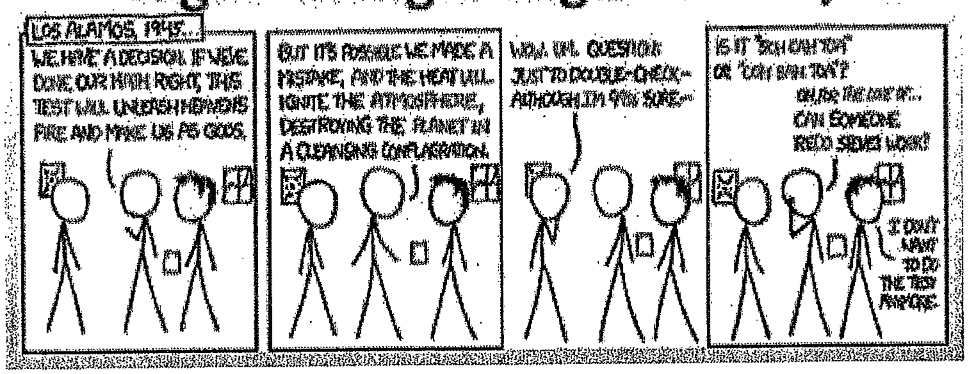
$\frac{20}{1} = \frac{5}{x}$ $\quad 20x=5$ $\quad \boxed{x=\frac{1}{4}}$

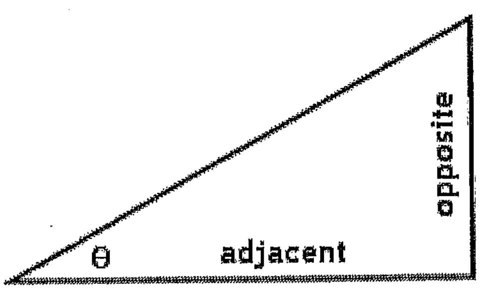
3. How do you solve for x?

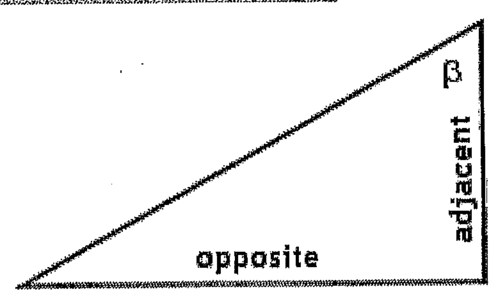
$x + 7 = 19$ $\quad \boxed{x=12}$

$x^2 = 30$ $\quad \boxed{x=\pm\sqrt{30}}$

Right Triangle Trigonometry

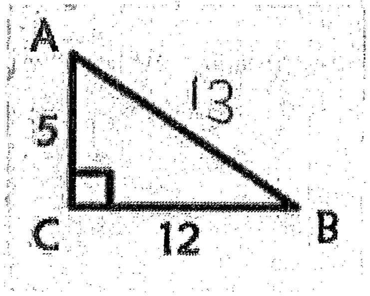






Examples: Find sine, cosine, and tangent of Angle A. *pythagorean theorem: $a^2+b^2=c^2$

1.

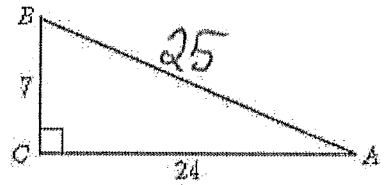


$\sin A = \frac{12}{13}$

$\cos A = 5/13$

$\tan A = 12/5$

2.



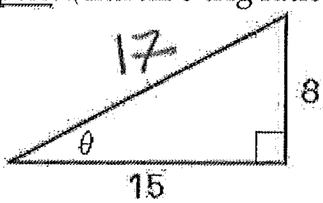
$\sin A = 7/25$

$\cos A = 24/25$

$\tan A = 7/24$

Examples: Find all 6 trig ratios from Angle A.

1.



$\sin\theta = 8/17$ $\quad \csc\theta = 17/8$

$\cos\theta = 15/17$ $\quad \sec\theta = 17/15$

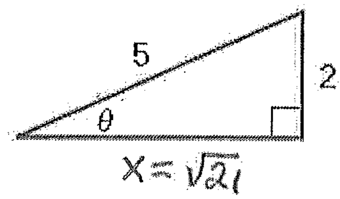
$\tan\theta = 8/15$ $\quad \cot\theta = 15/8$

2.

$x^2+2^2=5^2$

$x^2=21$

$x=\sqrt{21}$



$\sin\theta = 2/5$ $\quad \csc\theta = 5/2$

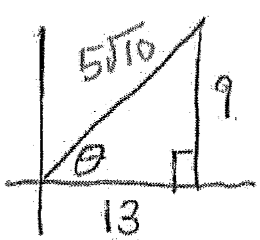
$\cos\theta = \frac{\sqrt{21}}{5}$ $\quad \sec\theta = \frac{5\sqrt{21}}{21}$

$\tan\theta = \frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}}$ $\quad \cot\theta = \frac{\sqrt{21}}{2}$

$\boxed{\frac{2\sqrt{21}}{21}}$

Example: Given $\cot\theta = \frac{13}{9}$, find the other 5 trig ratios from θ.

$\cot\theta \rightarrow \frac{adj}{opp.} \rightarrow \frac{13}{9}$



$13^2+9^2=c^2$

$250=c^2$

$\sqrt{c^2}=\sqrt{250}$

$c=\sqrt{250}$

$25 \cdot 10$

$c=5\sqrt{10}$

$\frac{9\sqrt{10}}{50}$

$\sin\theta = \frac{9}{5\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$ $\quad \csc\theta = \frac{5\sqrt{10}}{9}$

$\cos\theta = \frac{13\sqrt{10}}{50}$ $\quad \sec\theta = \frac{5\sqrt{10}}{13}$

$\tan\theta = \frac{9}{13}$ $\quad \cot\theta = \frac{13}{9}$

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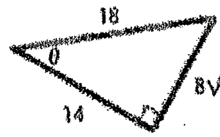
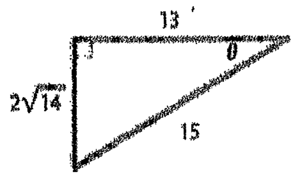
Name: ______________________
Date: ______________________ Period: ____

Find the exact values of the six trigonometric functions of θ.

1.


2.


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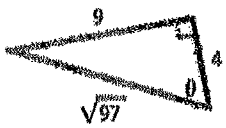
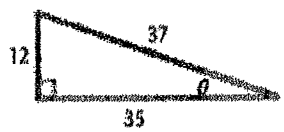
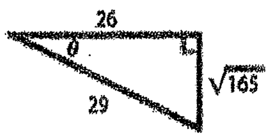


4.


5.


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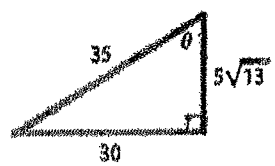
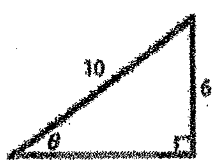
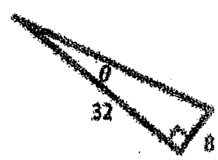


7.


8.


1. $\sin\theta = \frac{4\sqrt{2}}{9}$ $\csc\theta = \frac{9\sqrt{2}}{8}$
$\cos\theta = \frac{7}{9}$ $\sec\theta = \frac{9}{7}$
$\tan\theta = \frac{4\sqrt{2}}{9}$ $\cot\theta = \frac{7\sqrt{2}}{8}$

2. $\sin\theta = \frac{2\sqrt{14}}{15}$ $\csc\theta = \frac{15\sqrt{14}}{28}$
$\cos\theta = \frac{13}{15}$ $\sec\theta = \frac{15}{13}$
$\tan\theta = \frac{2\sqrt{14}}{13}$ $\cot\theta = \frac{13\sqrt{14}}{28}$

3. $\cos\theta = \frac{4\sqrt{97}}{97}$ $\sec\theta = \frac{\sqrt{97}}{4}$
$\sin\theta = \frac{9\sqrt{97}}{97}$ $\csc\theta = \frac{\sqrt{97}}{9}$
$\tan\theta = \frac{9}{4}$ $\cot\theta = \frac{4}{9}$

4. $\sin\theta = \frac{12}{37}$ $\csc\theta = \frac{37}{12}$
$\cos\theta = \frac{35}{37}$ $\sec\theta = \frac{37}{35}$
$\tan\theta = \frac{12}{35}$ $\cot\theta = \frac{35}{12}$

5. $\sin\theta = \frac{\sqrt{165}}{29}$ $\csc\theta = \frac{29\sqrt{165}}{165}$
$\cos\theta = \frac{26}{29}$ $\sec\theta = \frac{29}{26}$
$\tan\theta = \frac{\sqrt{165}}{26}$ $\cot\theta = \frac{26\sqrt{165}}{165}$

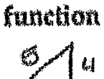
6. $C = \frac{\sqrt{13}}{7}$ $\sec = \frac{7\sqrt{13}}{13}$
$S = \frac{6}{7}$ $\csc = \frac{7}{6}$
$t = \frac{\sqrt{13}}{6}$ $\cot = \frac{6\sqrt{13}}{13}$

7. $S = \frac{3}{5}$ $\csc = \frac{5}{3}$
$C = \frac{4}{5}$ $\sec = \frac{5}{4}$
$t = \frac{3}{4}$ $\cot = \frac{4}{3}$

8. $S = \frac{\sqrt{17}}{17}$ $\csc = \sqrt{17}$
$C = \frac{4\sqrt{17}}{17}$ $\sec = 4\sqrt{17}$
$t = \frac{1}{4}$ $\cot = 4$

Use the given trigonometric function value of the acute angle θ to find the exact values of the five remaining trigonometric function values of θ. (Example 2)

9. $\sin\theta = \frac{4}{5}$



10. $\cos\theta = \frac{6}{7}$

11. $\tan\theta = 3$

12. $\sec\theta = 8$

13. $\cos\theta = \frac{5}{9}$

14. $\tan\theta = \frac{1}{4}$

15. $\cot\theta = 5$

16. $\csc\theta = 6$

17. $\sec\theta = \frac{9}{2}$

18. $\sin\theta = \frac{8}{13}$

9. $\cos\theta = \frac{3}{5}$ $\sec = \frac{5}{3}$
$\tan = \frac{4}{3}$ $\cot = \frac{3}{4}$
$\csc = \frac{5}{4}$

11. $\sin\theta = \frac{3\sqrt{10}}{10}$ $\csc\theta = \frac{\sqrt{10}}{3}$
$\cos\theta = \frac{\sqrt{10}}{10}$ $\sec\theta = \sqrt{10}$
$\cot = \frac{1}{3}$

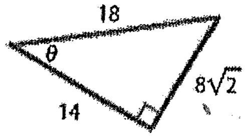
Accel Pre-Calculus

1.02 Practice Right Triangle Trig Ratios

Date: ____________

Find the exact values of the six trigonometric functions of θ. (Example 1)

1.

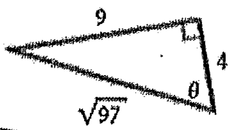


$\sin\theta = \frac{8\sqrt{2}}{18} = \frac{4\sqrt{2}}{9}$ $\csc\theta = \frac{9}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{9\sqrt{2}}{8}$

$\cos\theta = 7/9$ $\sec\theta = 9/7$

$\tan\theta = \frac{8\sqrt{2}}{14} = \frac{4\sqrt{2}}{7}$ $\cot\theta = \frac{7}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{8}$

3.

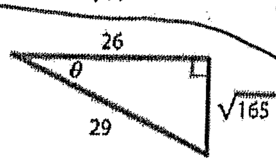


$\sin\theta = \frac{9}{\sqrt{97}} = \frac{9\sqrt{97}}{97}$ $\csc\theta = \frac{\sqrt{97}}{9}$

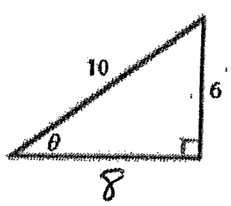
$\cos\theta = \frac{4}{\sqrt{97}} = \frac{4\sqrt{97}}{97}$ $\sec\theta = \frac{\sqrt{97}}{4}$

$\tan\theta = 9/4$ $\cot\theta = 4/9$

5.



7.



5) $\sin\theta = \frac{\sqrt{165}}{29}$ $\csc\theta = \frac{29}{\sqrt{165}} = \frac{29\sqrt{165}}{165}$

$\cos\theta = \frac{26}{29}$ $\sec\theta = 29/26$

$\tan\theta = \frac{\sqrt{165}}{26}$ $\cot\theta = \frac{26}{\sqrt{165}} \rightarrow \frac{26\sqrt{165}}{165}$

Use the given trigonometric function value of the acute angle θ to find the exact values of the five remaining trigonometric function values of θ. (Example 2)

9. $\sin\theta = \frac{4}{5}$

11. $\tan\theta = 3$

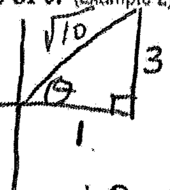
13. $\cos\theta = \frac{5}{9}$

7) $\sin\theta = \frac{6}{10} = \frac{3}{5}$ $\csc\theta = 5/3$

$\cos\theta = 4/5$ $\sec\theta = 5/4$

$\tan\theta = 3/4$ $\cot\theta = 4/3$

11)



$1^2 + 3^2 = c^2$

$10 = c^2$

$\sqrt{10} = c$

$\tan\theta = \frac{3}{1}$

$\sin\theta = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$\cos\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

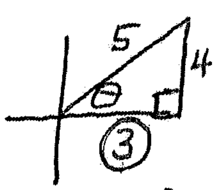
$\tan\theta = \frac{3}{1}$

$\csc\theta = \frac{\sqrt{10}}{3}$

$\sec\theta = \sqrt{10}$

$\cot\theta = 1/3$

9) $\sin\theta = 4/5$ $\csc\theta = 5/4$



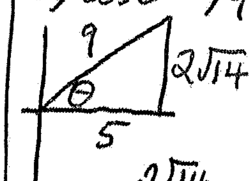
$\cos\theta = 3/5$ $\sec\theta = 5/3$

$\tan\theta = 4/3$ $\cot\theta = 3/4$

13) $\cos\theta = 5/9$

$5^2 + x^2 = 9^2$

$x^2 = 56$ $x = 2\sqrt{14}$



$\sin\theta = \frac{2\sqrt{14}}{9}$ $\csc\theta = \frac{9\sqrt{14}}{28}$

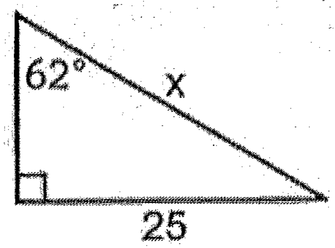
$\cos\theta = 5/9$ $\sec\theta = 9/5$

$\tan\theta = \frac{2\sqrt{14}}{5}$ $\cot\theta = \frac{5\sqrt{14}}{28}$

Date: ______________________

Examples: Find the missing side length using trigonometry (solve for x).

