Introduction to Matrices

Matrix: A rectangular array of numbers

Uses: to solve systems of equations (next week) and to record data (think Excel)

Example: $A = \begin{array}{c} \\ \left[\begin{array}{ccc} 5 & 0 & -1 \\ -3 & 1 & 9 \end{array}\right] \\ \text{Column 1} \quad \text{Column 2} \quad \text{Column 3} \end{array} \begin{array}{l} \text{Row 1} \\ \text{Row 2} \end{array}$

Dimensions size of a matrix: measured as rows × columns

$\boxed{\text{Ex: } 2 \times 3 \text{ matrix}}$

Elements values in a matrix, locations are described as rows, columns

$[A] \quad a_{13} \rightarrow \boxed{-1}$

Row 1 column 3

More examples:

$B = \begin{bmatrix} -2 \\ 4 \\ 0 \\ 7 \end{bmatrix}$

B is known as a "column matrix".
What are the dimensions of matrix B?

4×1 matrix

4 Rows 1 column

How about a "row matrix"?
Create a 1x3 matrix C.

$C = \begin{bmatrix} 4 & 6 & 9 \end{bmatrix}$

How about a "square matrix"?
Create a 4x4 matrix D.

$D = \begin{bmatrix} 3 & 4 & -1 & 5 \\ 0 & 4 & 6 & 9 \\ 10 & -3 & 0 & 6 \\ -1 & 5 & 7 & 8 \end{bmatrix}$

Example $\rightarrow 2T = 2\begin{bmatrix} -5 & -4 & 1 \\ 7 & -8 & 4 \end{bmatrix} = \begin{bmatrix} -10 & -8 & 2 \\ 14 & -16 & 8 \end{bmatrix}$

Operations

Addition and subtraction can only be done with matrices of the same size (dimensions).
Scalar multiplication can be done with a matrix of any size.

$$S = \begin{bmatrix} 2 & -1 & 3 \\ -4 & -2 & -8 \end{bmatrix} \quad T = \begin{bmatrix} -5 & -4 & 1 \\ 7 & -8 & 4 \end{bmatrix} \quad U = \begin{bmatrix} 9 & -2 \\ -3 & 0 \end{bmatrix}$$

Evaluate each:

S + U = not possible

S - T = $\begin{bmatrix} 2 & -1 & 3 \\ -4 & -2 & -8 \end{bmatrix} - \begin{bmatrix} -5 & -4 & 1 \\ 7 & -8 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 7 & 3 & 2 \\ -11 & 6 & -12 \end{bmatrix}}$

2T = $2\begin{bmatrix} -5 & -4 & 1 \\ 7 & -8 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} -10 & -8 & 2 \\ 14 & -16 & 8 \end{bmatrix}}$

½U = $\frac{1}{2}\begin{bmatrix} 9 & -2 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 9/2 & -2/2 \\ -3/2 & 0 \end{bmatrix}$ or $\boxed{\begin{bmatrix} 9/2 & -1 \\ -3/2 & 0 \end{bmatrix}}$

Use matrices A, B, and C to solve for X.

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 7 \end{bmatrix} \quad B = \begin{bmatrix} -4 \\ 12 \end{bmatrix} \quad C = \begin{bmatrix} -2 & 8 \\ 5 & -6 \end{bmatrix}$$

X = 2B

$X = 2\begin{bmatrix} -4 \\ 12 \end{bmatrix}$

$\boxed{X = \begin{bmatrix} -8 \\ 24 \end{bmatrix}}$

A + X = C

$X = C - A$

$X = \begin{bmatrix} -2 & 8 \\ 5 & -6 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ -1 & 7 \end{bmatrix}$

$\boxed{X = \begin{bmatrix} -5 & 8 \\ 6 & -13 \end{bmatrix}}$

5.01 Practice

State the dimensions of each matrix.

1. $\begin{bmatrix} 1 & -8 \\ 6 & -2 \end{bmatrix}$

2. $\begin{bmatrix} -9 & -8 \\ 2 & 17 \\ 11 & -6 \end{bmatrix}$

1) 2×2 2) 3 × 2

Find the value of each element in $A = \begin{bmatrix} -3 & 45 & 28 \\ 24 & 36 & -22 \\ -15 & 4 & 29 \end{bmatrix}$

3) $a_{22} = \boxed{36}$

Row 2
Column 2

4. $a_{31} = \boxed{-15}$

Row 3
Column 1

Find each of the following for $Q = \begin{bmatrix} 13 & -6 \\ 2 & -10 \\ -4 & 8 \end{bmatrix}$, $R = \begin{bmatrix} 1 & -3 \\ -5 & 9 \\ 12 & 7 \end{bmatrix}$, $S = \begin{bmatrix} 5 & -2 & 1 \\ -6 & 14 & 8 \end{bmatrix}$, and $T = \begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix}$.

If the matrix does not exist, write *impossible.*

5. Q + R

$\begin{bmatrix} 14 & -9 \\ -3 & -1 \\ 8 & 15 \end{bmatrix}$

6. $T - R$

not possible

7. $T - S$

$\begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -2 & 1 \\ -6 & 14 & 8 \end{bmatrix}$

$\boxed{\begin{bmatrix} -16 & 5 & 6 \\ 10 & -23 & 8 \end{bmatrix}}$

8. $2R + Q$

$\begin{bmatrix} 2 & -6 \\ -10 & 18 \\ 24 & 14 \end{bmatrix} + \begin{bmatrix} 13 & -6 \\ 2 & -10 \\ -4 & 8 \end{bmatrix} = \boxed{\begin{bmatrix} 15 & -12 \\ -8 & 8 \\ 20 & 22 \end{bmatrix}}$

9. $\frac{1}{2}(T + S)$

$\frac{1}{2}\begin{bmatrix} -6 & 1 & 8 \\ -2 & 5 & 24 \end{bmatrix} = \begin{bmatrix} -3 & 1/2 & 4 \\ -1 & 5/2 & 12 \end{bmatrix}$

Use matrices Q, R, S, and T to solve for X. If the matrix does not exist, write *impossible.*

10. $\left(Q - R = \frac{1}{3}X\right)3$

$3(Q-R) = X$

$3\begin{bmatrix} 12 & -3 \\ 7 & -19 \\ -16 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 36 & -9 \\ 21 & -57 \\ -48 & 3 \end{bmatrix}}$

11. $3S - X = T$

$3S - T = X$

$3\begin{bmatrix} 5 & -2 & 1 \\ -6 & 14 & 8 \end{bmatrix} - \begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix}$

$\begin{bmatrix} 15 & -6 & 3 \\ -18 & 42 & 24 \end{bmatrix} - \begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix}$

$\boxed{\begin{bmatrix} 26 & -9 & -4 \\ -22 & 51 & 8 \end{bmatrix}}$

12. $\left(2(Q - X) = -T\right)\frac{1}{2}$

$Q - X = -\frac{1}{2}T$

$Q + \frac{1}{2}T = X$

not possible
(dimensions are not the same)

13. Jessica took her two children to the community swimming pool once a week for six weeks. The daily admission fees are \$4.50 for a child and \$6.75 for an adult. Write a 1 x 3 matrix with a scalar multiple that represents the total cost of admission. What is the total cost?

$6\begin{bmatrix} 4.50 & 4.50 & 6.75 \end{bmatrix} \rightarrow \begin{bmatrix} 27 & 27 & 40.50 \end{bmatrix}$

Total cost is \$94.50

5.02 Matrix Multiplication Notes

Scenario: Catherine and Meg spend their spare time working at Chick-Fil-A, babysitting, and working on homework. Chick-Fil-A pays them each \$8.00/hour, they make \$10/hour babysitting, and their homework will pay them in dividends when they're older (translation: they don't get paid). When working at Chick-Fil-A, they can't use their phones at all. While babysitting, they can send roughly 30 texts/hour. While working on homework, they can send about 40 texts/hour. Create hours/schedules for each student. How can we set up a matrix multiplication problem to calculate the pay and texts/hour for each activity for both Catherine and Meg?

$$\begin{array}{c|c} & \begin{array}{ccc} \text{CFA} & \text{babysitting} & \text{HW} \end{array} \\ \begin{array}{c} \text{Catherine} \\ \text{Meg} \end{array} & \begin{bmatrix} 10 & 15 & 4 \\ 20 & 2 & 8 \end{bmatrix} \end{array} \quad \begin{array}{c|c} & \begin{array}{cc} \text{pay} & \text{text} \end{array} \\ \begin{array}{c} \text{CFA} \\ \text{babysit} \\ \text{HW} \end{array} & \begin{bmatrix} 8 & 0 \\ 10 & 30 \\ 0 & 40 \end{bmatrix} \end{array}$$

$\boxed{2} \times 3 \qquad 3 \times \boxed{2}$

	pay	text
Catherine	$10(8)+15(10)+4(0)$	$10(0)+15(30)+4(40)$
Meg	$20(8)+2(10)+8(0)$	$20(0)+2(30)+8(40)$

$$\boxed{\begin{array}{c|c} & \begin{array}{cc} \text{pay} & \text{text} \end{array} \\ \begin{array}{c} \text{Cath} \\ \text{Meg} \end{array} & \begin{bmatrix} \$230 & 610 \\ \$180 & 380 \end{bmatrix} \end{array}}$$ ← Answer

To multiply matrices, multiply every element from one row in the 1st matrix by every element from one column in the 2nd matrix and find the sum of the products. This gives you the element in that row and column in the answer matrix. (It's easier than it sounds! Just watch)