1.



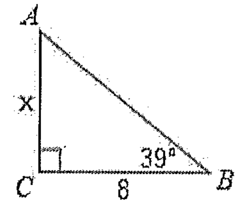
$\sin 62 = \frac{25}{x}$

$x \sin 62 = 25$

$x = \frac{25}{\sin 62}$

$\boxed{x = 28.31}$

2.



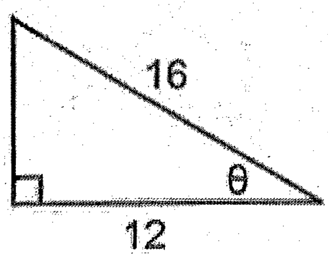
$\tan 39 = \frac{x}{8}$

$x = 8 \tan 39$

$\boxed{x = 6.48}$

Examples: Find the missing angle measure using trigonometry (solve for θ).

3.



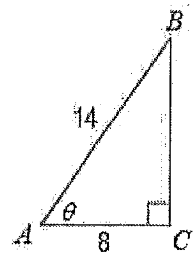
$\theta = \cos^{-1}\left(\frac{12}{16}\right)$

$\boxed{\theta = 41.41°}$

$\cos\theta = \frac{12}{16}$

$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\frac{12}{16}\right)$

4.



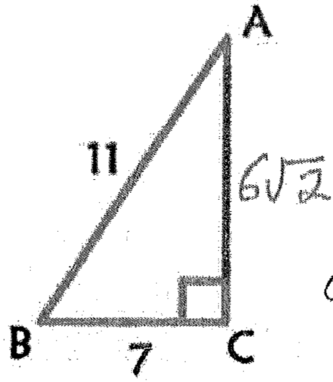
$\boxed{\theta = 55.15°}$

$\cos\theta = \frac{8}{14}$

$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\frac{8}{14}\right)$

Examples: Solve the triangle (find all side lengths and all angle measures).

5.



$\cos\theta = \frac{7}{11}$

$7^2 + y^2 = 11^2$

$y^2 = 72$

$y = \sqrt{72} = 6\sqrt{2}$

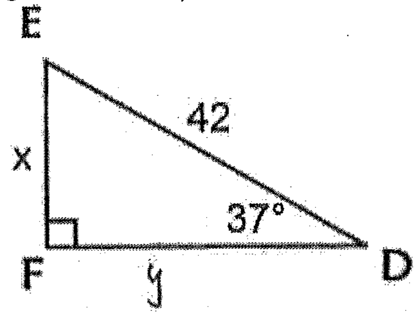
$\theta = 50.48°$

$\boxed{\angle B = 50.48°}$

$\angle A = 180 - 90 - 50.48$

$\boxed{\angle A = 39.52°}$

6.



$\sin 37 = \frac{x}{42}$

$x = 42 \sin 37$

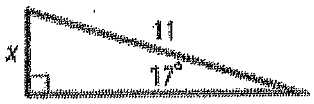
$\boxed{x = 25.28}$

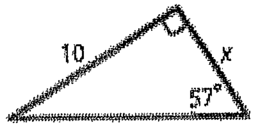
$25.28^2 + y^2 = 42^2$

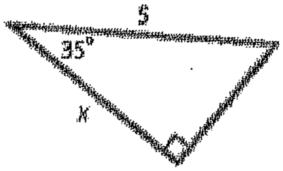
$\boxed{y = 33.54}$

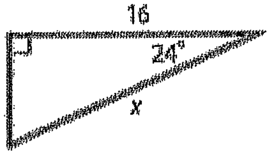
$\boxed{\angle E = 53°}$

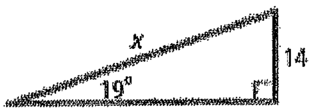
Find the value of x. Round to the nearest tenth, if necessary.
(Example 3)

19.


20.


21.


22.


23.


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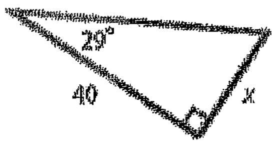

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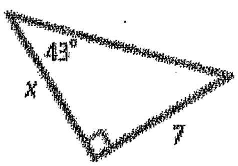

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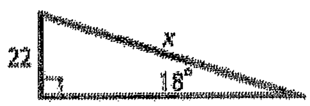


19) $x=3.2$ 20) $x=6.5$
21) $x=4.16$ 22) $x=17.5$
23) $x=43.0$ 24) $x=72.2$
25) $x=7.5$ 26) $x=71.2$

27. MOUNTAIN CLIMBING A team of climbers must determine the width of a ravine in order to set up equipment to cross it. If the climbers walk 25 feet along the ravine from their crossing point, and sight the crossing point on the far side of the ravine to be at a 35° angle, how wide is the ravine? (Example 4)

27) $x = 17.51$ ft

$\boxed{x = 17.51 \text{ ft}}$

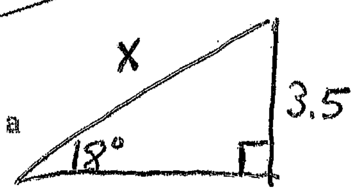
$\tan 35 = \frac{x}{25}$

$x = 25 \tan 35$

28. SNOWBOARDING Brad built a snowboarding ramp with a height of 3.5 feet and an 18° incline. (Example 4)

a. Draw a diagram to represent the situation.

b. Determine the length of the ramp.



$\sin 18 = \frac{3.5}{x}$

$x \sin 18 = 3.5$

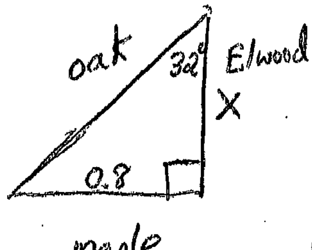
$x = \frac{3.5}{\sin 18}$

$\boxed{x = 11.33 \text{ ft}}$

29. DETOUR Traffic is detoured from Elwood Ave., left 0.8 mile on Maple St., and then right on Oak St., which intersects Elwood Ave. at a 32° angle. (Example 4)

a. Draw a diagram to represent the situation.

b. Determine the length of Elwood Ave. that is detoured.

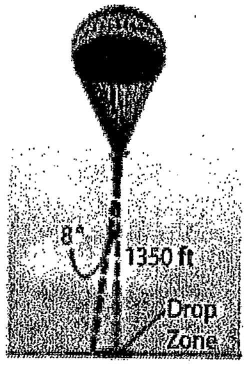


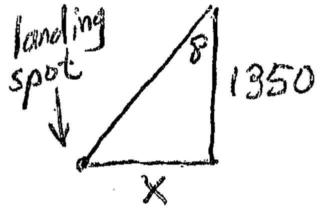
$\tan 32 = \frac{0.8}{x}$

$x \tan 32 = 0.8$

$x = \frac{0.8}{\tan 32} = \boxed{1.3 mi}$

30. **PARACHUTING** A paratrooper encounters stronger winds than anticipated while parachuting from 1350 feet, causing him to drift at an 8° angle. How far from the drop zone will the paratrooper land? (Example 4)



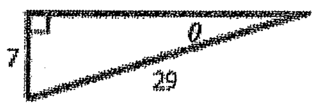


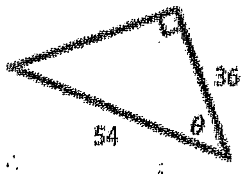
$\tan 8 = \frac{x}{1350}$

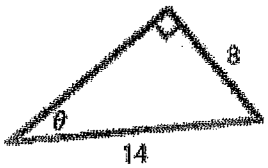
$x = 1350 \tan 8$

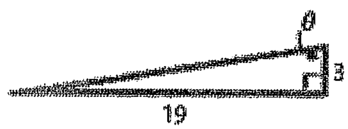
$\boxed{x = 190 \text{ ft.}}$

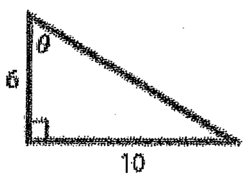
Find the measure of angle θ. Round to the nearest degree, if necessary. (Example 5)

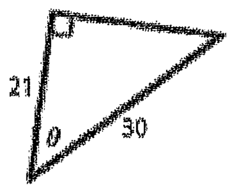
31. 

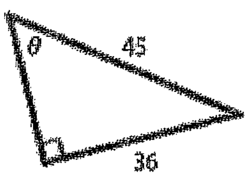
32. 

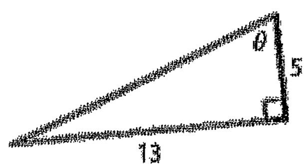
33. 

34. 

35. 

36. 

37. 

38. 

31) $\theta = 14°$ 32) $\theta = 48°$

33) $\theta = 35°$ 34) $\theta = 81°$

35) $\theta = 59°$ 36) $\theta = 46°$

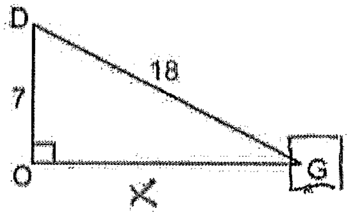
37) $\theta = 53°$ 38) $\theta = 69°$

Accel Pre-Calculus
1.04 Trig Applications Notes

Date: ____________________

A Quick Rewind:

1. Solve the triangle.



$\sin G = \frac{7}{18}$

$G = \sin^{-1}\left(\frac{7}{18}\right)$

$\boxed{G = 22.89^\circ}$

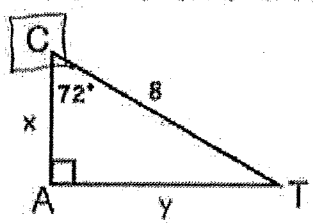
$\angle D = 90 - 22.89 = \boxed{67.115^\circ}$

$x^2 + 7^2 = 18^2$

$x^2 = 275$

$x = \sqrt{275} = \boxed{5\sqrt{11}}$

2. Solve the triangle.



$\cos 72 = \frac{x}{8}$

$\frac{\cos 72}{1} = \frac{x}{8}$

$x = 8\cos 72$

$\boxed{x = 2.472}$

$m\angle T = 90 - 72 = 18^\circ$

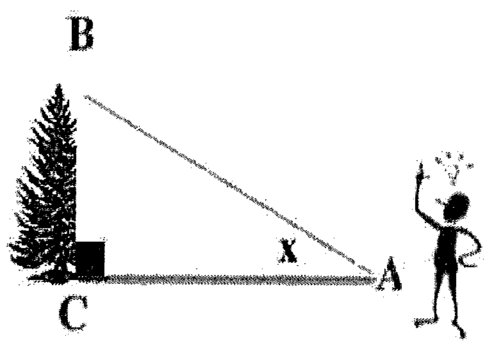
$x^2 + y^2 = 8^2$

$(2.47)^2 + y^2 = 64$

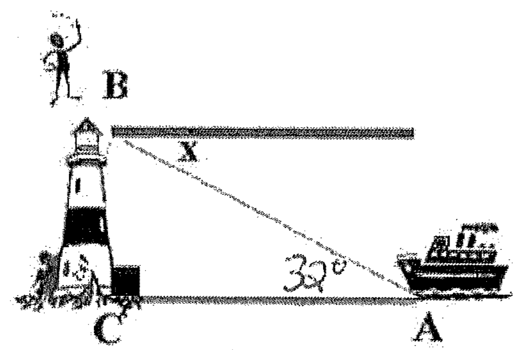
$y^2 = 57.89$

$\boxed{y = 7.608}$

Trigonometric ratios have many practical real-world examples. Angles of elevation and depression are formed by the horizontal lines that a person's lines of sight to an object form. If a person is looking up, the angle is an elevation angle. If a person is looking down, the angle is a depression angle.

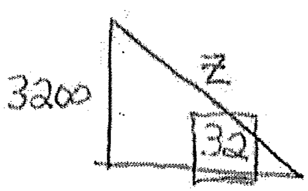


x = angle of elevation from ground to top of tree



x = angle of depression from lighthouse to boat

Example #3: A plane is coming in for a landing with an angle of depression of 32°. The plane is currently 3200 feet in the air. How far does the plane have to travel before it hits the runway?



$\sin\theta = \frac{opp}{hyp}$

$\sin 32 = \frac{3200}{z}$

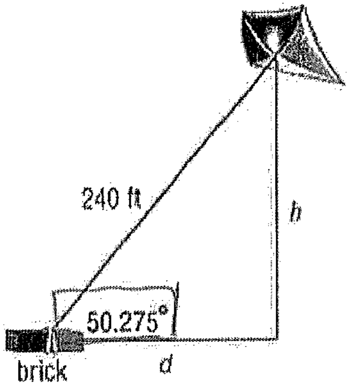
$\frac{\sin 32}{1} = \frac{3200}{z}$

$z\sin 32 = 3200$

$z = \frac{3200}{\sin 32}$

$\boxed{z = 6038.66\text{ ft}}$

Example #4: A child holding on to the string of a kite gets tired and decides to put the string on the ground and secure it with a brick. The length of the string from the brick to the kite is 240 feet.



a. If the angle formed by the string and the ground is 50.275°, how high is the kite?

$\sin 50.275 = \frac{h}{240}$

$h = 240 \sin 50.275$

$\boxed{h = 184.59\text{ ft}}$

b. What is the horizontal distance between the kite and the brick?

$\cos 50.275 = \frac{d}{240}$

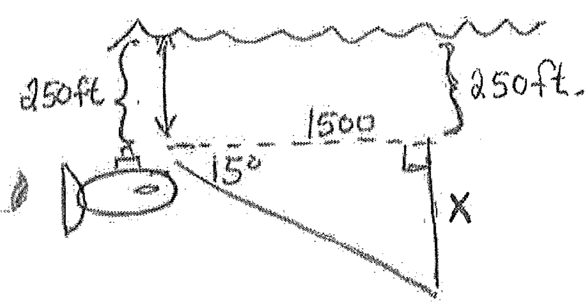
$d = 240 \cos 50.275 = \boxed{153.39\text{ ft}}$

OR $h^2 + d^2 = 240^2$

$184.59^2 + d^2 = 240^2$

$\boxed{d = 153.39\text{ ft}}$

Example #5: A submersible traveling at a depth of 250 feet dives at an angle of 15° with respect to a line parallel to the water's surface. It travels a horizontal distance of 1500 feet during the dive. What is the depth of the submersible after the dive?



$\tan 15 = \frac{x}{1500}$

$x = 1500 \tan 15$

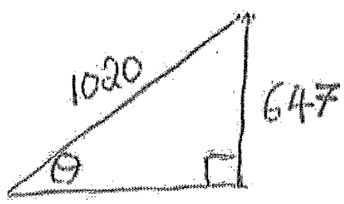
$x = 401.924$

Total depth $= 250 + 401.92$

$\boxed{= 651.923\text{ ft below the water surface.}}$

Example #6: The steepest railway in the world is the Katoomba Scenic Railway in Australia. The passenger car is pulled up the mountain by twin steel cables. It travels along the tract 1020 feet to obtain a change in altitude of 647 feet.

a. Find the angle of elevation of the railway.