Examples: $\begin{bmatrix} 4 & 8 \\ 0 & 2 \\ 1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 \\ 9 & 4 \end{bmatrix} =$

$\boxed{3} \times \underbrace{2 \quad 2}_{\checkmark} \times \boxed{2}$

$$\begin{bmatrix} 4 & 8 \\ 0 & 2 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 9 & 4 \end{bmatrix} \qquad \begin{bmatrix} 4(5)+8(9) & 4(2)+8(4) \\ 0(5)+2(9) & 0(2)+2(4) \\ 1(5)+6(9) & 1(2)+6(4) \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} 92 & 40 \\ 18 & 8 \\ 59 & 26 \end{bmatrix}}$$

$[-2 \quad 8 \quad 4] \cdot \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 5 & -6 \end{bmatrix} =$

$\boxed{1} \times \underbrace{3 \quad 3}_{\checkmark} \times \boxed{2}$

1 Row   2 columns

$$\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 5 & -6 \end{bmatrix}$$

$$[-2 \quad 8 \quad 4] \quad -2(3)+8(-1)+4(5) \qquad -2(4)+8(2)+4(-6)$$

$$\boxed{[6 \quad -16]}$$

5.02 Notes continued

$$\begin{bmatrix}6&3&0\\2&5&1\\9&8&6\end{bmatrix}\cdot\begin{bmatrix}7&4\\6&7\\5&0\end{bmatrix}=$$

$\underbrace{3\times 3 \quad 3}_{\checkmark}\times 2$

$$\begin{array}{c|c} & \begin{bmatrix}7&4\\6&7\\5&0\end{bmatrix} \\ \hline \begin{bmatrix}6&3&0\\2&5&1\\9&8&6\end{bmatrix} & \begin{matrix}6(7)+3(6)+0(5) & 6(4)+3(7)+0(0)\\ 2(7)+5(6)+1(5) & 2(4)+5(7)+1(0)\\ 9(7)+6(8)+6(5) & 9(4)+8(7)+6(0)\end{matrix}\end{array}$$

$$\begin{matrix}42+18+0 & 24+21+0\\ 14+30+5 & 8+35+0\\ 63+48+30 & 36+56+0\end{matrix} = \boxed{\begin{bmatrix}60&45\\49&43\\141&92\end{bmatrix}}$$

Matrix Multiplication and Dimensions:

What happens if we switch the order of the matrices?

3×3 and 2×3

* may not be possible if the inner dimensions do not match.

What happens if we switch the dimensions of one matrix?

* may become not possible

* may change answer (dimensions may change)

So what has to be true for matrix multiplication to work?

$\underbrace{3\times 3 \quad 3}_{\checkmark}\times 4$

\# of columns from the first matrix must equal the \# of rows from the 2nd matrix

(inner dimensions of the 2 matrices must be the same)

Does matrix multiplication result in the same product when the order is reversed?

Not the same product if order is switched

$$\begin{bmatrix}2&-3\\-5&1\end{bmatrix}\begin{bmatrix}6&-2\\1&-1\end{bmatrix}=$$

$$\begin{bmatrix}6&-2\\1&-1\end{bmatrix}\begin{bmatrix}2&-3\\-5&1\end{bmatrix}=$$

$$\begin{array}{c|c} & \begin{bmatrix}6&-2\\1&-1\end{bmatrix} \\ \hline \begin{bmatrix}2&-3\\-5&1\end{bmatrix} & \begin{matrix}2(6)+(-3)(1) & 2(-2)+(-3)(-1)\\ -5(6)+1(1) & -5(-2)+1(-1)\end{matrix}\end{array}$$

$$\begin{array}{c|c} & \begin{bmatrix}2&-3\\-5&1\end{bmatrix} \\ \hline \begin{bmatrix}6&-2\\1&-1\end{bmatrix} & \begin{matrix}6(2)+(-2)(-5) & 6(-3)+(-2)(1)\\ 1(2)+(-5)(-1) & 1(-3)+(-1)(1)\end{matrix}\end{array}$$

$$\begin{bmatrix}9&-1\\-29&9\end{bmatrix} \neq \begin{bmatrix}22&-20\\7&-4\end{bmatrix}$$

5.02 Practice

Find AB and BA, if possible.

1. $A = \begin{bmatrix} 8 & 1 \end{bmatrix}; B = \begin{bmatrix} 3 & -7 \\ -5 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & 9 \\ -7 & 3 \end{bmatrix}; B = \begin{bmatrix} 6 & -4 \\ 0 & 3 \end{bmatrix}$

3. $A = \begin{bmatrix} 3 & -5 \end{bmatrix}; B = \begin{bmatrix} 4 & 0 & -2 \\ 1 & -3 & 2 \end{bmatrix}$

4. $A = \begin{bmatrix} 4 \\ 5 \end{bmatrix}; B = \begin{bmatrix} 6 & 1 & -10 & 9 \end{bmatrix}$

5. $A = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix}; B = \begin{bmatrix} 6 & 0 & -1 \\ -4 & 9 & 8 \end{bmatrix}$

6. $A = \begin{bmatrix} 2 & 0 \\ -4 & -3 \\ 1 & -2 \end{bmatrix}; B = \begin{bmatrix} 0 & 6 & -5 \\ 2 & -7 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 12 & -10 \\ -6 & -3 & 17 \\ -4 & 20 & -7 \end{bmatrix}$

7. Different point values are awarded for different shots in basketball. Use the information to determine the total points scored by each player. $\boxed{3} \times 3$ and $3 \times \boxed{1}$

Player	Free Throw	2-pointer	3-pointer
Rey	44	32	25
Chris	37	24	31
Jerry	35	39	29

Shots	Points
Free Throw	1
2-pointer	2
3-pointer	3

Points

Rey $44(1) + 32(2) + 25(3)$

Chris $37(1) + 24(2) + 31(3)$

Jerry $35(1) + 39(2) + 29(3)$

$\longrightarrow$

$$\begin{array}{c|c} & \text{Points} \\ \text{Ray} & \\ \text{Chris} & \begin{bmatrix} 183 \\ 178 \\ 200 \end{bmatrix} \\ \text{Jerry} & \end{array}$$

9b

5.02b Matrix Multiplication Extra Practice

State the dimensions of each matrix and, if multiplication is possible, give the dimensions of the answer.

1. $\begin{bmatrix} 8 & -7 & 0 \\ 3 & 4 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -2 \\ 1 \end{bmatrix} = ?$

   A · B = AB

   A = ______________

   B = ______________

   AB = ______________

2. $\begin{bmatrix} 6 & -2 \\ 0 & -9 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -7 \\ 8 \end{bmatrix} = ?$

   A · B = AB

   A = ______________

   B = ______________

   AB = ______________

3. $\begin{bmatrix} 11 \\ -3 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 7 & 2 & -5 \end{bmatrix} = ?$

   A · B = AB

   A = ______________

   B = ______________

   AB = ______________

Evaluate.

4. $\begin{bmatrix} -6 & -2 \\ -4 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -6 \end{bmatrix}$

5. $\begin{bmatrix} -2 & 6 \\ 3 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 4 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 4 & 2 \\ -1 & 3 & -4 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 4 \\ -1 \end{bmatrix}$

7. $\begin{bmatrix} 6 & -5 \\ 4 & -1 \\ 0 & -5 \end{bmatrix} \cdot \begin{bmatrix} -2 & 6 \\ 4 & 5 \\ 5 & 6 \end{bmatrix}$

8. $\begin{bmatrix} -3 & -1 \\ 0 & -1 \\ -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & -4 \\ 6 & 3 \end{bmatrix}$

9. $\begin{bmatrix} -5 & 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 4 & -1 \\ -5 & -6 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 0 \\ 0 & 5 & 5 \end{bmatrix} \right)$

Use matrix multiplication to determine if the matrices are inverses.

10. $A = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$ $and\ B = \begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$

11. $A = \begin{bmatrix} 5 & 11 \\ -4 & -9 \end{bmatrix}$ $and\ B = \begin{bmatrix} 9 & 11 \\ -4 & -5 \end{bmatrix}$

Key 9c

5.02 Matrix Multiplication Extra Practice

State the dimensions of each matrix and, if multiplication is possible, give the dimensions of the answer.

1. $\begin{bmatrix} 8 & -7 & 0 \\ 3 & 4 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -2 \\ 1 \end{bmatrix} = ?$

A · B = AB

$\boxed{2} \times \underbrace{3 \quad 3}_{\checkmark} \times \boxed{1}$

A = 2×3

B = 3×1

AB = 2×1

2. $\begin{bmatrix} 6 & -2 \\ 0 & -9 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -7 \\ 8 \end{bmatrix} = ?$

A · B = AB

$3 \times \underbrace{2 \quad 3}_{\times} \times 1$

A = 3×2

B = 3×1

AB = not possible

3. $\begin{bmatrix} 11 \\ -3 \\ 0 \end{bmatrix} \cdot [7 \quad 2 \quad -5] = ?$

A · B = AB

$\boxed{3} \times \underbrace{1 \quad 1}_{\checkmark} \times \boxed{3}$

A = 3×1

B = 1×3

AB = 3×3

Evaluate.