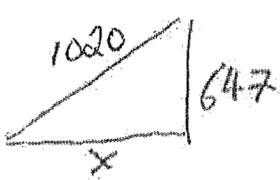


$\sin\theta = \frac{647}{1020}$

$\theta = \sin^{-1}\left(\frac{647}{1020}\right)$

$\boxed{\theta = 39.369°}$

b. How far does the car travel in a horizontal direction?



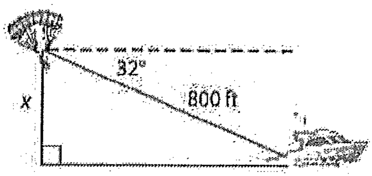
$x^2 + 647^2 = 1020^2$

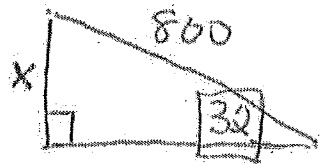
$\boxed{x = 788.537\text{ ft.}}$

Accel Pre-Calculus

1.04 Practice Trig Applications   Date: ____________

39. PARASAILING Kayla decided to try parasailing. She was strapped into a parachute towed by a boat. An 800-foot line connected her parachute to the boat, which was at a 32° angle of depression below her. How high above the water was Kayla? (Example 6)





$\sin 32 = \frac{x}{800}$

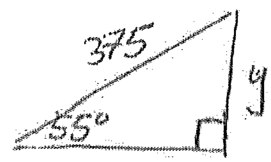
$x = 800\sin 32$

$\boxed{x = 423.935\text{ ft}}$

41. ROLLER COASTER On a roller coaster, 375 feet of track ascend at a 55° angle of elevation to the top before the first and highest drop. (Example 6)

a. Draw a diagram to represent the situation.

b. Determine the height of the roller coaster.

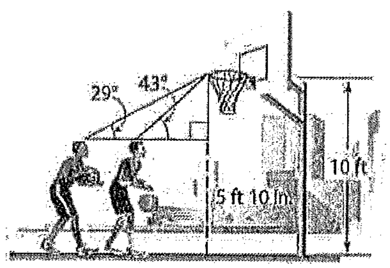


$\sin 55 = \frac{y}{375}$

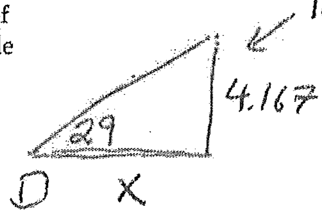
$y = 375\sin 55$

$\boxed{y = 307.182\text{ ft.}}$

43. BASKETBALL Both Derek and Sam are 5 feet 10 inches tall. Derek looks at a 10-foot basketball goal with an angle of elevation of 29°, and Sam looks at the goal with an angle of elevation of 43°. If Sam is directly in front of Derek, how far apart are the boys standing? (Example 7)



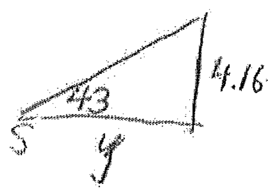
$10 - 5.833 = 4.167$



$\tan 29 = \frac{4.167}{x}$

$x\tan 29 = 4.167$

$x = 7.517$



$\tan 43 = \frac{4.167}{y}$

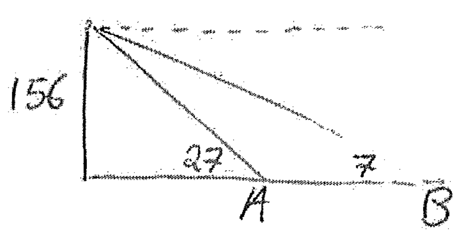
$y = \frac{4.167}{\tan 43} = 4.469$

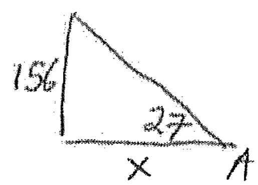
$7.517 - 4.469 = \boxed{3.048\text{ ft}}$

45. LIGHTHOUSE Two ships are spotted from the top of a 156-foot lighthouse. The first ship is at a 27° angle of depression, and the second ship is directly behind the first at a 7° angle of depression. (Example 7)

a. Draw a diagram to represent the situation.

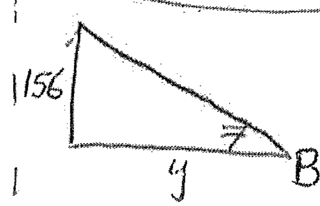
b. Determine the distance between the two ships.





$\tan 27 = \frac{156}{x}$

$x = \frac{156}{\tan 27} = 306.167$



$\tan 7 = \frac{156}{y}$

$y = \frac{156}{\tan 7}$

$y = 1270.518$

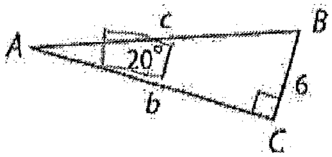
$1270.518 - 306.167$

$= \boxed{964.351\text{ ft.}}$

apart

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree. (Example 8)

47.



$\sin 20 = \frac{6}{c}$

$c \sin 20 = 6$

$c = \frac{6}{\sin 20}$

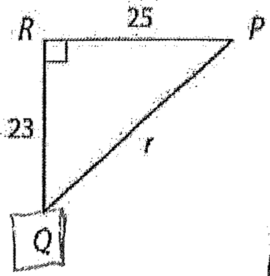
$\boxed{c = 17.543}$

$b^2 + 6^2 = 17.543^2$

$\boxed{b = 16.485}$

$m\angle B = 90 - 20 = \boxed{70°}$

49.



$\tan Q = \frac{25}{23}$

$Q = \tan^{-1}\left(\frac{25}{23}\right)$

$\boxed{Q = 47.386°}$

$23^2 + 25^2 = r^2$

$\boxed{r = 33.971}$

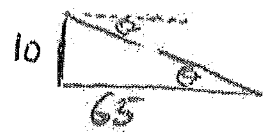
$m\angle P = 90 - 47.386$

$\boxed{m\angle P = 42.614°}$

55. BASEBALL Michael's seat at a game is 65 feet behind home plate. His line of vision is 10 feet above the field.

a. Draw a diagram to represent the situation.

b. What is the angle of depression to home plate? 9°



$\tan \theta = \frac{10}{65}$

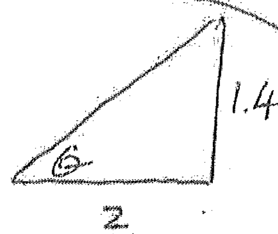
$\theta = \tan^{-1}\left(\frac{10}{65}\right)$

$\boxed{\theta = 8.746°}$

56. HIKING Jessica is standing 2 miles from the center of the base of Pikes Peak and looking at the summit of the mountain, which is 1.4 miles from the base.

a. Draw a diagram to represent the situation.

b. With what angle of elevation is Jessica looking at the summit of the mountain?



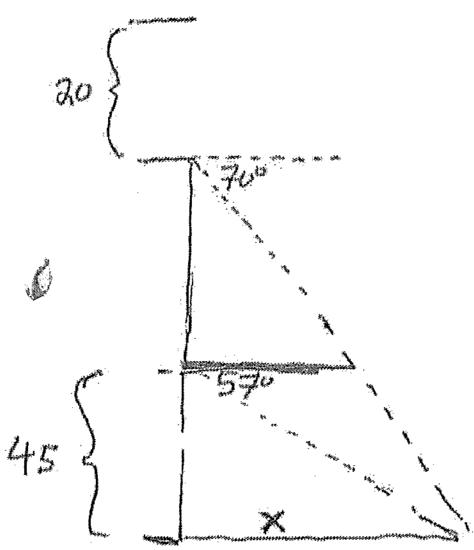
$\tan \theta = \frac{1.4}{2}$

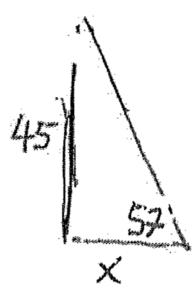
$\theta = \tan^{-1}\left(\frac{1.4}{2}\right)$

$\boxed{\theta = 35°}$

71. SCUBA DIVING A scuba diver located 20 feet below the surface of the water spots a shipwreck at a 70° angle of depression. After descending to a point 45 feet above the ocean floor, the diver sees the shipwreck at a 57° angle of depression. Draw a diagram to represent the situation, and determine the depth of the shipwreck. 100 ft

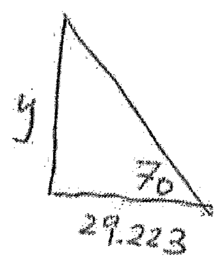
depth of shipwreck is 20 ft + 80.290 ft $= \boxed{100.290 \text{ ft}}$





$\tan 57 = \frac{45}{x}$

$\boxed{x = 29.223 \text{ ft}}$



$\tan 70 = \frac{y}{29.223}$

$y = 29.223 \tan 70$

$\boxed{y = 80.290 \text{ ft}}$

1.05 Radian Investigation

Date: ____________

Materials: Circles Handout, Protractor, Ruler, 5 Twizzler strands

Part 1: Defining a Radian:

Measuring the Radii

Work on one circle at a time

Step 1: Use a Twizzler strand to measure the radius of the circle. Cut your Twizzler to that length.

Step 2: Wrap your radius Twizzler along the circle, starting at the line and in either direction.

Step 3: Make a mark on your paper where the Twizzler ends.

Repeat for the other circles. In Step 2, make sure to wrap the Twizzler strand around in the same direction as you did for the first circle.

Measuring the Angle Formed

Step 1: From the center, draw a line of best fit passing between your three points.

Step 2: Using a protractor, measure the angle that is created: ________

Step 3: Share your measurement with the class.

Definition of Radian

The angle that you submitted is measured in degrees. **Radian** is another unit that we can use to express angle measurement. More specifically, a radian is defined as

Radian is the unit of measure of an angle, equal to an angle at the center of a circle whose arc is equal in length to the radius.

Part 2: Converting Between Degrees and Radians:

Degrees vs. Radians

So we know: $C = 2\pi r$

$360° = 2\pi$ radians

$180° = \pi$ radians

Therefore, each radian is how many degrees?

1 radian $= \frac{180}{\pi}$

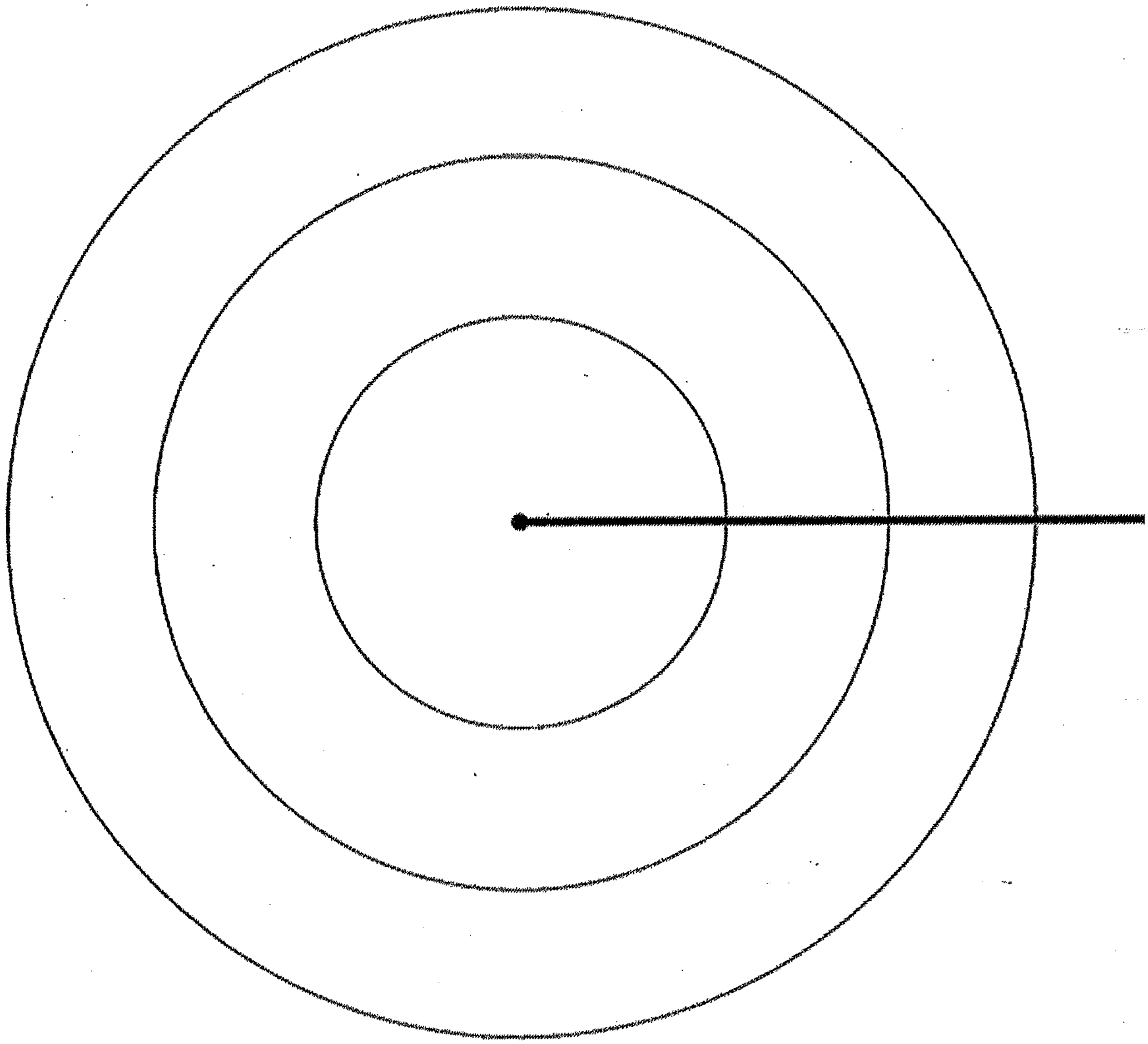
Convert from Degrees to Radians:

multiply by $\frac{\pi}{180}$

Convert from Radians to Degrees:

multiply by $\frac{180°}{\pi}$

*dimensional analysis



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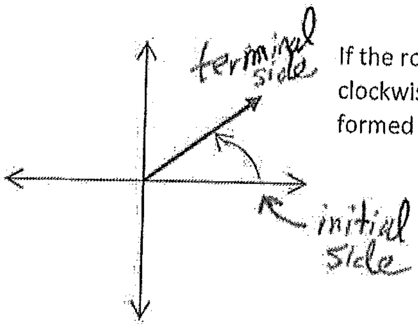
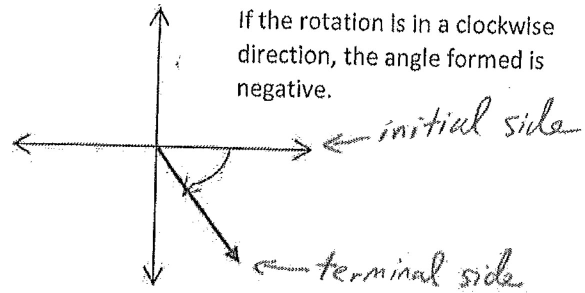
1.06 Angle Measures in Degrees and Radians

An **angle** is formed by two rays that share a fixed endpoint known as the **vertex**. One of the rays is fixed to form the **initial side** of the angle and the other ray rotates to form its **terminal side**. An angle with its vertex at the origin and its initial side along the positive x-axis is in **standard position.**

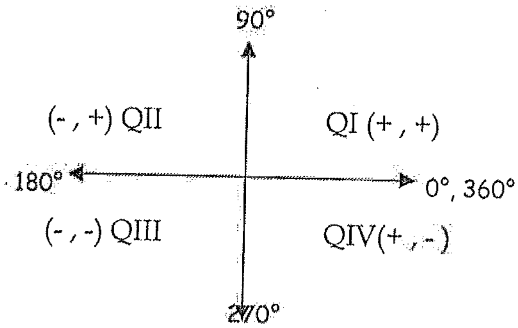


If the rotation is in a counter-clockwise direction, the angle formed is positive.