4. $\begin{bmatrix} -6 & -2 \\ -4 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -6 \end{bmatrix}$

$\boxed{2} \times \underbrace{2 \quad 2}_{} \times \boxed{1}$

$\begin{bmatrix} -6 & -2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ -6 \end{bmatrix}$

$-6(-1) + -2(-6)$

$-4(-1) + 3(-6)$

$\rightarrow \boxed{\begin{bmatrix} 18 \\ -14 \end{bmatrix}}$

5. $\begin{bmatrix} -2 & 6 \\ 3 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 4 & 1 \end{bmatrix}$

$\boxed{2} \times \overbrace{2 \quad 2}^{\checkmark} \times \boxed{2}$

$\begin{bmatrix} -2 & 6 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 4 & 1 \end{bmatrix}$

$-2(-2)+6(4) \quad -2(1)+6(1)$

$3(-2)+-3(4) \quad 3(1)+-3(1)$

$\rightarrow \boxed{\begin{bmatrix} 28 & 4 \\ -18 & 0 \end{bmatrix}}$

6. $\begin{bmatrix} 1 & 4 & 2 \\ -1 & 3 & -4 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 4 \\ -1 \end{bmatrix}$

$\boxed{2} \times \underbrace{3 \quad 3}_{\checkmark} \times \boxed{1}$

$\begin{bmatrix} 1 & 4 & 2 \\ -1 & 3 & -4 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \\ -1 \end{bmatrix}$

$1(-3) + 4(4) + 2(-1)$

$-1(-3) + 3(4) + -4(-1)$

$\boxed{\begin{bmatrix} 11 \\ 19 \end{bmatrix}}$

7. $\begin{bmatrix} 6 & -5 \\ 4 & -1 \\ 0 & -5 \end{bmatrix} \cdot \begin{bmatrix} -2 & 6 \\ 4 & 5 \\ 5 & 6 \end{bmatrix}$

$3 \times \underbrace{2 \quad 3}_{\times} \times 2$

$\boxed{\text{not possible}}$

8. $\begin{bmatrix} -3 & -1 \\ 0 & -1 \\ -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & -4 \\ 6 & 3 \end{bmatrix}$

$\boxed{3} \times \underbrace{2 \quad 2}_{\checkmark} \times \boxed{2}$

$$\begin{array}{c|cc} & 0 & -4 \\ & 6 & 3 \\ \hline \begin{bmatrix} -3 & -1 \\ 0 & -1 \\ -4 & 2 \end{bmatrix} & \begin{matrix} -3(0)+-1(6) \\ 0(0)+-1(6) \\ -4(0)+2(6) \end{matrix} & \begin{matrix} -3(-4)+-1(3) \\ 0(-4)+-1(3) \\ -4(-4)+2(3) \end{matrix} \end{array}$$

$$\boxed{\begin{bmatrix} -6 & 9 \\ -6 & -3 \\ 12 & 22 \end{bmatrix}}$$

9. $\begin{bmatrix} -5 & 1 \end{bmatrix} \cdot \left(\underbrace{\begin{bmatrix} 4 & -1 \\ -5 & -6 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 0 \\ 0 & 5 & 5 \end{bmatrix}}_{\text{multiply this first!}} \right)$

$\boxed{2} \times \underbrace{2 \quad 2}_{\checkmark} \times \boxed{3}$

$$\begin{array}{c|ccc} & 0 & 2 & 0 \\ & 0 & 5 & 5 \\ \hline \begin{bmatrix} 4 & -1 \\ -5 & -6 \end{bmatrix} & \begin{matrix} 4(0)+-1(0) \\ -5(0)-6(0) \end{matrix} & \begin{matrix} 4(2)-5 \\ -10-30 \end{matrix} & \begin{matrix} 0-5 \\ 0-30 \end{matrix} \end{array} \rightarrow \begin{bmatrix} 0 & 3 & -5 \\ 0 & -40 & -30 \end{bmatrix}$$

$\begin{bmatrix} -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 & -5 \\ 0 & -40 & -30 \end{bmatrix} \rightarrow \boxed{1} \times \underbrace{2 \quad 2}_{\checkmark} \times \boxed{3}$

$$\begin{array}{c|ccc} & 0 & 3 & -5 \\ & 0 & -40 & -30 \\ \hline \begin{bmatrix} -5 & 1 \end{bmatrix} & -5(0)+1(0) & 3(-5)+1(-40) & -5(-5)+1(-30) \end{array}$$

$$\rightarrow \boxed{\begin{bmatrix} 0 & -55 & -5 \end{bmatrix}}$$

Use matrix multiplication to determine if the matrices are inverses.

10. $A = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$ *and* $B = \begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$

$$\begin{array}{c|cc} & 5 & 3 \\ & -3 & 2 \\ \hline \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} & \begin{matrix} 2(5)+-3(-3) \\ -3(5)+5(-3) \end{matrix} & \begin{matrix} 2(3)+-3(2) \\ -3(3)+5(2) \end{matrix} \end{array} \rightarrow \begin{bmatrix} 19 & 0 \\ -30 & 1 \end{bmatrix}$$

No, matrix A and B are not inverses since $[A][B] \neq I$

identity matrix

11. $A = \begin{bmatrix} 5 & 11 \\ -4 & -9 \end{bmatrix}$ *and* $B = \begin{bmatrix} 9 & 11 \\ -4 & -5 \end{bmatrix}$

$$\begin{array}{c|cc} & 9 & 11 \\ & -4 & -5 \\ \hline \begin{bmatrix} 5 & 11 \\ -4 & -9 \end{bmatrix} & \begin{matrix} 5(9)+11(-4) \\ -4(9)+-9(-4) \end{matrix} & \begin{matrix} 5(11)+11(-5) \\ -4(11)+-9(-5) \end{matrix} \end{array} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Yes, matrix A and B are inverses since

$[A][B] = I$

Identity matrix

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5.03 Notes: Introduction to Matrix Inverses

What's created when you multiply inverses together?

$2\left(\frac{1}{2}\right) = 1$ $\quad 3\left(\frac{1}{3}\right) = 1$ $\quad 25\left(\frac{1}{25}\right) = 1$

Multiply matrix inverses together and you get….. $[A][B] = I$ and $[B][A] = I$

Identity Matrix: a square matrix with diagonal elements = 1 and all other elements = 0

$I_1 = [1]$ $\quad I_2 = \boxed{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}$ $\quad I_3 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ $\quad I_4 = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$

Multiplying any matrix by the identity matrix is like multiplying a number by 1 Ex: $3 \times 1 = 3$

Ex. $10 \times 1 = 10$

Example: $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}6 & -2\\1 & -1\end{bmatrix}$

$$\begin{bmatrix}6 & -2\\1 & -1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}6(1)+0(1) & -2(1)+0(-1)\\6(0)+1(1) & 0(-2)+1(-1)\end{bmatrix} = \begin{bmatrix}6 & -2\\1 & -1\end{bmatrix}$$

Why? * Every number not part of the original matrix gets zeroed out, leaving us with the original matrix unchanged.

Find the product of the given matrices to determine if they are inverses.

$\boldsymbol{A} = \begin{bmatrix}-7 & 2\\-8 & 2\end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix}1 & -1\\4 & -3.5\end{bmatrix}$

$$\begin{bmatrix}1 & -1\\4 & -3.5\end{bmatrix}$$

$$\begin{bmatrix}-7 & 2\\-8 & 2\end{bmatrix}\begin{bmatrix}-7+8 & 7+2(-3.5)\\-8(1)+2(4) & -8(-1)+2(-3.5)\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

yes, matrix A and B are inverses.

$\boldsymbol{C} = \begin{bmatrix}1 & 1 & 2\\-1 & 0 & 3\\-1 & -2 & -8\end{bmatrix}$ and $\boldsymbol{D} = \begin{bmatrix}-6 & -4 & 3\\11 & 6 & -5\\-2 & -1 & 1\end{bmatrix}$

$$\begin{bmatrix}-6 & -4 & 3\\11 & 6 & -5\\-2 & -1 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 1 & 2\\-1 & 0 & 3\\-1 & -2 & -8\end{bmatrix}\begin{bmatrix}-6+11-4 & -4+6-2 & 3-5+2\\6+0-6 & 4+0-3 & -3+0+3\\6-22+16 & 4-12+8 & -3+10-8\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{bmatrix}$$

C and D are not inverses of each other

important component for finding inverse of matrix

Determinant of a 2x2 Matrix: $\begin{bmatrix}a & b\\c & d\end{bmatrix} = ad - bc$

For $\boldsymbol{E} = \begin{bmatrix}6 & 1\\-2 & 5\end{bmatrix}$, find det($E$)

$\det[E] = |E| = 6(5) - (-2)(1)$

$= 30 + 2 = \boxed{32}$

For $\boldsymbol{F} = \begin{bmatrix}3 & 9\\-2 & -6\end{bmatrix}$, find $|F|$

$|F| = 3(-6) - (-2)(9) = -18 - (-18)$

$= -18 + 18 = \boxed{0}$ *matrix F does not have inverse

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5.03 Practice

Determine whether A and B are inverse matrices.

1. $A = \begin{bmatrix} 12 & -7 \\ -5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 7 \\ 5 & 12 \end{bmatrix}$

$$\begin{bmatrix} 3 & 7 \\ 5 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 12 & -7 \\ -5 & 3 \end{bmatrix} \begin{matrix} 12(3)+-7(5) & 12(7)-7(12) \\ -5(3)+3(5) & -5(7)+3(12) \end{matrix}$$

Identity Matrix $\rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ yes since $[A][B] = I$

2. $A = \begin{bmatrix} 2 & 3 & -4 \\ 3 & 6 & -5 \\ -2 & -8 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -34 & 29 & 9 \\ 7 & -6 & -2 \\ -12 & 10 & 3 \end{bmatrix}$

yes, since $[A][B] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} -34 & 29 & 9 \\ 7 & -6 & -2 \\ -12 & 10 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & -4 \\ 3 & 6 & -5 \\ -2 & -8 & 1 \end{bmatrix} \begin{matrix} -68+21+48 & 58-18-40 & 18-6-12 \\ -102+42+60 & 87-36-50 & 27-12-15 \\ 68-56-12 & -58+48+10 & -18+16+3 \end{matrix}$$

Find the determinant of each matrix.

3. $\begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}$

$\det(A) = 6(-2)-(-5)(3)$
$= -12+15$
$= \boxed{3}$

4. $\begin{bmatrix} -2 & 7 \\ 1 & 8 \end{bmatrix}$

$\det(A) = -2(8)-7(1)$
$= -16-7$
$= \boxed{-23}$

5. $\begin{bmatrix} -4 & -7 \\ 6 & 9 \end{bmatrix}$

$\det(A) = -4(9)-(-7)(6)$
$= -36+42$
$= \boxed{6}$

6. $\begin{bmatrix} 12 & -9 \\ -4 & 3 \end{bmatrix}$

$\det(A) = 12(3)-(-9)(-4)$
$= 36-36 = \boxed{0}$

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5.04 Matrix Inverses Notes

Things to Know:

Matrix Multiplicative Identity: $X \cdot I = X$ and $I \cdot X = X$

Determinants: Value used to find the inverse

Inverses: (notation: A^{-1}) $A \cdot A^{-1} = I$ $A^{-1} \cdot A = I$

* the "-b" and "-c" means those values will be the opposite signs of the original starting matrix. (not necessarily negative values)

Finding the inverse of a matrix:

-A matrix is considered non-singular if it has an inverse and singular if it does not have an inverse

-A matrix is singular and has no inverse if the determinant = 0

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -2 \\ 8 & -7 \end{bmatrix} =$

Ex: $A = \begin{bmatrix} -7 & 2 \\ -8 & 2 \end{bmatrix}$ det = -14-(-16), det = 2

$\boxed{A^{-1} = \begin{bmatrix} 1 & -1 \\ 4 & -3.5 \end{bmatrix}}$

Find the determinant. Then find the inverse of the matrix, if it exists.

1. $A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$ det = 8-(-12) = 20

$A^{-1} = \frac{1}{20}\begin{bmatrix} 4 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1/5 & 3/20 \\ -1/5 & \frac{1}{10} \end{bmatrix}$

2. $B = \begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}$ det B = 36-36 = 0

No inverse for matrix B b/c matrix B is singular.

3. $C = \begin{bmatrix} -4 & 6 \\ 8 & -12 \end{bmatrix}$ det C = 48-48 = 0

no inverse for matrix C

4. $D = \begin{bmatrix} 2 & -3 \\ 2 & 2 \end{bmatrix}$ det(D) = 4-(-6) = 10

$D^{-1} = \frac{1}{10}\begin{bmatrix} 2 & 3 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 1/5 & 3/10 \\ -1/5 & 1/5 \end{bmatrix}$

Given $A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$ and $AB = \begin{bmatrix} 30 & -20 \\ 21 & -14 \end{bmatrix}$ find B. (Hint: remember that the product of inverses is the identity matrix)

$[A] \cdot [B] = AB$

$[B] = [A^{-1}] \cdot AB$

det(A) = 2(4) - 6(1) = 8 - 6 = 2

$A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -\frac{1}{2} & 1 \end{bmatrix}$

$B = \begin{bmatrix} 2 & -3 \\ -1/2 & 1 \end{bmatrix}\begin{bmatrix} 30 & -20 \\ 21 & -14 \end{bmatrix}$

$\begin{bmatrix} 2 & -3 \\ -1/2 & 1 \end{bmatrix}\begin{bmatrix} 2(30)-3(21) & 2(-20)+3(14) \\ -1/2(30)+21 & -1/2(-20)+1(-14) \end{bmatrix} = \boxed{\begin{bmatrix} -3 & 2 \\ 6 & -4 \end{bmatrix}}$

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5.04 Practice

$A^{-1} = \frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

In #1–4, use the determinants you found yesterday in #3–6 to find the inverse of the given matrix, if it exists.

1. $\begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}$ $\det(A) = 3$

$A^{-1} = \frac{1}{3}\begin{bmatrix} -2 & 5 \\ -3 & 6 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -2/3 & 5/3 \\ -1 & 2 \end{bmatrix}$

2. $\begin{bmatrix} -2 & 7 \\ 1 & 8 \end{bmatrix}$ $\det(A) = -23$

$A^{-1} = \frac{-1}{23}\begin{bmatrix} 8 & -7 \\ -1 & -2 \end{bmatrix}$

$\boxed{A^{-1} = \begin{bmatrix} -8/23 & 7/23 \\ 1/23 & 2/23 \end{bmatrix}}$

3. $\begin{bmatrix} -4 & -7 \\ 6 & 9 \end{bmatrix}$ $\det(A) = 6$

$A^{-1} = \frac{1}{6}\begin{bmatrix} 9 & 7 \\ -6 & -4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 9/6 & 7/6 \\ -1 & -4/6 \end{bmatrix}$ or $\boxed{\begin{bmatrix} 3/2 & 7/6 \\ -1 & -2/3 \end{bmatrix}}$

4. $\begin{bmatrix} 12 & -9 \\ -4 & 3 \end{bmatrix}$ $\det(A) = 0$

$\boxed{\text{no inverse exists for matrix}}$

5. Given A and AB, find B. $A = \begin{bmatrix} 8 & -4 \\ 3 & 6 \end{bmatrix}$, and $AB = \begin{bmatrix} 36 & 48 \\ -24 & 48 \end{bmatrix}$

$[A]\cdot[B] = AB$

$[B] = [A]^{-1}\cdot AB$

$\det(A) = 8(6) - (-4)(3)$
$= 60$

$A^{-1} = \boxed{\frac{1}{60}\begin{bmatrix} 6 & 4 \\ -3 & 8 \end{bmatrix}} = \begin{bmatrix} 1/10 & 1/15 \\ -1/20 & 2/15 \end{bmatrix}$

$[B] = \frac{1}{60}\begin{bmatrix} 6 & 4 \\ -3 & 8 \end{bmatrix}\begin{bmatrix} 36 & 48 \\ -24 & 48 \end{bmatrix}$

$\begin{bmatrix} 6 & 4 \\ -3 & 8 \end{bmatrix}\begin{bmatrix} 36 & 48 \\ -24 & 48 \end{bmatrix} = \begin{bmatrix} 6(36)+4(-24) & 6(48)+4(48) \\ -3(36)+8(-24) & -3(48)+8(48) \end{bmatrix} \to \begin{bmatrix} 120 & 480 \\ -300 & 240 \end{bmatrix}$

$[B] = \frac{1}{60}\begin{bmatrix} 120 & 480 \\ -300 & 240 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & 8 \\ -5 & 4 \end{bmatrix}}$

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Accelerated Precalculus
5.01-5.04 Quiz Review WS #1: Matrix Operations and Inverses

Perform the given operation. If it is not possible, write undefined and explain why.

1. $\begin{bmatrix} 1 & 7 & 0 \\ -8 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 7 \\ 2 & 2 \\ 5 & 0 \end{bmatrix}$

____

2. $3\begin{bmatrix} -3 & -2 & 1 \\ 2 & 6 & -1 \end{bmatrix} - \begin{bmatrix} 4 & -1 & 11 \\ -5 & 3 & 1 \end{bmatrix}$

____

3. $\begin{bmatrix} 5 & 1 & 4 \\ -9 & 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 7 & 4 \\ 8 & 11 \end{bmatrix} - \begin{bmatrix} -5 & 8 \\ 4 & -3 \end{bmatrix}$

____

4. $\begin{bmatrix} 2 & 3 & -1 & 5 & 3 & -1 \end{bmatrix} \cdot 2\begin{bmatrix} 4 \\ -2 \\ 1 \\ 4 \\ -1 \\ -6 \end{bmatrix}$

____

5. $2\begin{bmatrix} 1 & -1 & 4 & 2 \\ 6 & 1 & 5 & -2 \end{bmatrix}\begin{bmatrix} -3 & -1 & 0 & 5 \\ -5 & 7 & 2 & 3 \end{bmatrix}$

____

Determine if [A] and [B] are inverses by using matrix multiplication and explain why.

6. $A = \begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$

Evaluate the following.

7. $det\begin{bmatrix} -2 & 2 \\ 5 & 7 \end{bmatrix}$ __________

8. $\begin{vmatrix} 2 & 9 \\ 0 & -1 \end{vmatrix}$ __________

Find the inverse of the following matrices. If it's not possible, state not possible and why.

9. $R = \begin{bmatrix} -2 & 0 \\ -5 & 1 \end{bmatrix}$

____

10. $B = \begin{bmatrix} 3 & 7 \\ -1 & -3 \end{bmatrix}$

____

11. $W = \begin{bmatrix} 5 & 7 \\ -4 & -5 \end{bmatrix}$

____

Find the value for the missing element that would matrix F singular.

12. $G = \begin{bmatrix} -4 & x \\ -1 & 4 \end{bmatrix}$ ________________

Key

(13c)

Accelerated Precalculus
5.01-5.04 Quiz Review WS #1: Matrix Operations and Inverses

Perform the given operation. If it is not possible, write undefined and explain why.