If the rotation is in a clockwise direction, the angle formed is negative.

Know the quadrants and the signs for x and y in each quadrant! That is very important in trigonometry. Degree or angle measures are read with respect to the quadrants starting at the positive x-axis (standard position) and moving in a counter-clockwise direction.



If the terminal side of an angle falls on one of the axes, the angle is a **quadrantal** angle.

There are four (4) quadrantal angles on a unit circle. They are:
0° (360°) on the x-axis, 90° on the y-axis, 180° on the –x-axis, and 270° on the –y-axis.

More Vocabulary from Geometry:

Right Angle: an angle whose measure is exactly 90°

Acute Angle: an angle whose measure is less than 90°

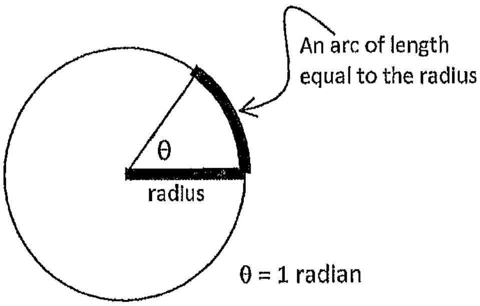
Obtuse Angle: an angle whose measure is greater than 90°

Complementary Angles: two angles the sum of whose measure is 90°

Supplementary Angles: two angles the sum of whose measure is 180°

new *unit of measurement* for angles: a **radian** is the measure of an angle in a circle whose intercepted arc has a length equal to the radius.

Remember the formula relating the length of a radius (r) of a circle to the circumference of the circle (C):
$C = 2\pi r$



About the center of a circle is 360° (degrees).

Also about the center of a circle is 2π radians.

Therefore: 360° is equivalent to 2π radians

So $360° = 2\pi$ radians

Or simplified, **$180° = \pi$ radians**

(This is our conversion unit!)

Degrees → Radians

Convert from degrees to radians.
State the quadrant in which the angle lies.

a. 120° Multiply by $\frac{\pi}{180°}$ to cancel the degrees and get radians.

$120° \cdot \frac{\pi}{180°} = \frac{120\pi}{180} = \boxed{\frac{2\pi}{3} \text{ radians}}$

Radians → Degrees

Convert from radians to degrees.
State the quadrant in which the angle lies.

b. $\frac{5\pi}{6}$ radians Multiply by $\frac{180°}{\pi}$ to cancel the radians and get degrees.

$\frac{5\cancel{\pi}}{6} \cdot \frac{180°}{\cancel{\pi}} = \boxed{150° \text{ degrees}}$

1.06 Degree & Radian Conversion Practice

Directions: Complete #10 – 17 all

Write each degree measure in radians as a multiple of π and each radian measure in degrees. (Example 2)

10. 30° $\cdot \frac{\pi}{180} = \pi/6$

11. 225° $\cdot \frac{\pi}{180} = 5\pi/4$

12. −165° $\cdot \frac{\pi}{180} = -\frac{11\pi}{12}$

13. −45° $\cdot \frac{\pi}{180} = -\pi/4$

14. $\frac{2\pi}{3} \cdot \frac{180}{\pi} = 120°$

15. $\frac{5\pi}{2} \cdot \frac{180}{\pi} = 450°$

16. $-\frac{\pi}{4} \cdot \frac{180}{\pi} = -45°$

17. $-\frac{7\pi}{6} \cdot \frac{180}{\pi} = -210°$

Accel Pre-Calculus
1.07 Arc Length Notes

Date: ____________________

Review:

a) Convert 315° to radians.

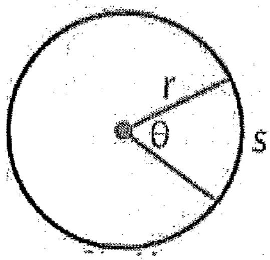
$315° \cdot \frac{\pi}{180} = \boxed{\frac{7\pi}{4} \text{ rad.}}$

b) Convert $\frac{11\pi}{6}$ to degrees.

$\frac{11\pi}{6} \cdot \frac{180}{\pi} = \boxed{330°}$

If θ is the measure in **radians** of a central angel in a circle with radius r, then the length, s, of the arc intercepted by θ is given by $s = r\theta$. NOTE: θ must be in radians.

notation for angle ∡



* This formula is meant for the ∡ to be in radians.

* If Θ (the angle) is in degrees, then be sure to convert to radians before using $\boxed{s = r\Theta}$

<u>Example 1</u>: Find the arc length if the radius is 8 cm and the central angle measures $\frac{3\pi}{4}$ radians.

$r = 8$
$\Theta = 3\pi/4$
$s = \_\_$

$s = r\Theta$

$s = 8 \cdot (3\pi/4) = 6\pi \rightarrow s \approx \boxed{18.850\text{cm}}$

<u>Example 2</u>: Find the arc length if the radius is 12 cm and the central angle measures $\frac{5\pi}{6}$ radians.

$r = 12$
$\Theta = 5\pi/6$
$s = \_\_$

$s = 12 \cdot \frac{5\pi}{6} \rightarrow s = 10\pi \approx \boxed{31.416\text{ cm}}$

<u>Example 3</u>: Find the arc length if the radius is 2.5 mi and the central angle measures 300°.

$r = 2.5$
$\Theta = 300° \cdot \frac{\pi}{180} = \frac{5\pi}{3}$
$s = \_\_$

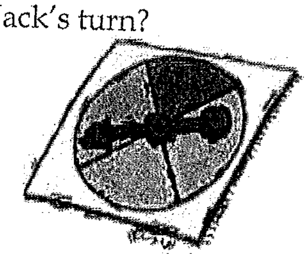
$s = r\Theta$

$s = 2.5\left(\frac{5\pi}{3}\right) = \frac{5}{2} \cdot \frac{5\pi}{3} = \frac{25\pi}{6} \approx \boxed{13.090\text{ mi}}$

<u>Example 4</u>: While playing a game of chance, Jack flicks a spinner with a radius of 2 inches. If the spinner swings through 2665°, how far did the arrowhead travel during Jack's turn?

$r = 2$
$\Theta = 2665 \cdot \frac{\pi}{180} \approx 46.513$
$s = \_\_$

$s = r\Theta$
$s = 2(46.513)$
$\boxed{s = 93.026\text{ in.}}$



1.07 Practice- Applications with Arc Length

Date: ______________

Directions: Complete #1,2 51-60, 63, 33 $s = r\theta$

1. Find the length of an arc that subtends a central angle of 3 radians in a circle with radius 2 in.

$\theta = 3$ $\quad s = 2(3) = \boxed{6 \text{ in.}}$

$r = 2$

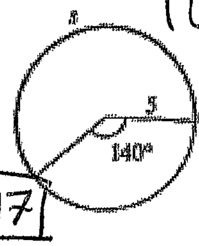
2. Find the length of a radius of a circle if an arc of length 7 cm is subtended by an angle of 2 rad.

$s =$ $s = 7$ | $s = r\theta$ | $\frac{7}{2} = r$

$\theta = 2$ rad | $7 = r(2)$ | $\boxed{r = 3.5 cm}$

$r =$ ___

51. Find the length of the arc s in the figure.

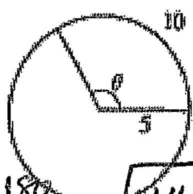


$r = 5$

$\theta = 140 \cdot \frac{\pi}{180} = \frac{7\pi}{9}$

$s = (5)\left(\frac{7\pi}{9}\right) = \frac{35\pi}{9} = \boxed{12.217}$

52. Find the angle θ in the figure.



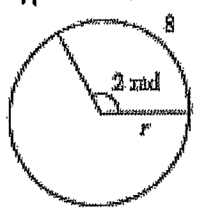
$r = 5$ | $s = r\theta$

$s = 10$ | $10 = 5(\theta)$

$\theta =$ ___ | $2 = \theta$

$\theta = 2 \text{ rad} \cdot \frac{180}{\pi} = \boxed{114.592°}$

53. Find the radius r of the circle in the figure. $s = r\theta$



$s = 8$ | $8 = r(2)$

$r =$ ___ | $\boxed{4 = r}$

$\theta = 2$ rad

54. Find the length of an arc that subtends a central angle of 45° in a circle of radius 10 m.

55. Find the length of an arc that subtends a central angle of 2 rad in a circle of radius 2 mi.

56. A central angle θ in a circle of radius 5 m is subtended by an arc of length 6 m. Find the measure of θ in degrees and in radians.

57. An arc of length 100 m subtends a central angle θ in a circle of radius 50 m. Find the measure of θ in degrees and in radians.

58. A circular arc of length 3 ft subtends a central angle of 25°. Find the radius of the circle.

59. Find the radius of the circle if an arc of length 6 m on the circle subtends a central angle of $\pi/6$ rad.

60. Find the radius of the circle if an arc of length 4 ft on the circle subtends a central angle of 135°.

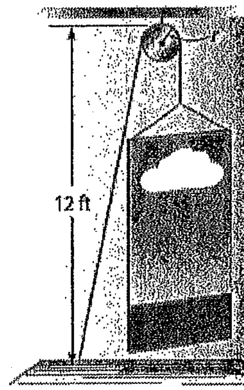
54) $s = 10 \cdot \frac{\pi}{4} = \frac{5\pi}{2} \approx \boxed{7.854m}$

55) $s = 2 \cdot 2 = \boxed{4 \text{ mi}}$

63. **DRAMA** A pulley with radius r is being used to remove part of the set of a play during intermission. The height of the pulley is 12 feet.

a. If the radius of the pulley is 6 inches and it rotates 180°, how high will the object be lifted?

b. If the radius of the pulley is 4 inches and it rotates 900°, how high will the object be lifted?



a) $s = 6 \cdot \pi$

$\boxed{s \approx 18.850 \text{ in.}}$

b) $r = 4$

$\theta = 900 \cdot \frac{\pi}{180}$

$s = 4.5\pi$

$\boxed{s = 62.832 \text{ in.}}$

33. **AMUSEMENT PARK** A carousel at an amusement park rotates 3024° per ride. (Example 4)

a. How far would a rider seated 13 feet from the center of the carousel travel during the ride?

b. How much farther would a second rider seated 18 feet from the center of the carousel travel during the ride than the rider in part a?

a) $s =$ ___ | $\theta = 3024 \cdot \frac{\pi}{180} = 52778$

$r = 13$ | $s = (13)(52.778) = \boxed{686.124 ft}$

b) $s = (18)(52.778) = 950.018$

Difference is $950.018 - 686.124 =$ $\boxed{263.894 ft}$

56) $6 = 5\theta$ | $1.2 \cdot \frac{180}{\pi} = \boxed{68.755°}$

$\boxed{\theta = 1.2 \text{ rad}}$

57) $100 = 50\theta$

$\boxed{2 = \theta}$ $\quad \theta = 2 \cdot \frac{180}{\pi} = \boxed{114.592°}$

58) $s = 3$ $\quad \theta = 25 \cdot \frac{\pi}{180} = 0.43633$

$3 = r(0.43633)$ $\boxed{r = 6.875 ft}$

59) $s = 6$ | $6 = r \cdot \pi/6$

$\theta = \pi/6$ | $r = 6 \cdot \frac{6}{\pi} = \frac{36}{\pi} = \boxed{11.459 m}$

60) $s = 4$

$\theta = 135 \cdot \pi/180$

$4 = \frac{3\pi}{4} \cdot r$

$\boxed{r = 1.698 ft}$

Accelerated Pre-Calculus

1.08 Co-terminal Angles and Reference Angles

Co-terminal angles are angles that have the same terminal (ending) side. Not only are co-terminal angles created by measuring an angle both in the negative and in the positive directions, but they can be created by doing more than one revolution (360°). Yes, angles can measure more than 360°!

Co-terminal angles can be found at the same location just another revolution more or less. So, to find co-terminal angles, we must add or subtract 360° and we will end in the same location for the terminal side.

Example 1: Find one positive and one negative co-terminal angle for the given angle.

a. 30°

$30+360 = \boxed{390^\circ}$

$30-360 = \boxed{-330^\circ}$

b. -34°

$-34+360 = \boxed{326^\circ}$

$-34-360 = \boxed{-394^\circ}$

c. $\frac{7\pi}{6}$

$\frac{7\pi}{6} + \frac{12\pi}{6} = \boxed{\frac{19\pi}{6}}$

$\frac{7\pi}{6} - \frac{12\pi}{6} = \boxed{\frac{-5\pi}{6}}$

Can we ever identify *all* co-terminal angles? **<u>No, there are infinitely many!</u>**

We can use this process for angles larger than 360° by subtracting 360 from the larger angle measure until we find a positive and a negative co-terminal angle. [in the range of (-360°, 360°)]

Example 2: Find one positive and one negative co-terminal angle for the given angle. State the quadrant in which the terminal side lies.

a. 800°

-720

$\boxed{80^\circ}$

Q1

80
-360

$\boxed{-280^\circ}$

b. -3732°

10(360)

$+3600$

$\boxed{-132^\circ}$

Q3

-132
$+360$

$\boxed{228^\circ}$

c. 3945°

10(360)

-3600

$\boxed{345^\circ}$

Q4

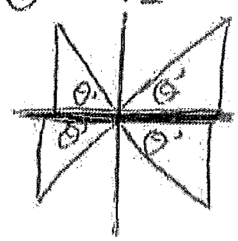
345
-360

$\boxed{-15^\circ}$

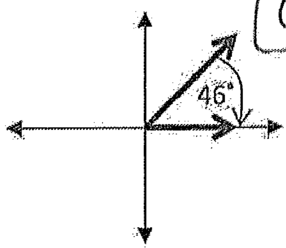
Reference Angles

A Reference angle (θ') is the angle formed by the *terminal side* of the angle and the closest part of the x-axi

Examples: Find the measure of the reference angle (θ') for each angle. *Reference angle is $0° < x < 90°$, $0 < \theta < \pi/2$

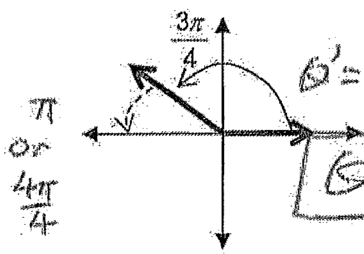


a. 46°



$\boxed{\theta' = 46°}$

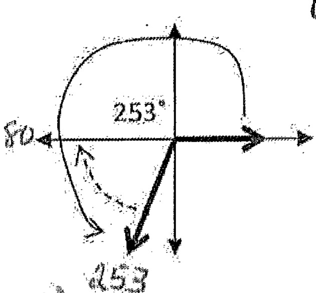
b. $\frac{3\pi}{4}$



$\theta' = \frac{4\pi}{4} - \frac{3\pi}{4}$

$\boxed{\theta' = \pi/4}$

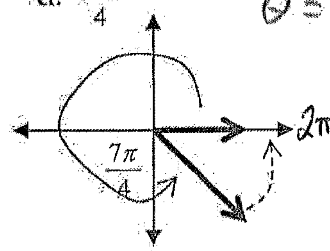
c. 253°



$\theta' = 253 - 180$

$\boxed{\theta' = 73°}$

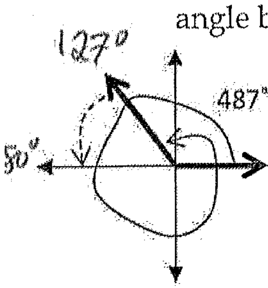
d. $\frac{7\pi}{4}$



$\theta' = \frac{8\pi}{4} - \frac{7\pi}{4}$

$\boxed{\theta' = \pi/4}$

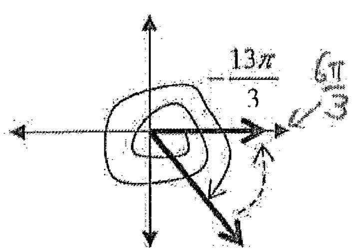
e. 487° (first find a positive co-terminal angle between 0° and 360°)



$\theta' = 180 - 127$

$\boxed{\theta' = 53°}$

f. $-\frac{13\pi}{3}$ (first find a positive co-terminal angle between 0 and 2π)



$\frac{-13\pi}{3} + \frac{6\pi}{3} = \frac{-7\pi}{3} + \frac{6\pi}{3} = \frac{-\pi}{3}$

$\frac{-\pi}{3} + \frac{6\pi}{3} = \frac{5\pi}{3}$

$\boxed{\theta' = \frac{6\pi}{3} - \frac{5\pi}{3} = \frac{\pi}{3}}$

How large can the *reference angle* be? **Up to 90° - reference angles are always acute and positive!**

In summary, to find the reference angle (θ') based on the quadrant in which the terminal side of θ lies,

QII	QI
$\theta' = 180-\theta$ $\theta' = \theta - 180°$	$\theta' = \theta$
QIII	**QIV**
$\theta' = \theta - 180°$ $\theta' = 180 - \theta$	$\theta' = 360 - \theta$ $\theta' = -\theta$

Reminder: to use the rules in this table, the angle θ must be between 0° and 360° (or between 0 and 2π).

If this is not the case, then find a positive, co-terminal angle for θ between 0° and 360° to use the table.

is an element of

$n \in \mathbb{Z}$ ← set of all integers

$360n$ ← multiples of 360

$2\pi n$ ← multiples of 2π

1.08 Coterminal & Reference Angles

Directions: Complete #18 - 26 all

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle. (Example 3)

18. 120° $120° + 360°n \quad n \in \mathbb{Z}$

19. $-75°$ $-75 + n360° \quad n \in \mathbb{Z}$

20. 225° $225 + 360°n \quad n \in \mathbb{Z}$

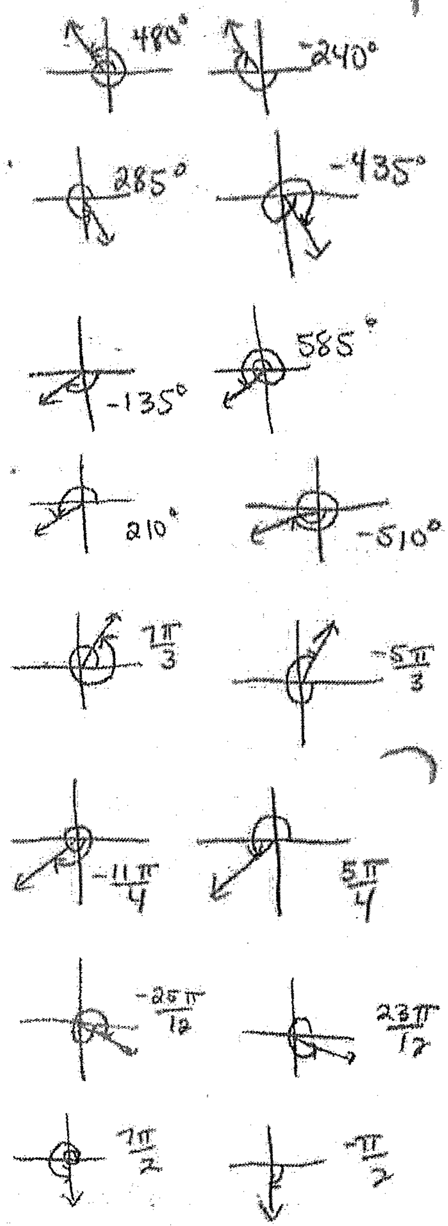
21. $-150°$ $-150° + n360° \quad n \in \mathbb{Z}$

22. $\frac{\pi}{3}$ $\frac{\pi}{3} + 2\pi n \quad n \in \mathbb{Z}$

23. $-\frac{3\pi}{4}$ $\frac{-3\pi}{4} + 2\pi n \quad n \in \mathbb{Z}$

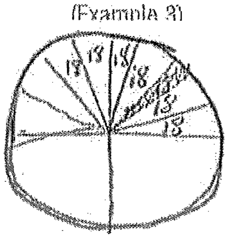
24. $-\frac{\pi}{12}$ $\frac{-\pi}{12} + 2\pi n \quad n \in \mathbb{Z}$

25. $\frac{3\pi}{2}$ $\frac{3\pi}{2} + 2\pi n \quad n \in \mathbb{Z}$



26. GAME SHOW Sofia is spinning a wheel on a game show. There are 20 values in equal-sized spaces around the circumference of the wheel. The value that Sofia needs to win is two spaces above the space where she starts her spin, and the wheel must make at least one full rotation for the spin to count. Describe a spin rotation in degrees that will give Sofia a winning result. (Example 3)

$\frac{360}{20} = 18$



$360 + 2(18) = 396°$

Directions: Complete #17 – 24 all

Sketch each angle. Then find its reference angle. (Example 3)

17. $135°$

18. $210°$

19. $\frac{7\pi}{12}$

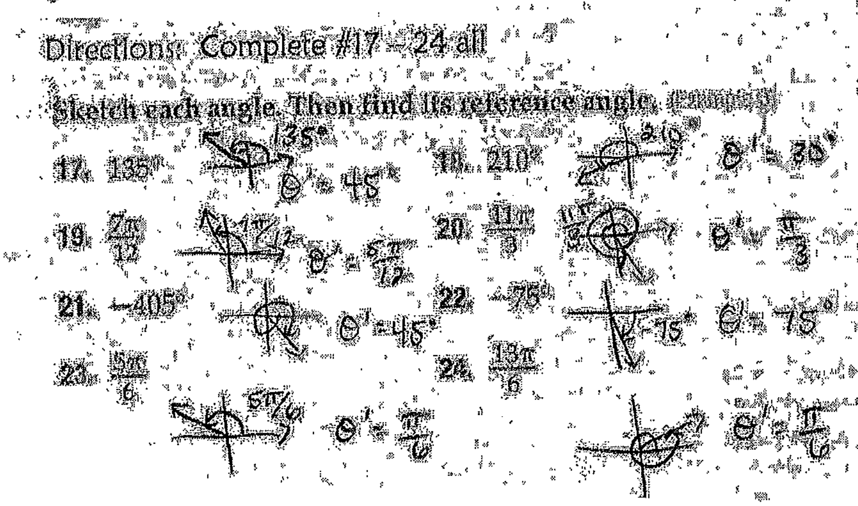
20. $\frac{11\pi}{3}$

21. $-405°$

22. $-75°$

23. $\frac{5\pi}{6}$

24. $\frac{13\pi}{6}$


Directions: Complete #17 – 24 all

Sketch each angle. Then find its reference angle. (Example 3)

17. $135°$ $\theta' = 45°$

18. $210°$ $\theta' = 30°$

19. $\frac{7\pi}{12}$ $\theta' = \frac{5\pi}{12}$

20. $\frac{11\pi}{3}$ $\theta' = \frac{\pi}{3}$

21. $-405°$ $\theta' = 45°$

22. $-75°$ $\theta' = 75°$

23. $\frac{5\pi}{6}$ $\theta' = \frac{\pi}{6}$

24. $\frac{13\pi}{6}$ $\theta' = \frac{\pi}{6}$

1.09 Coterminal and Reference Angles

Directions: Complete #17 – 24 all

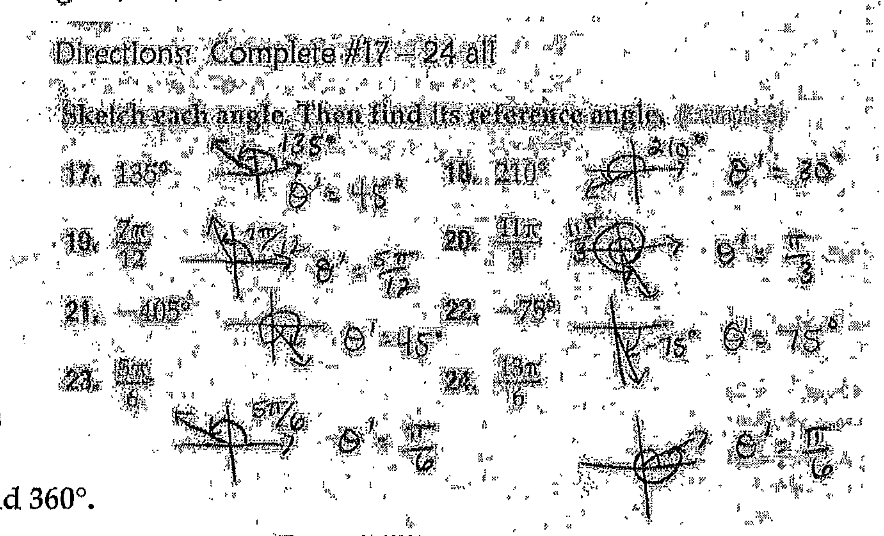
Sketch each angle. Then find its reference angle. (Example 3)

17. 135° **18.** 210°

19. $\frac{7\pi}{12}$ **20.** $\frac{11\pi}{3}$

21. −405° **22.** −75°

23. $\frac{5\pi}{6}$ **24.** $\frac{13\pi}{6}$



1.09 Coterminal and Reference Angles

Find a coterminal angle between 0° and 360°.

1) 885°

$$\begin{array}{r} 885 \\ -720 \\ \hline \boxed{165^\circ} \end{array}$$

2) -435°

$$\begin{array}{r} -435 \\ +720 \\ \hline \boxed{285^\circ} \end{array}$$

0 and 2π

Find a coterminal angle between 0 and 2p for each given angle.

3) $\frac{17\pi}{6}$ $\frac{17\pi}{6} - \frac{12\pi}{6} = \boxed{\frac{5\pi}{6}}$

4) $-\frac{\pi}{4}$ $-\frac{\pi}{4} + \frac{8\pi}{4} = \boxed{\frac{7\pi}{4}}$

Find a positive and a negative coterminal angle for each given angle.

5) 240° $240+360 = 600^\circ$

$240-360 = -120^\circ$

6) -166° $-166+360 = 194^\circ$

$-166-360 = -526^\circ$

7) $-\frac{5\pi}{2}$ $-\frac{5\pi}{2} + \frac{4\pi}{2} = -\frac{\pi}{2}$

$-\frac{5\pi}{2} + \frac{4\pi}{2} + \frac{4\pi}{2} = \frac{3\pi}{2}$

or $-\frac{5\pi}{2} - \frac{4\pi}{2} = -\frac{9\pi}{2}$

8) $\frac{17\pi}{12}$ $\frac{17\pi}{12} - \frac{24\pi}{12} = -\frac{7\pi}{12}$

$\frac{17\pi}{12} + \frac{24\pi}{12} = \frac{41\pi}{12}$

State if the given angles are coterminal.