1. $\begin{bmatrix} 1 & 7 & 0 \\ -8 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 7 \\ 2 & 2 \\ 5 & 0 \end{bmatrix}$

not possible since dimensions are not alike

2. $3\begin{bmatrix} -3 & -2 & 1 \\ 2 & 6 & -1 \end{bmatrix} - \begin{bmatrix} 4 & -1 & 11 \\ -5 & 3 & 1 \end{bmatrix}$

$\begin{bmatrix} -9 & -6 & 3 \\ 6 & 18 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -1 & 11 \\ -5 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -13 & -5 & -8 \\ 11 & 15 & -4 \end{bmatrix}$

$\begin{bmatrix} -13 & -5 & -8 \\ 11 & 15 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 5 & 1 & 4 \\ -9 & 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 7 & 4 \\ 8 & 11 \end{bmatrix} - \begin{bmatrix} -5 & 8 \\ 4 & -3 \end{bmatrix}$

$\boxed{2} \times \underbrace{3 \quad 3}_{\checkmark} \times \boxed{2}$

$\begin{bmatrix} 1 & 5 \\ 7 & 4 \\ 8 & 11 \end{bmatrix}$

$\begin{bmatrix} 5 & 1 & 4 \\ -9 & 3 & 0 \end{bmatrix}$ $\begin{matrix} 5(1)+1(7)+4(8) & 5(5)+1(4)+4(11) \\ -9(1)+3(7)+0(8) & -9(5)+3(4)+(0)11 \end{matrix}$

$\begin{bmatrix} 44 & 73 \\ 12 & -33 \end{bmatrix} \rightarrow \begin{bmatrix} 44 & 73 \\ 12 & -33 \end{bmatrix} - \begin{bmatrix} -5 & 8 \\ 4 & -3 \end{bmatrix} =$

$\begin{bmatrix} 49 & 65 \\ 8 & -30 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 3 & -1 & 5 & 3 & -1 \end{bmatrix} \cdot 2\begin{bmatrix} 4 \\ -2 \\ 1 \\ 4 \\ -1 \\ -6 \end{bmatrix}$

$\boxed{1} \times \underbrace{6 \quad 6}_{\checkmark} \times \boxed{1}$

$\begin{bmatrix} 48 \end{bmatrix}$

$\begin{bmatrix} 8 \\ -4 \\ 2 \\ 8 \\ -2 \\ -12 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 & -1 & 5 & 3 & -1 \end{bmatrix}$ $2(8)+3(-4)-1(2)+5(8)+3(-2)+1(12)$

2×4 and 2×4

5. $2\begin{bmatrix} 1 & -1 & 4 & 2 \\ 6 & 1 & 5 & -2 \end{bmatrix}\begin{bmatrix} -3 & -1 & 0 & 5 \\ -5 & 7 & 2 & 3 \end{bmatrix}$

not possible, dimensions are not compatible for multiplication.

Determine if [A] and [B] are inverses by using matrix multiplication and explain why.

6. $A = \begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$

$\boxed{2} \times 2 \quad 2 \times \boxed{2}$ ✓

$\begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix} \quad \begin{matrix} 3(1)+5(-2) & 3(5)+5(3) \\ 2(1)-2(1) & 2(5)+3(1) \end{matrix} = \begin{bmatrix} -7 & 30 \\ 0 & 13 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Therefore matrices A and B are not inverses.

Evaluate the following. (* Just find the determinant)

7. $det \begin{bmatrix} -2 & 2 \\ 5 & 7 \end{bmatrix} \rightarrow -2(7)-5(2) = -24$ $\boxed{-24}$

8. $\begin{vmatrix} 2 & 9 \\ 0 & -1 \end{vmatrix} \rightarrow 2(-1)-9(0) = -2$ $\boxed{-2}$

Find the inverse of the following matrices. If it's not possible, state not possible and why.

9. $R = \begin{bmatrix} -2 & 0 \\ -5 & 1 \end{bmatrix}$ $det(R) = -2-0 = -2$

$R^{-1} = \frac{1}{-2} \begin{bmatrix} 1 & 0 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} -1/2 & 0/2 \\ -5/2 & -2/-2 \end{bmatrix}$

$\begin{bmatrix} -1/2 & 0 \\ -5/2 & 1 \end{bmatrix}$

10. $B = \begin{bmatrix} 3 & 7 \\ -1 & -3 \end{bmatrix}$ $det(B) = 3(-3)-7(-1) = -2$

$B^{-1} = \frac{1}{-2} \begin{bmatrix} -3 & -7 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} +3/2 & +7/2 \\ -1/2 & -3/2 \end{bmatrix}$

$\begin{bmatrix} +3/2 & +7/2 \\ -1/2 & -3/2 \end{bmatrix}$

11. $W = \begin{bmatrix} 5 & 7 \\ -4 & -5 \end{bmatrix}$ $det(W) = 5(-5)-7(-4) = 3$

$W^{-1} = \frac{1}{3} \begin{bmatrix} -5 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -5/3 & -7/3 \\ 4/3 & 5/3 \end{bmatrix}$

$\begin{bmatrix} -5/3 & -7/3 \\ 4/3 & 5/3 \end{bmatrix}$

*set ad-bc =

(*inverse does not exist) means that determinant = 0

Find the value for the missing element that would matrix F singular.

12. $G = \begin{bmatrix} -4 & x \\ -1 & 4 \end{bmatrix}$

*set $a(d)-b(c) = 0$

$-4(4) - x(-1) = 0$

$-16 + x = 0$

$x = 16$

$\boxed{x = 16}$

13d

Accelerated Precalculus

5.01-5.04 Quiz Review WS #2: Matrix Operations and Inverses

Perform the given operation. If it is not possible, write undefined and explain why.

1. $\begin{bmatrix} -5 & 8 \\ 4 & -3 \end{bmatrix} - \left(2\begin{bmatrix} 5 & 1 & 4 \\ -9 & 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 7 & 4 \\ 8 & 11 \end{bmatrix} \right)$ ____________________

2. $2\begin{bmatrix} -1 & -4 & 3 \\ 2 & 7 & -1 \end{bmatrix} - 4\begin{bmatrix} 4 & -5 & 6 \\ -1 & 0 & 1 \end{bmatrix}$ ____________________

3. $5\begin{bmatrix} 6 & 7 \\ 2 & 2 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ -8 & 4 & 1 \end{bmatrix}$ ____________________

4. $\begin{bmatrix} 2 & -1 & 5 & 3 & -1 \end{bmatrix} \cdot 2\begin{bmatrix} 4 \\ -2 \\ 1 \\ 4 \\ -1 \\ -6 \end{bmatrix}$ ____________________

5. $2\begin{bmatrix} 1 & -1 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} -3 & -1 & 0 & 5 \\ -5 & 7 & 2 & 3 \end{bmatrix}$ ____________________

Determine if [A] and [B] are inverses by using matrix multiplication and explain why.

6. $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & -5 \\ 2 & 2 \end{bmatrix}$

Evaluate the following.

7. $det\begin{bmatrix} -2 & 2 \\ 5 & 6 \end{bmatrix}$ ________

8. $\begin{vmatrix} 5 & -12 \\ 3 & -1 \end{vmatrix}$ ________

Find the inverse of the following matrices. If it's not possible, state not possible and why.

9. $R = \begin{bmatrix} -4 & 2 \\ -5 & 1 \end{bmatrix}$

____

10. $B = \begin{bmatrix} 4 & 7 \\ -1 & -3 \end{bmatrix}$

____

11. $W = \begin{bmatrix} 3 & 7 \\ -2 & -1 \end{bmatrix}$

____

Find the value for the missing element that would matrix F singular.

12. $G = \begin{bmatrix} -3 & x \\ -8 & 4 \end{bmatrix}$ ____________

13e

Key

Accelerated Precalculus

5.01-5.04 Quiz Review WS #2: Matrix Operations and Inverses

Perform the given operation. If it is not possible, write undefined and explain why.

1. $\begin{bmatrix} -5 & 8 \\ 4 & -3 \end{bmatrix} - \left(2\begin{bmatrix} 5 & 1 & 4 \\ -9 & 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 7 & 4 \\ 8 & 11 \end{bmatrix}\right)$

$\boxed{2} \times \underbrace{3 \quad 3}_{\checkmark} \times \boxed{2}$

$\begin{bmatrix} 10 & 2 & 8 \\ -18 & 6 & 0 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 7 & 4 \\ 8 & 11 \end{bmatrix}$

$\begin{matrix} 10(1)+2(7)+8(8) & 10(5)+2(4)+8(11) \\ -18(1)+6(7)+8(0) & -18(5)+4(6)+11(0) \end{matrix} \rightarrow \begin{bmatrix} 88 & 146 \\ 24 & -66 \end{bmatrix}$

$\begin{bmatrix} -5 & 8 \\ 4 & -3 \end{bmatrix} - \begin{bmatrix} 88 & 146 \\ 24 & -66 \end{bmatrix} = \begin{bmatrix} -93 & -138 \\ -20 & 63 \end{bmatrix}$

$\begin{bmatrix} -93 & -138 \\ -20 & 63 \end{bmatrix}$

2. $2\begin{bmatrix} -1 & -4 & 3 \\ 2 & 7 & -1 \end{bmatrix} - 4\begin{bmatrix} 4 & -5 & 6 \\ -1 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} -2 & -8 & 6 \\ 4 & 14 & -2 \end{bmatrix} - \begin{bmatrix} 16 & -20 & 24 \\ -4 & 0 & 4 \end{bmatrix}$

$\begin{bmatrix} -18 & 12 & -18 \\ 8 & 14 & -6 \end{bmatrix}$

3. $5\begin{bmatrix} 6 & 7 \\ 2 & 2 \\ 5 & 0 \end{bmatrix} - 3\begin{bmatrix} 1 & 7 & 0 \\ -8 & 4 & 1 \end{bmatrix}$

not possible since dimensions are not alike.