9) 115°, 475°

$$\begin{array}{r} 475 \\ -360 \\ \hline 115^\circ \end{array}$$ ✓

$\boxed{\text{yes}}$

10) $\frac{5\pi}{6}, \frac{23\pi}{6}$

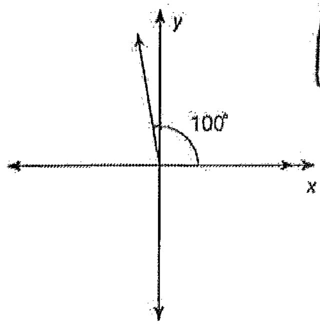
$\frac{23\pi}{6} - \frac{12\pi}{6} = \boxed{\frac{11\pi}{6}}$

$\boxed{\text{No}}$

$\frac{11\pi}{6} - \frac{12\pi}{6} = \boxed{-\frac{\pi}{6}}$

Find the reference angle. $0 < \theta' < 90°$ or $0 < \theta' < \pi/2$

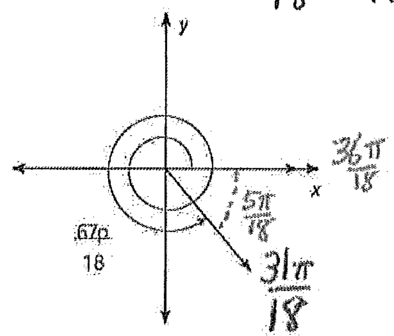
11)



$\boxed{\theta' = 80°}$

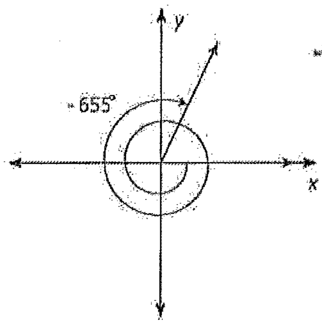
12)

$\frac{67\pi}{18} - \frac{36\pi}{18} = \frac{31\pi}{18}$



$\boxed{\theta' = \frac{5\pi}{18}}$

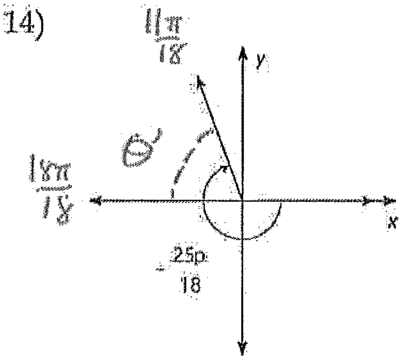
13)



$$\begin{array}{r} -655 \\ +720 \\ \hline \end{array}$$

$\boxed{\theta' = 65°}$

14)

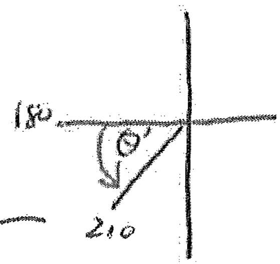


$-\frac{25\pi}{18} + \frac{36\pi}{18} = \frac{11\pi}{18}$

$\boxed{\theta' = \frac{7\pi}{18}}$

15) -510°

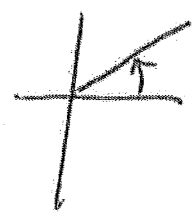
$$\begin{array}{r} -510 \\ +360 \\ +360 \\ \hline 210 \end{array}$$



$\boxed{\theta' = 30°}$

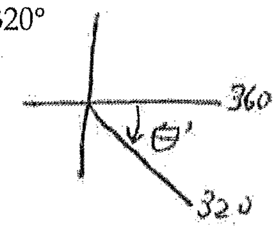
16) $\frac{19\pi}{9}$

$\frac{19\pi}{9} - \frac{18\pi}{9} = \frac{\pi}{9}$



$\boxed{\theta' = \pi/9}$

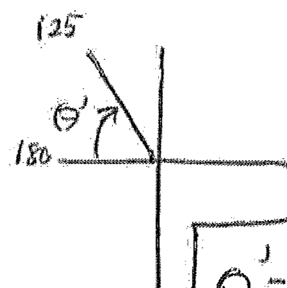
17) 320°



$\boxed{40° = \theta'}$

18) -595°

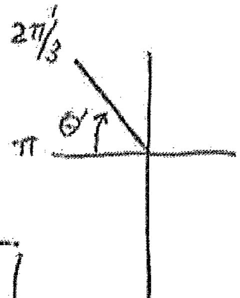
$$\begin{array}{r} -595 \\ +360 \\ \hline -235 \\ +360 \\ \hline 125° \end{array}$$



$\boxed{\theta' = 55°}$

19) $-\frac{4\pi}{3}$

$-\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{2\pi}{3}$

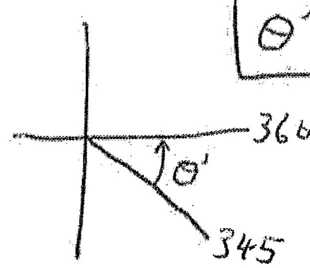


$\boxed{\theta' = \frac{\pi}{3}}$

20) 345°

345°

$\boxed{\theta' = 15°}$



1.16 Unit Circle Trigonometry Extension Worksheet

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

1. (1, -8)

2. (-8, 15)

State the quadrant or axis where the terminal side of θ is found.

3. $\sin\theta < 0$ and $\cos\theta < 0$ Q3

4. $\tan\theta > 0$ and $\sec\theta > 0$ Q1

5. $\cos\theta > 0$ and $\cot\theta < 0$ Q4

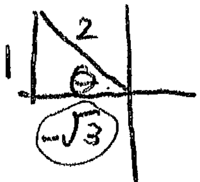
6. $\sec\theta < 0$ and $\sin\theta = 0$ negative x-axis
   cos < 0 x-axis

7. $\cos\theta = 0$ and $\csc\theta > 0$ positive y-axis

8. $\cot\theta < 0$ and $\cos\theta < 0$ Q2

First, state the quadrant or axis where the terminal side of θ is found. Then, find the exact value of the specified trigonometric function using the given information.

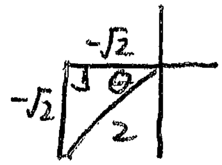
9. Find $\cos\theta$ if $\sin\theta = \frac{1}{2}$ and $\tan\theta < 0$.



Quadrant: 2

$\cos\theta = -\sqrt{3}/2$

10. Find $\tan\theta$ if $\cos\theta = -\frac{\sqrt{2}}{2}$ and $\sin\theta < 0$.



Quadrant: 3

$\tan\theta =$ 1

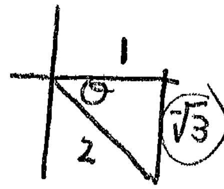
11. Find $\sin\theta$ if $\sec\theta$ is undefined and csc < 0.

(0, -1)

Quadrant: neg. y-axis

$\sin\theta =$ -1

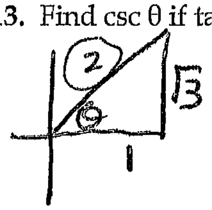
12. Find $\cot\theta$ if $\sec\theta = 2$ and $\csc\theta < 0$.



Quadrant: 4

$\cot\theta = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

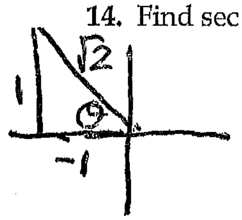
13. Find $\csc\theta$ if $\tan = \sqrt{3}$ and $\sec\theta > 0$



Quadrant: 1

$\csc\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

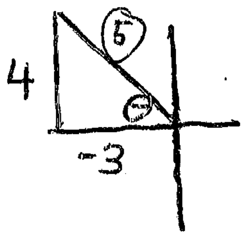
14. Find $\sec\theta$ if $\cot\theta = -1$ and $\sin\theta > 0$



Quadrant: 2

$\sec\theta = \frac{-\sqrt{2}}{1} = -\sqrt{2}$

15. Find $\sec\theta$ and $\csc\theta$ if $\tan\theta = -\frac{4}{3}$ and $\cos\theta < 0$.



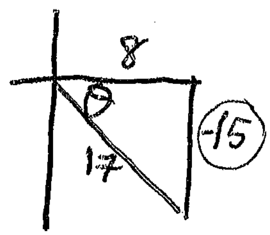
Quadrant: 2

$\sec\theta = 4/5$

$\csc\theta = 5/4$

16. Find csc x and cos x if $\sec\theta = \frac{17}{8}$ and $\sin\theta < 0$.

Cos(+)
Sin(-)

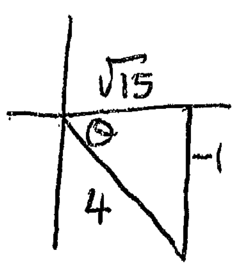


Quadrant: 4

$\csc\theta = \frac{17}{-15}$

$\cos\theta = 8/17$

17. Find $\cos\theta$ and $\cot\theta$ if $\sin\theta = -\frac{1}{4}$ and $\tan\theta < 0$.

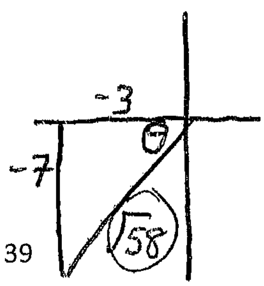


Quadrant: 4

$\cos\theta = \sqrt{15}/4$

$\cot\theta = -\sqrt{15}$

18. Find $\sin\theta$ and $\cos\theta$ if $\cot\theta = \frac{3}{7}$ and $\sec\theta < 0$.



Quadrant: 3

$\sin\theta = -7/\sqrt{58}$ $\quad \frac{-7\sqrt{58}}{58}$

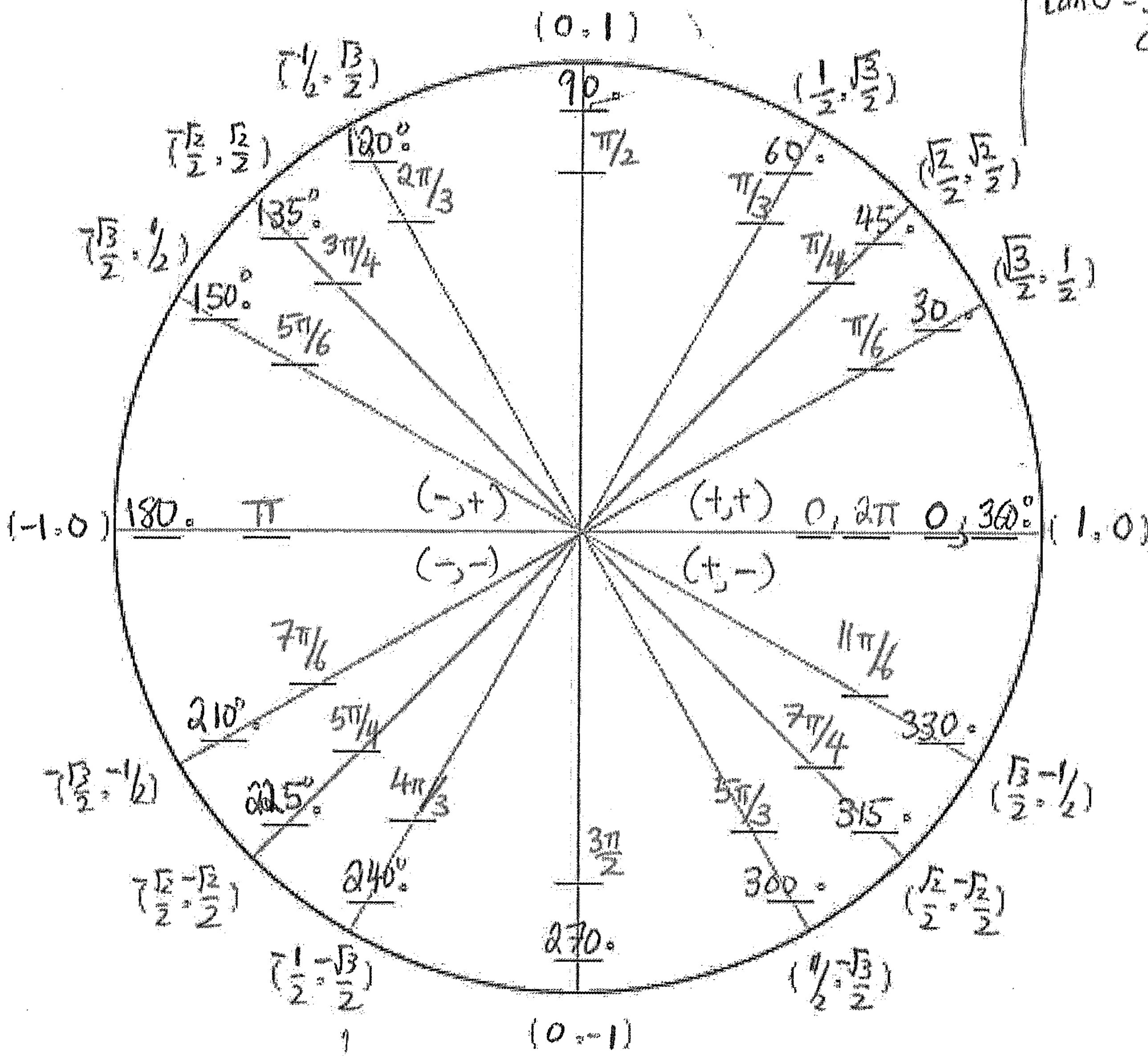
$\cos\theta = \frac{-3}{\sqrt{58}}$ $\quad \frac{-3\sqrt{58}}{58}$

(Radius = 1)

1.12 HW: Try filling out the unit circle

$$(\underset{\cos\theta}{x}, \underset{\sin\theta}{y})$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$



$$\cos\theta = x$$

$$\sin\theta = y$$

$$\tan\theta = \frac{y}{x} \rightarrow \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{1}{x} \rightarrow \frac{1}{\cos\theta}$$

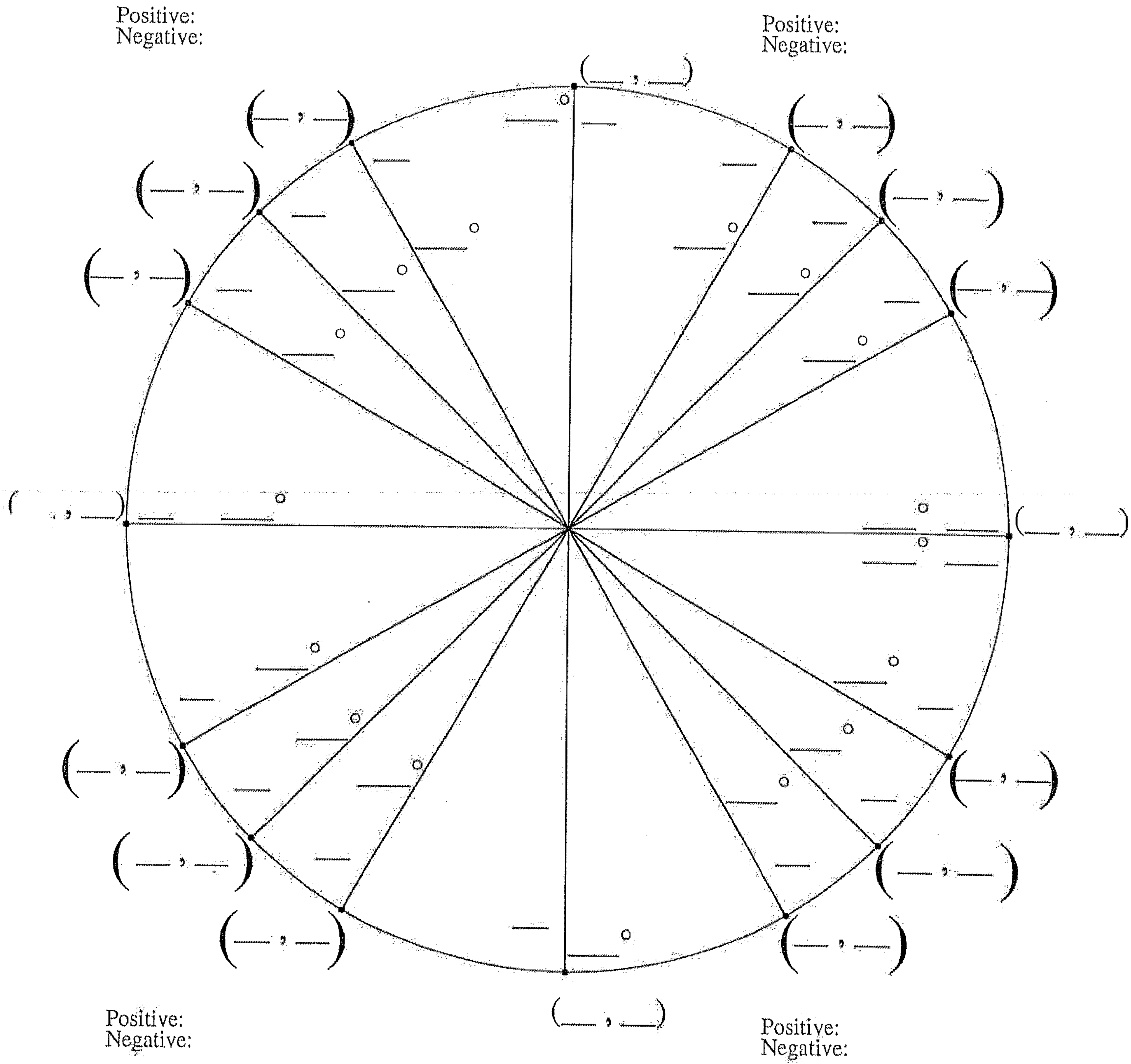
$$\csc\theta = \frac{1}{y} \rightarrow \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{x}{y} \rightarrow \frac{\cos\theta}{\sin\theta}$$

The Unit Circle Table of Values

			x	y	y/x	flip cos	flip sin	flip tan
Degree	Radian	Coordinates	cos θ	sin θ	tan θ	sec θ	csc θ	cot θ
0°	0	$(1,0)$	1	0	0	1	und	und
30°	$\pi/6$	$(\frac{\sqrt{3}}{2}, \frac{1}{2})$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	2	$\sqrt{3}$
45°	$\pi/4$	$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\pi/3$	$(\frac{1}{2}, \frac{\sqrt{3}}{2})$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	2	$\frac{2\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$
90°	$\pi/2$	$(0,1)$	0	1	und	und	1	0
120°	$2\pi/3$	$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$
135°	$3\pi/4$	$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
150°	$5\pi/6$	$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\sqrt{3}$
180°	π	$(-1,0)$	-1	0	0	-1	und	und
210°	$7\pi/6$	$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	-2	$\sqrt{3}$
225°	$5\pi/4$	$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240°	$4\pi/3$	$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\sqrt{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$
270°	$3\pi/2$	$(0,-1)$	0	-1	und	und	1	0
300°	$5\pi/3$	$(\frac{1}{2}, -\frac{\sqrt{3}}{2})$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	2	$-\frac{2\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$
315°	$7\pi/4$	$(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
330°	$11\pi/6$	$(\frac{\sqrt{3}}{2}, -\frac{1}{2})$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\sqrt{3}$
360°	2π	$(1,0)$	1	0	0	1	und	und

Fill in The Unit Circle



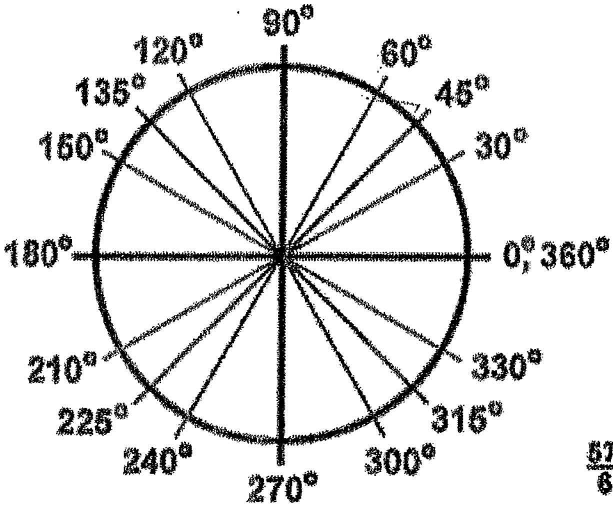
EmbeddedMath.com

The Unit Circle Table of Values

Degree	Radian	Coordinates	$\cos\theta$	$\sin\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
0°								
30°								
45°								
60°								
90°								
120°								
135°								
150°								
180°								
210°								
225°								
240°								
270°								
300°								
315°								
330°								
360°								

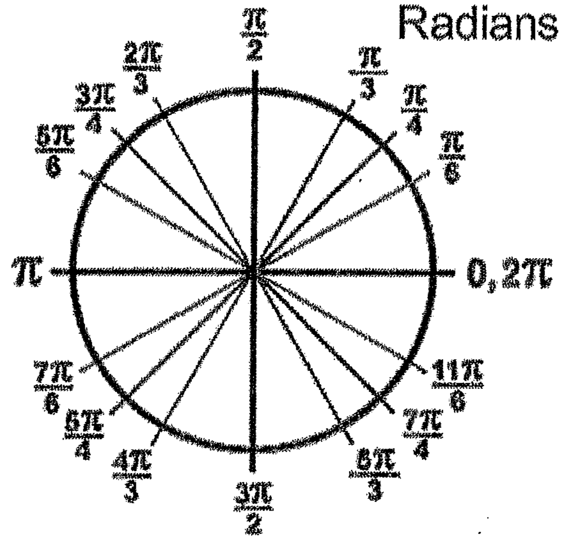
1.12 Intro to Unit Circle

Date: ____________________

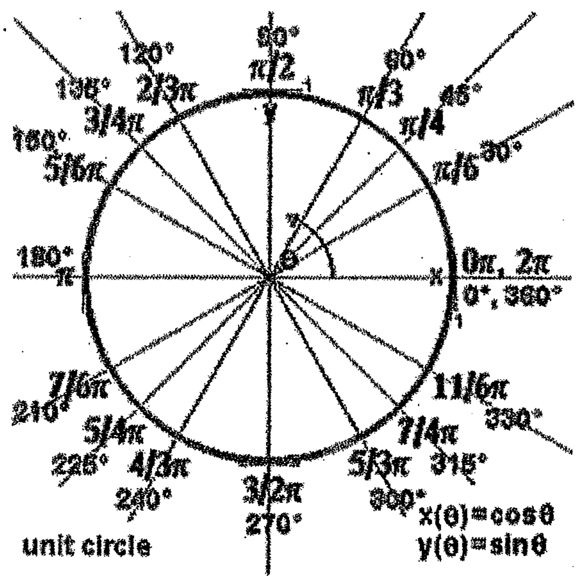


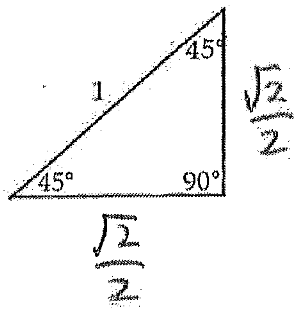
Degrees

Radians



Degrees and Radians Together

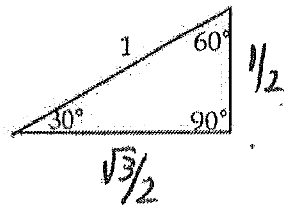




$\sin 45° = \frac{\sqrt{2}}{2}$

$\cos 45° = \sqrt{2}/2$

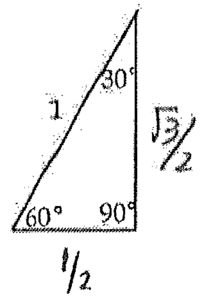
$\tan 45° = 1$



$\sin 30° = 1/2$

$\cos 30° = \sqrt{3}/2$

$\tan 30° = \frac{1}{\sqrt{3}} \rightarrow \frac{\sqrt{3}}{3}$

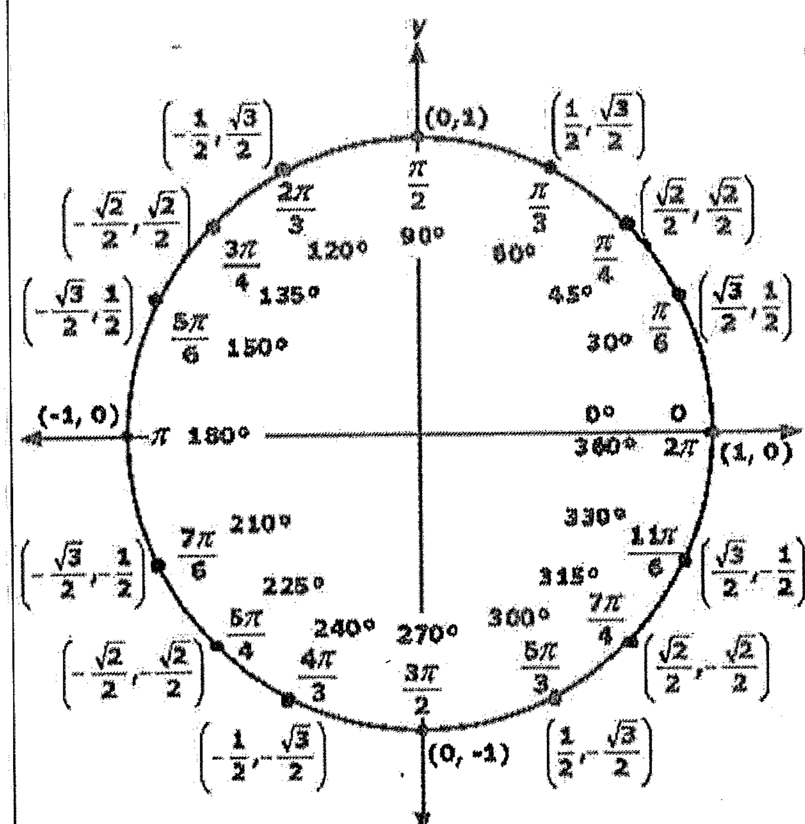


$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

Unit Circle



coordinates = (x,y)

$\cos\theta = x$
$\sin\theta = y$
$\tan\theta = y/x$

$\sec\theta = 1/x$
$\csc\theta = 1/y$
$\cot\theta = x/y$

Reciprocals

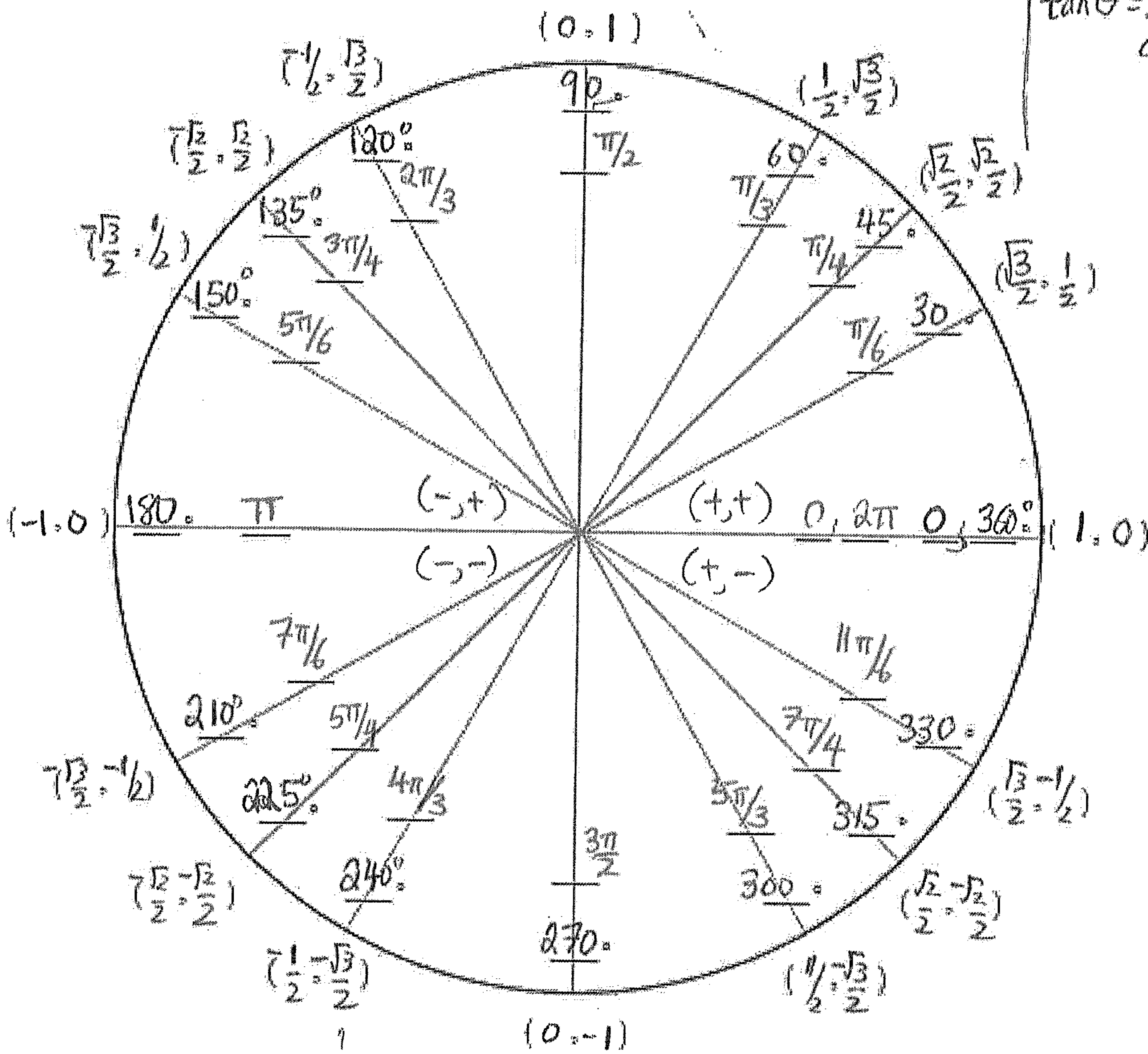
cosine	secant
sine	cosecant
tangent	cotangent

(Radius = 1)

1.12 HW: Try filling out the unit circle

$$\left(\underset{\cos\theta}{x},\ \underset{\sin\theta}{y}\right)$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$



$\cos\theta = x$

$\sin\theta = y$

$\tan\theta = \frac{y}{x} \rightarrow \frac{\sin\theta}{\cos\theta}$

$\sec\theta = \frac{1}{x} \rightarrow \frac{1}{\cos\theta}$

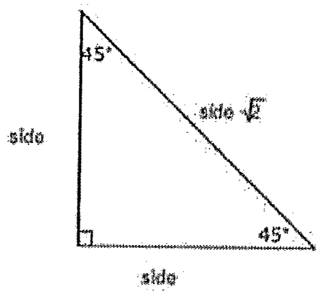
$\csc\theta = \frac{1}{y} \rightarrow \frac{1}{\sin\theta}$

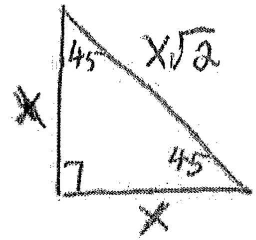
$\cot\theta = \frac{x}{y} \rightarrow \frac{\cos\theta}{\sin\theta}$

Name: ______________

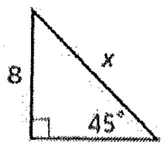
You may remember special right triangles from Geometry. Here's a refresher in case you don't. ☺

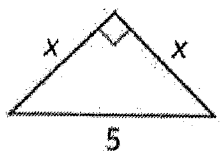
45°- 45°- 90° Triangle



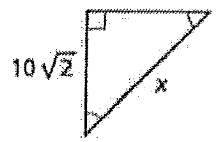


$x = 8\sqrt{2}$





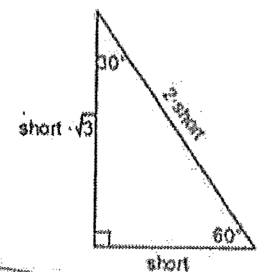
$x = \frac{5}{\sqrt{2}} = \boxed{\frac{5\sqrt{2}}{2}}$

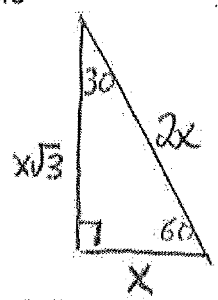


$x = 10\sqrt{2} \cdot \sqrt{2}$

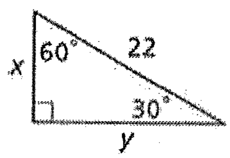
$\boxed{x = 20}$

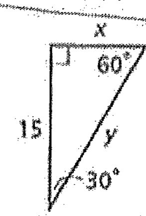
30°- 60°- 90° Triangle





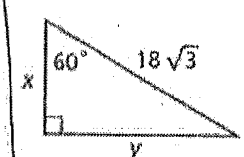
$\boxed{\begin{array}{l} x = 11 \\ y = 11\sqrt{3} \end{array}}$





$x = \frac{15}{\sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3}$

$y = 10\sqrt{3}$

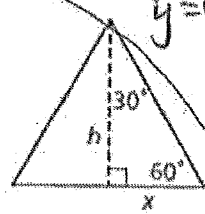


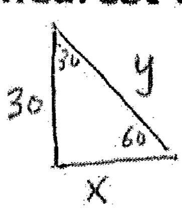
$x = 9\sqrt{3}$

$y = 9\sqrt{3} \cdot \sqrt{3}$

$y = 27$

A manufacturer wants to make a larger clock with a height of 30 centimeters. What is the length of each side of the frame? Round to the nearest tenth.