4. $\begin{bmatrix} 2 & -1 & 5 & 3 & -1 \end{bmatrix} \cdot 2\begin{bmatrix} 4 \\ -2 \\ 1 \\ 4 \\ -1 \\ -6 \end{bmatrix}$

$1 \times \underbrace{5 \quad 6}_{\times} \times 1$

Not possible. Inner dimensions do not match to allow for multiplication

5. $2\begin{bmatrix} 1 & -1 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} -3 & -1 & 0 & 5 \\ -5 & 7 & 2 & 3 \end{bmatrix} \rightarrow \boxed{2} \times \underbrace{2 \quad 2}_{\checkmark} \times \boxed{4}$

$\begin{bmatrix} 2 & -2 \\ 12 & 2 \end{bmatrix}\begin{bmatrix} -3 & -1 & 0 & 5 \\ -5 & 7 & 2 & 3 \end{bmatrix}$

$\begin{matrix} 2(-3)+5(2) & 2(-1)-2(7) & 2(0)+2(-2) & 2(5)+3(-2) \\ -3(12)+2(-5) & 12(-1)+2(7) & 12(0)+2(2) & 12(5)+2(3) \end{matrix}$

$\boxed{\begin{bmatrix} 4 & -16 & -4 & 4 \\ -46 & 2 & 4 & 66 \end{bmatrix}}$

*If $[A][B] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then matrices A and B would be inverses of each other.

Determine if [A] and [B] are inverses by using matrix multiplication and explain why.

6. $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & -5 \\ 2 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & -5 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix} \begin{matrix} 2(1)+5(2) & 2(-5)+5(2) \\ 2(1)+1(2) & 2(-5)+1(2) \end{matrix} \rightarrow \begin{bmatrix} 12 & 0 \\ 4 & -8 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrix A and B are not inverses.

Evaluate the following.

7. $det \begin{bmatrix} -2 & 2 \\ 5 & 6 \end{bmatrix}$ $\quad 6(-2)-5(2) = -12-10 = \boxed{-22}$ $\quad$ -22

8. $\begin{vmatrix} 5 & -12 \\ 3 & -1 \end{vmatrix}$ $\quad 5(-1)-3(-12) = -5+36 = \boxed{31}$ $\quad$ 31

Find the inverse of the following matrices. If it's not possible, state not possible and why.

9. $R = \begin{bmatrix} -4 & 2 \\ -5 & 1 \end{bmatrix}$ $\quad det(R) = -4(1)-2(-5) = -4+10 = 6$

$$R^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -2 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 1/6 & -2/6 \\ 5/6 & -4/6 \end{bmatrix}$$

$$\begin{bmatrix} 1/6 & -1/3 \\ 5/6 & -2/3 \end{bmatrix}$$

10. $B = \begin{bmatrix} 4 & 7 \\ -1 & -3 \end{bmatrix}$ $\quad det(B) = 4(-3)-7(-1) = -12+7 = -5$

$$B^{-1} = \frac{1}{-5}\begin{bmatrix} -3 & -7 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 3/5 & 7/5 \\ -1/5 & -4/5 \end{bmatrix}$$

$$\begin{bmatrix} 3/5 & 7/5 \\ -1/5 & -4/5 \end{bmatrix}$$

11. $W = \begin{bmatrix} 3 & 7 \\ -2 & -1 \end{bmatrix}$ $\quad det(W) = 3(-1)-7(-2) = -3+14 = 11$

$$W^{-1} = \frac{1}{11}\begin{bmatrix} -1 & -7 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -1/11 & -7/11 \\ 2/11 & 3/11 \end{bmatrix}$$

Find the value for the missing element that would matrix F singular.

12. $G = \begin{bmatrix} -3 & x \\ -8 & 4 \end{bmatrix}$ $\quad$ *set $ad-bc=0$

$-3(4)-(-8)(x) = 0$

$-12+8x = 0$

$8x = 12$

$x = \frac{12}{8} = \boxed{\frac{3}{2}}$

$\boxed{x = 3/2}$

13P

Accelerated Precalculus 5.01-5.04 Quiz Review WS #3: Matrix Operations and Inverses

Perform the given operation. If it is not possible, write undefined and explain why.

1. $\left(2\begin{bmatrix} 0 & 1 & 2 \\ -5 & 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 4 \\ 2 & -3 \end{bmatrix}\right) - 3\begin{bmatrix} -4 & 2 \\ 1 & -3 \end{bmatrix}$ ________________

2. $2\begin{bmatrix} -1 & -4 & 3 \\ 2 & 4 & -1 \end{bmatrix} - 1\begin{bmatrix} 4 & -1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ ________________

3. $3\begin{bmatrix} 6 & 7 \\ 2 & 2 \\ 5 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 7 & 0 \\ -8 & 4 & 1 \end{bmatrix}$ ________________

4. $9\begin{bmatrix} 2 & -1 & 5 & 3 & -1 \end{bmatrix} \cdot 2\begin{bmatrix} 4 \\ -2 \\ 1 \\ 4 \\ -1 \\ -6 \end{bmatrix}$ ________________

5. $2\begin{bmatrix} -2 & -1 & 0 & 5 \\ -5 & 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 1 \\ 0 & 2 \\ 4 & 0 \end{bmatrix}$ ________________

Determine if [A] and [B] are inverses by using matrix multiplication <u>and explain why</u>.

6. $A = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix}$

Evaluate the following.

7. $det \begin{bmatrix} -2 & -3 \\ -5 & 4 \end{bmatrix}$ ____________

8. $\begin{vmatrix} 4 & -7 \\ 2 & -1 \end{vmatrix}$ ____________

Find the inverse of the following matrices. If it's not possible, state not possible <u>and why</u>.

9. $R = \begin{bmatrix} -3 & 2 \\ -6 & 1 \end{bmatrix}$

____

10. $B = \begin{bmatrix} 2 & 5 \\ -1 & -3 \end{bmatrix}$

____

11. $W = \begin{bmatrix} 5 & 2 \\ -6 & -1 \end{bmatrix}$

____

Find the value for the missing element that would make matrix F singular.

12. $G = \begin{bmatrix} -2 & x \\ -8 & 5 \end{bmatrix}$ ____________

13g

Key

Accelerated Precalculus 5.01-5.04 Quiz Review WS #3: Matrix Operations and Inverses

Perform the given operation. If it is not possible, write undefined and explain why.

1. $\left(2\begin{bmatrix}0&1&2\\-5&3&0\end{bmatrix}\begin{bmatrix}1&-2\\-1&4\\2&-3\end{bmatrix}\right)-3\begin{bmatrix}-4&2\\1&-3\end{bmatrix}$

$\boxed{2}\times \underbrace{3 \quad 3} \times\boxed{2}$

$$\begin{bmatrix}0&2&4\\-10&6&0\end{bmatrix}\begin{bmatrix}1&-2\\-1&4\\2&-3\end{bmatrix}$$

$$\begin{matrix}1(0)+2(-1)+4(2) & -2(0)+2(4)+4(-3)\\ -10(1)+6(-1)+0(2) & -10(-2)+6(4)+0(-3)\end{matrix} \rightarrow \begin{bmatrix}6&-4\\-16&44\end{bmatrix}-\begin{bmatrix}-12&6\\3&-9\end{bmatrix}=$$

$$\begin{bmatrix}18&-10\\-19&53\end{bmatrix}$$

2. $2\begin{bmatrix}-1&-4&3\\2&4&-1\end{bmatrix}-1\begin{bmatrix}4&-1&2\\-1&0&1\end{bmatrix}$

$$\begin{bmatrix}-2&-8&6\\4&8&-2\end{bmatrix}-\begin{bmatrix}4&-1&2\\-1&0&1\end{bmatrix}$$

$$\begin{bmatrix}-6&-7&4\\5&8&-3\end{bmatrix}$$

3. $3\begin{bmatrix}6&7\\2&2\\5&0\end{bmatrix}-2\begin{bmatrix}1&7&0\\-8&4&1\end{bmatrix}$

not possible. Dimensions are not alike.

4. $9\begin{bmatrix}2&-1&5&3&-1\end{bmatrix}\cdot 2\begin{bmatrix}4\\-2\\1\\4\\-1\\-6\end{bmatrix}$

1×5 and 6×1

Not possible. Inner dimensions do not match.

5. $2\begin{bmatrix}-2&-1&0&5\\-5&1&2&4\end{bmatrix}\begin{bmatrix}1&-2\\3&1\\0&2\\4&0\end{bmatrix}$

$\boxed{2}\times \underbrace{4 \quad 4} \times\boxed{2}$

$$\begin{bmatrix}-4&-2&0&10\\-10&2&4&8\end{bmatrix}\begin{bmatrix}1&-2\\3&1\\0&2\\4&0\end{bmatrix}$$

$$\begin{matrix}-4(1)+3(-2)+0(0)+4(10) & -4(-2)+1(-2)+2(0)+10(0)\\ -10(1)+2(3)+4(0)+8(4) & -10(-2)+2(1)+4(2)+8(0)\end{matrix} = \begin{bmatrix}30&6\\28&30\end{bmatrix}$$

$$\begin{bmatrix}30&6\\28&30\end{bmatrix}$$

$\boxed{\text{WS\#3}}$

* If $[A][B] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then matrices A and B are inverses of each other.