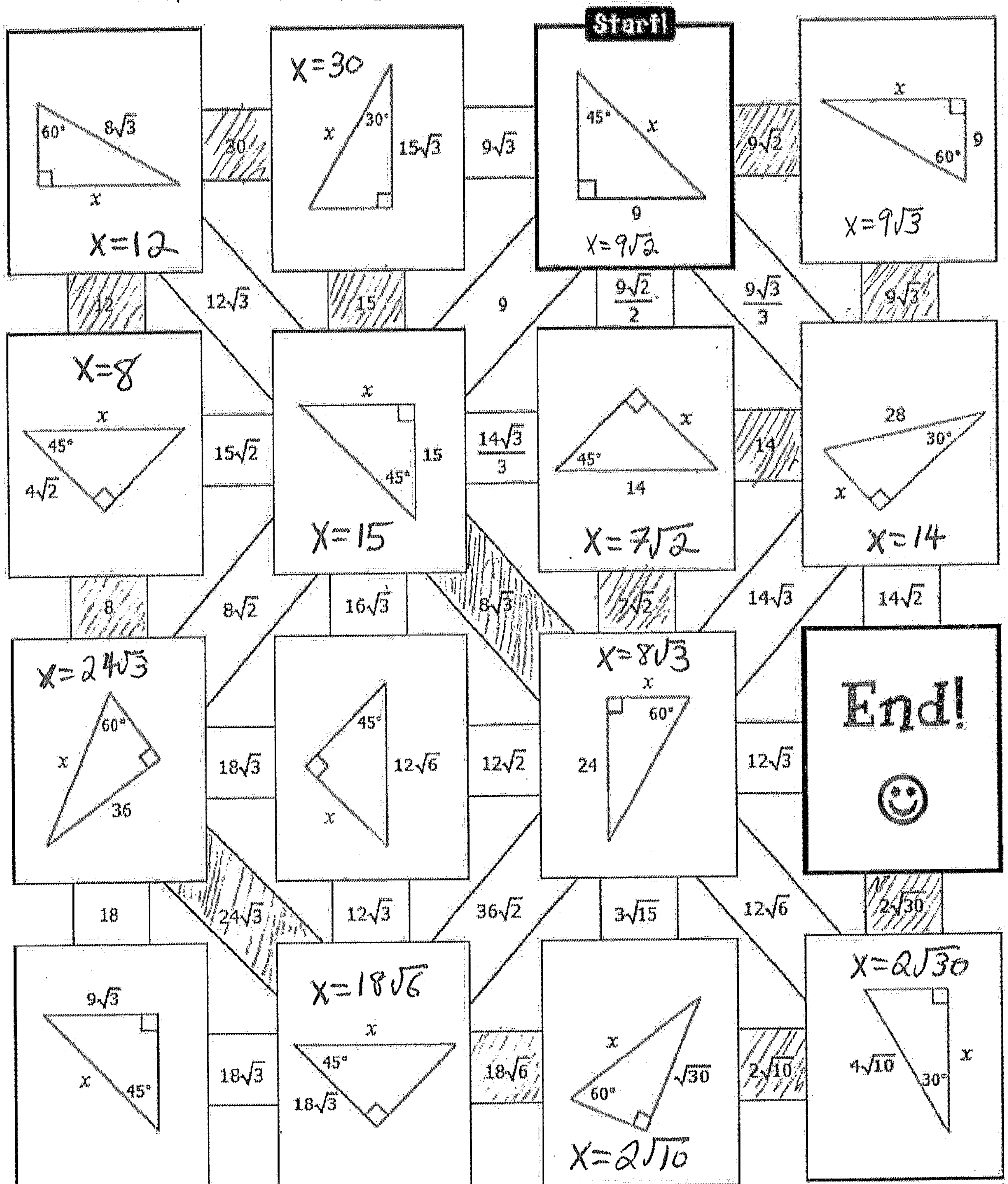


$x = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$

$\boxed{y = 20\sqrt{3} \text{ is the length of each side of frame.}}$

Special Right Triangles!

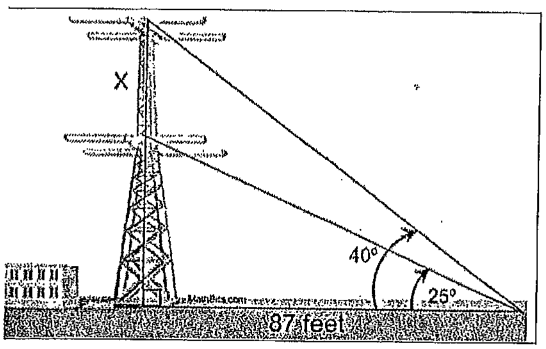
Directions: Find each missing side. Write all answers in simplest radical form. Use your solutions to navigate through the maze. **Staple all work to this paper!**



Version 1: Simplest Radical Form

Accel Pre-Calc: Trig Word Problem Practice WS (Double Triangles)

A radio station tower was built in two sections. From a point 87 feet from the base of the tower, the angle of elevation of the top of the first section is 25°, and the angle of elevation of the top of the second section is 40°. To the *nearest foot*, what is the height of the top section of the tower?



2) You are standing on a plateau that is 800 feet above a basin where you see two hikers. If the angle of depression to the hikers is 25° and 15°, how far apart are the two hikers, rounded to the nearest hundredth of a foot?

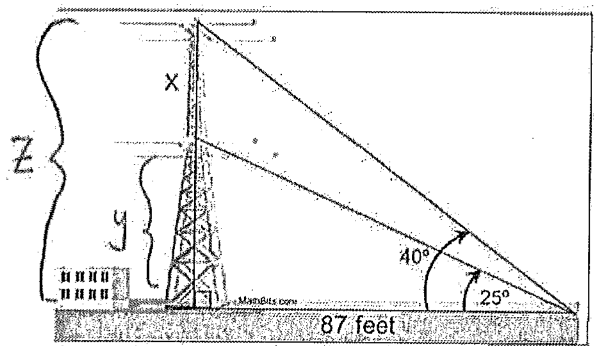
3) An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. ($\sqrt{3} = 1.732$)

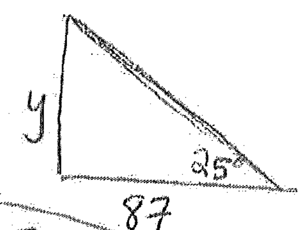
4) Brad is standing on a 40-foot ocean bluff. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?

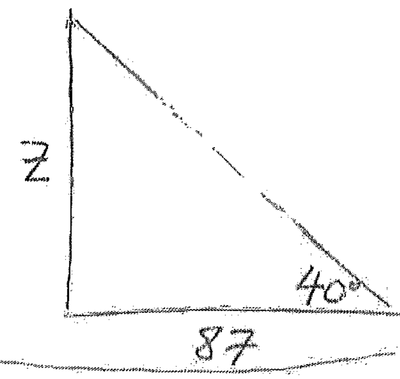
Accel Pre-Calc: Trig Word Problem Practice WS (Double Triangles)

Key

1. A radio station tower was built in two sections. From a point 87 feet from the base of the tower, the angle of elevation of the top of the first section is 25°, and the angle of elevation of the top of the second section is 40°. To the *nearest foot*, what is the height of the top section of the tower?







$\tan(25) = \frac{y}{87}$

$y = 87\tan(25)$

$\underline{y = 40.569}$

$\tan(40) = \frac{z}{87}$

$z = 87\tan 40$

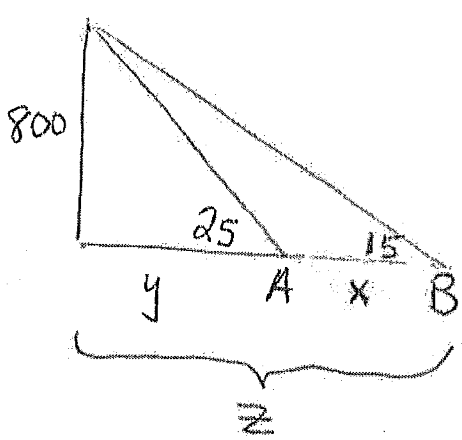
$\underline{z = 73.002}$

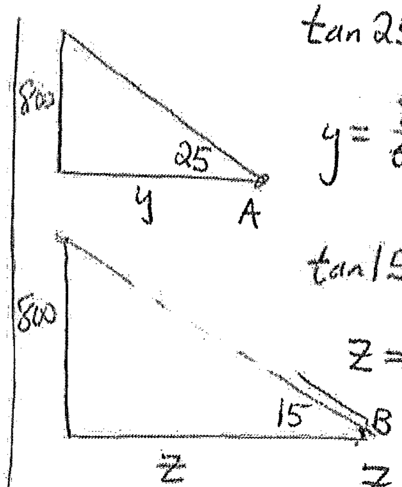
$x = z - y$

$x = 73.002 - 40.569$

$x = 32.432 \approx \boxed{32 \text{ feet}}$

2) You are standing on a plateau that is 800 feet above a basin where you see two hikers. If the angle of depression to the hikers is 25° and 15°, how far apart are the two hikers, rounded to the nearest hundredth of a foot?





$\tan 25 = \frac{800}{y}$

$y = \frac{800}{\tan 25} = \underline{1715.606}$

$\tan 15 = \frac{800}{z}$

$z = \frac{800}{\tan 15}$

$z = \underline{2985.641}$

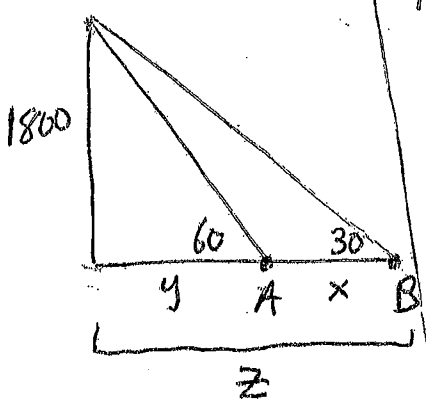
distance x:

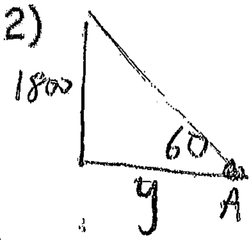
$x = z - y$

$x = 2985.641 - 1715.606$

$\boxed{x = 1270.035}$ ft.

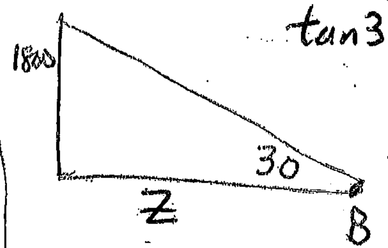
3) An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. ($\sqrt{3} = 1.732$)





$\tan 60 = \frac{1800}{y}$

$y = \frac{1800}{\tan 60} = 1039.231$



$\tan 30 = \frac{1800}{z}$

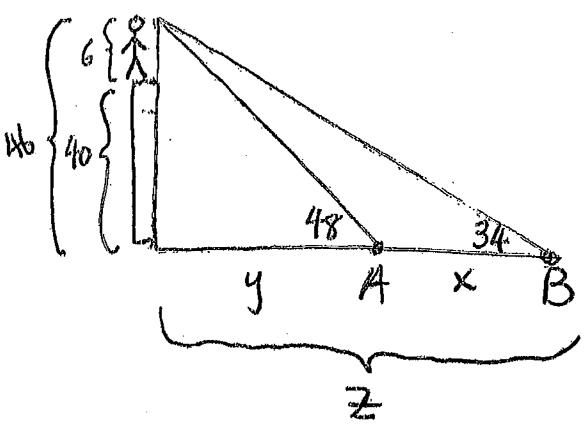
$z = \frac{1800}{\tan 30} = 3117.691$

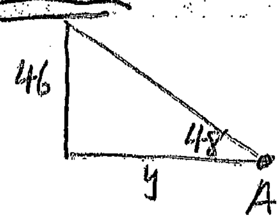
$x = z - y$

$x = 3117.691 - 1039.231$

$\boxed{x = 2078.46 \text{ meters}}$

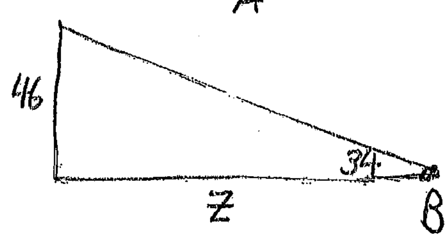
4) Brad is standing on a 40-foot ocean bluff. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?





$\tan 48 = \frac{46}{y}$

$y = \frac{46}{\tan 48} = 41.419$



$\tan 34 = \frac{46}{z}$

$z = \frac{46}{\tan 34} = 68.198$

$x = z - y$

$x = 68.198 - 41.419 = 26.779 \approx \boxed{27 \text{ ft}}$