Determine if [A] and [B] are inverses by using matrix multiplication and explain why.

6. $A = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix}$

$$\begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix} = \begin{bmatrix} 7(-1)+4(2) & 7(4)+4(-7) \\ 2(-1)+1(2) & 2(4)+1(-7) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Identity matrix

Since $[A][B] =$ Identity matrix, A and B are inverses.

Evaluate the following.

7. $\det\begin{bmatrix} -2 & -3 \\ -5 & 4 \end{bmatrix} = -2(4) - (-3)(-5) = -8 - 15 = \boxed{-23}$ ________

8. $\begin{vmatrix} 4 & -7 \\ 2 & -1 \end{vmatrix} \quad 4(-1) - 2(-7) = \boxed{10}$ ________

Find the inverse of the following matrices. If it's not possible, state not possible and why.

9. $R = \begin{bmatrix} -3 & 2 \\ -6 & 1 \end{bmatrix}$ $\quad \det(R) = -3(1) - 2(-6) = -3 + 12 = 9$

$$R^{-1} = \frac{1}{9}\begin{bmatrix} 1 & -2 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 1/9 & -2/9 \\ 6/9 & -3/9 \end{bmatrix}$$

$\begin{bmatrix} 1/9 & -2/9 \\ 2/3 & -1/3 \end{bmatrix}$ ________

10. $B = \begin{bmatrix} 2 & 5 \\ -1 & -3 \end{bmatrix}$ $\quad \det(B) = 2(-3) - 5(-1) = -6 + 5 = -1$

$$B^{-1} = \frac{1}{-1}\begin{bmatrix} -3 & -5 \\ 1 & 2 \end{bmatrix}$$

$\begin{bmatrix} 3 & 5 \\ -1 & -2 \end{bmatrix}$ ________

11. $W = \begin{bmatrix} 5 & 2 \\ -6 & -1 \end{bmatrix}$ $\quad \det(W) = 5(-1) - 2(-6) = -5 + 12 = 7$

$$W^{-1} = \frac{1}{7}\begin{bmatrix} -1 & -2 \\ 6 & 5 \end{bmatrix} = \boxed{\begin{bmatrix} -1/7 & -2/7 \\ 6/7 & 5/7 \end{bmatrix}}$$

____

set determinant = 0 since inverse does not exist

Find the value for the missing element that would make matrix F singular.

* set $ad - bc = 0$

12. $G = \begin{bmatrix} -2 & x \\ -8 & 5 \end{bmatrix}$ ________

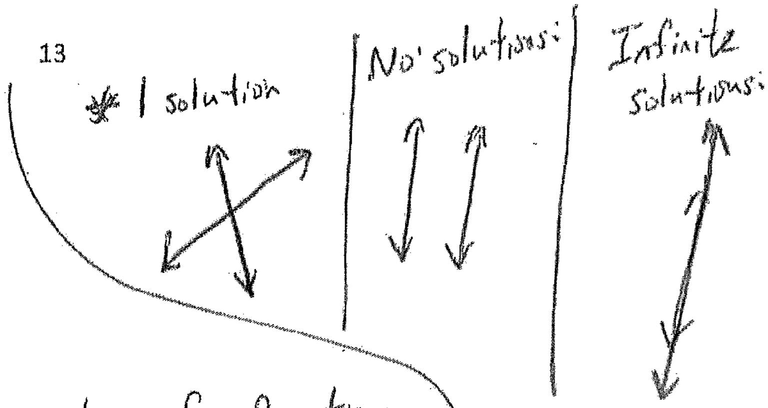
$-2(5) - (-8)(x) = 0$

$-10 + 8x = 0$

$8x = 10$

$x = \frac{10}{8} \longrightarrow \boxed{x = \frac{5}{4}}$

5.05 Solving a 2x2 System of Equations Notes



Ways to solve a 2-variable system of equations:

* 1) elimination

2) substitution

3) graphing

What does your solution represent?

point of intersection of functions

Review: Solve the system using the elimination method.

$\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases} \rightarrow \begin{matrix} -4x + 9y = 9 \\ 4(x - 3y = -6) \end{matrix} \rightarrow \begin{matrix} \cancel{-4x} + 9y = 9 \\ \cancel{4x} - 12y = -24 \end{matrix}$

$-3y = -15$

$\frac{-3y}{-3} = \frac{-15}{-3}$

$\underline{\underline{y = 5}}$

$x - 3y = -6$

$x - 3(5) = -6$

$x - 15 = -6$

$\underline{\underline{x = 9}}$ $\boxed{(9, 5)}$

This year, a NEW way to solve!

How can matrices help to solve a system of equations?

Matrix equations should take the form AX = B; where A = the coefficient matrix, X = the variable matrix, and B = the constant matrix

$[A] \cdot [X] = [B]$

Example:

As a system: $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$ As a matrix equation: $\begin{bmatrix} -4 & 9 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \end{bmatrix}$

If A·X = B, how do we solve for X? $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 & 9 \\ 1 & -3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 9 \\ -6 \end{bmatrix}$

Solve $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$ using a matrix equation. $\det(A) = -4(-3) - (1)(9) = 3$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -3 & -9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 9 \\ -6 \end{bmatrix}$

$\boxed{2} \times \underbrace{2 \quad\quad 2} \times \boxed{1}$

$\begin{bmatrix} 9 \\ -6 \end{bmatrix}$

$\begin{bmatrix} -3 & -9 \\ -1 & -4 \end{bmatrix} \begin{matrix} \underline{-3(9) + (-9)(-6)} \\ \underline{-1(9) + (-4)(-6)} \end{matrix} \rightarrow \begin{bmatrix} 27 \\ 15 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 27 \\ 15 \end{bmatrix} \rightarrow \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$\boxed{\begin{matrix} x = 9 \\ y = 5 \end{matrix} \rightarrow (9, 5)}$

Using a Matrix Equation to Solve a System

1. Write the equation, AX = B, made of the coefficient matrix A times the variable matrix X equal to the constant matrix B.

2. Solve for the variable matrix by multiplying the inverse of the coeffiecient matrix times the answer matrix. $X = A^{-1}B$

3. If A^{-1} does not exist, then there are either:
- NO SOLUTIONS
- or INFINITELY MANY SOLUTIONS

Examples Cont'd: Solve the following systems using matrix equations.

2. $\begin{cases} x - 4y = 12 \\ 3x + 2y = 8 \end{cases}$

$$\begin{bmatrix} 1 & -4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 12 \\ 8 \end{bmatrix}$$

$$det(A) = 2 - (-12) = 14$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 12 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix} \begin{matrix} 2(12) + 4(8) \\ 12(-3) + 1(8) \end{matrix} \rightarrow \begin{bmatrix} 56 \\ -28 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 56 \\ -28 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}}$$

3. $\begin{cases} 2x + 12y = 20 \\ 5x + 30y = -7 \end{cases}$

$$\begin{bmatrix} 2 & 12 \\ 5 & 30 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 5 & 30 \end{bmatrix}^{-1}\begin{bmatrix} 20 \\ -7 \end{bmatrix}$$

$$det(A) = 2(30) - 5(12) = 0$$

matrix A is singular. Determinant = 0. No inverse exists. No solution.

4. $\begin{cases} 5x + 4y = -30 \\ 3x - 9y = -18 \end{cases}$

$$\begin{bmatrix} 5 & 4 \\ 3 & -9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -30 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & -9 \end{bmatrix}^{-1}\begin{bmatrix} -30 \\ -18 \end{bmatrix}$$

$$det(A) = -45 - 12 = -57$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-57}\begin{bmatrix} -9 & -4 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} -30 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} -30 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} -9 & -4 \\ -3 & 5 \end{bmatrix} \begin{matrix} -9(-30) + 4(18) \\ -3(-30) + 5(-18) \end{matrix} \rightarrow \begin{bmatrix} 342 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{57}\begin{bmatrix} 342 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{matrix} x = -6 \\ y = 0 \end{matrix} \rightarrow (-6, 0)}$$

5.06 Practice

Solve each system of equations using the elimination method.

1. $5x = -3y - 31$
   $2y = -4x - 22$

$2(5x + 3y = -31)$
$-3(4x + 2y = -22)$

$10x + 6y = -62$
$-12x - 6y = 66$
$-2x = 4$
$\underline{\underline{x = -2}}$

$4x + 2y = -22$
$4(-2) + 2y = -22$
$-8 + 2y = -22$
$2y = -14$
$\underline{\underline{y = -7}}$

$\boxed{(-2, -7)}$

2. $4y + 17 = -7x$
   $8x + 5y = -19$

$5(7x + 4y = -17)$
$-4(8x + 5y = -19)$

$35x + 20y = -85$
$-32x - 20y = 76$
$3x = -9$
$\underline{\underline{x = -3}}$

$7x + 4y = -17$
$7(-3) + 4y = -17$
$-21 + 4y = -17$
$4y = 4$
$\underline{\underline{y = 1}}$

$\boxed{(-3, 1)}$

3. $12x = 21 - 3y$
   $2y = -4x - 22$

$2(12x + 3y = 21)$
$3(4x + 2y = -22)$

$-24x - \cancel{6y} = -42$
$12x + \cancel{6y} = -66$
$-12x = -108$
$x = 9$

$2y = -4x - 22$
$2y = -4(9) - 22$
$2y = -58$
$y = -29$

$\boxed{(9, -29)}$

4. $4y = 12x - 3$
   $9x = 20y - 2$

$5(-12x + 4y = -3)$
$9x - 20y = -2$

$-60x + \cancel{20y} = -15$
$9x - \cancel{20y} = -2$
$-51x = -17$
$\underline{\underline{x = \frac{1}{3}}}$

$9(\frac{1}{3}) = 20y - 2$
$3 = 20y - 2$
$5 = 20y$
$\frac{5}{20} = y$
$y = 1/4$

$\boxed{(\frac{1}{3}, 1/4)}$

Write each system of equations as a matrix equation. Solving using an inverse matrix.

5. $5x - 2y = 11$
$-4x + 7y = 2$

$$\begin{bmatrix} 5 & -2 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -4 & 7 \end{bmatrix}^{-1}\begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 7 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 \\ 4 & 5 \end{bmatrix} \begin{matrix} 7(11) + 2(2) \\ 11(4) + 5(2) \end{matrix} = \begin{bmatrix} 81 \\ 54 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 81 \\ 54 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\boxed{(3, 2)}$$

6. $2x + 3y = 2$
$x - 4y = -21$

$$\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ -21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -21 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -21 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} \begin{matrix} -4(2) - 21(-3) \\ 2(-1) + 2(-21) \end{matrix} = \begin{bmatrix} 55 \\ -44 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 55 \\ -44 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \end{bmatrix}}$$

7. $-3x + 5y = 33$
$2x - 4y = -26$

$$\begin{bmatrix} -3 & 5 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 33 \\ -26 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 2 & -4 \end{bmatrix}^{-1}\begin{bmatrix} 33 \\ -26 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -4 & -5 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 33 \\ -26 \end{bmatrix}$$

$$\begin{bmatrix} 33 \\ -26 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -5 \\ -2 & -3 \end{bmatrix} \begin{matrix} 33(-4) - 5(-26) \\ 33(-2) - 3(-26) \end{matrix} = \begin{bmatrix} -2 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -2 \\ 12 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}}$$

8. $-4x + y = 19$
$3x - 2y = -18$

$$\begin{bmatrix} -4 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 3 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 19 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -2 & -1 \\ -3 & -4 \end{bmatrix}\begin{bmatrix} 19 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} 19 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \cdot \begin{bmatrix} -2 & -1 \\ -3 & -4 \end{bmatrix} \begin{matrix} -2(19) + 1(18) \\ -3(19) - 18(-4) \end{matrix} = \begin{bmatrix} -20 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -20 \\ 15 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}}$$

$\boxed{18}$

5.07 Matrix Inverses (3x3) Notes

Determinant of a 3x3: If A $= \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ then $\det[A] = (aei + bfg + cdh) - (bdi + afh + ceg)$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{matrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{matrix}$$

Examples: Find the determinant of the following matrices, then use a calculator to find the inverse, if it exists.

1. $E = \begin{bmatrix} -3 & 2 & 4 \\ 1 & -1 & 2 \\ -1 & 4 & 0 \end{bmatrix}$

$$\begin{matrix} -3 & 2 & 4 & -3 & 2 \\ 1 & -1 & 2 & 1 & -1 \\ -1 & 4 & 0 & -1 & 4 \end{matrix}$$

$0 - 4 + 16 - (4 - 24 + 0)$

$12 - (-20)$

$\boxed{\det(E) = 32}$

2. $F = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$\begin{matrix} 3 & 1 & 2 & 3 & 1 \\ 1 & 2 & -1 & 1 & 2 \\ 2 & -1 & 3 & 2 & -1 \end{matrix}$$

$18 - 2 - 2 - (8 + 3 + 3)$

$\boxed{\det(F) = 0}$

3. $G = \begin{bmatrix} -1 & -2 & 1 \\ 4 & 0 & 3 \\ -3 & 1 & -2 \end{bmatrix}$

$$\begin{matrix} -1 & -2 & 1 & -1 & -2 \\ 4 & 0 & 3 & 4 & 0 \\ -3 & 1 & -2 & -3 & 1 \end{matrix}$$

$0 + 18 + 4 - (0 - 3 + 16)$

$\boxed{\det(G) = 9}$

4. Given A and AB, find B.

$A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 0 & 2 \\ 4 & 1 & -2 \end{bmatrix}$ and $AB = \begin{bmatrix} 17 & -2 \\ 3 & 3 \\ 12 & -4 \end{bmatrix}$

$[A][B] = [AB]$

$[B] = [A^{-1}][AB]$

$[B] = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 0 & 2 \\ 4 & 1 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 17 & -2 \\ 3 & 3 \\ 12 & -4 \end{bmatrix}$

$\boxed{[B] = \begin{bmatrix} 5 & -1 \\ 0 & 2 \\ 4 & 1 \end{bmatrix}}$

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5.07 Practice

Find the determinant of each matrix. Then, find the inverse of the matrix, if it exists.

1. $\begin{bmatrix} 3 & 1 & -2 \\ 8 & -5 & 2 \\ -4 & 3 & -1 \end{bmatrix}$

$det(A) = -11$

$A^{-1} = \begin{bmatrix} 1/11 & 5/11 & 8/11 \\ 0 & 1 & 2 \\ -4/11 & 13/11 & 23/11 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -1 & -2 \\ 5 & 9 & 3 \\ 2 & 7 & 4 \end{bmatrix}$

$det(A) = -5$

3. $\begin{bmatrix} 9 & 3 & 7 \\ -6 & -2 & -5 \\ 3 & 1 & 4 \end{bmatrix}$

$det(A) = 0$

4. $\begin{bmatrix} 2 & 3 & -1 \\ -4 & -5 & 2 \\ 6 & 1 & 3 \end{bmatrix}$

$det(A) = 12$

5. $\begin{bmatrix} -1 & 3 & 2 \\ 3 & -5 & -3 \\ 4 & 2 & 6 \end{bmatrix}$

$det(A) = -14$

6. $\begin{bmatrix} 6 & -1 & 2 \\ 1 & -2 & -4 \\ -3 & 1 & -5 \end{bmatrix}$

$det(A) = 57$

7. Given A and AB, find B. $A = \begin{bmatrix} 5 & 0 & 1 \\ 2 & -3 & 2 \\ 1 & -1 & 4 \end{bmatrix}, AB = \begin{bmatrix} 1 & 4 \\ -16 & -6 \\ -2 & -5 \end{bmatrix}$

$\boxed{B = \begin{bmatrix} 0 & 1 \\ 6 & 2 \\ 1 & -1 \end{bmatrix}}$

Find the determinant of each matrix.

8. $\begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & t \end{bmatrix} \begin{matrix} r & 0 \\ 0 & s \\ 0 & 0 \end{matrix}$

$rst + 0 + 0 - (0 - 0 - 0)$

$det(A) = \boxed{rst}$

9. $\begin{bmatrix} c & c & c \\ 0 & c & c \\ 0 & 0 & c \end{bmatrix} \begin{matrix} c & c \\ 0 & c \\ 0 & 0 \end{matrix}$

$c^3 + 0 - 0 - (0 + 0 + 0)$

$\boxed{c^3}$

Matrices on the Calculator

Entering matrices into your TI calculator:

1. Access the "Matrix" window by pressing [2nd] and the button showing "Matrix" in blue.
2. The first menu, "NAMES", is how you select matrices to work with. The second menu, "MATH", is where you will find operations to perform on matrices (like *determinant* and *transpose*!). The third menu, "EDIT", is where you will enter matrices.
3. Select a matrix under the "EDIT" menu. Enter the dimensions of the matrix (number of rows first and then number of columns). Enter all elements of the matrix. When you reach the end of the matrix, double check all elements are entered correctly.
4. If you need to enter another matrix, start the process all over, [2nd], "Matrix", go to the "EDIT" menu, select a matrix that is NOT the one you just entered, set its dimensions, and enter its elements.
5. Press [2nd] followed by [mode] to quit to the home screen.

Using matrices on your TI calculator:

- Find the determinant of A: det [A] or |A|
  1. Store the matrix in the "EDIT" menu (described above). Be sure to "quit" at the end.
  2. Press [2nd], "Matrix", go to the "MATH" menu. Select "determinant" from the "MATH" list of the matrix window. You should see "det(" on the home screen.
  3. Press [2nd], "Matrix". Select the matrix from the "NAMES" menu that was entered.
  4. Press [ENTER]. The calculator returns the determinant.

- Find the inverse of A: A^{-1}
  1. Store the matrix in the "EDIT" menu (described above). Be sure to "quit" at the end.
  2. Press [2nd], "Matrix". Select the matrix from the "NAMES" menu that was entered.
  3. Enter the 'inverse' exponent of -1. You should see "$[A]^{-1}$" on the home screen
     a. Graphing Calc use the button [x^{-1}]
     b. TI36xPro: inverse is in the matrix "MATH" menu OR use [$x^{\square}$] and enter -1.
  4. [ENTER] = WHOA! Ugly decimals. Convert that to fractions!
     a. Graphing Calc: [MATH], "1:▶Frac", [ENTER]
     b. TI-36xPro: [◀▶≈] is found right above [enter].

- Solve a system using a matrix equation:
  1. Write the matrix equation on paper: CV = A.
     - Not here, but if an equation in the system is missing variables, fill in 0 for the coefficient in the correct spot in the row (x is first, then y, and z). Also, if an equation is not in standard form (x first, then y, followed by z), then rearrange the equation so that it is.
  2. Store the *coefficient matrix* and the *answers matrix* in the matrix "EDIT" menu. Be sure to "quit" at the end of the process.
  3. Multiply the *inverse of the coefficient matrix times the answers matrix*. You should see something similar to this "$[A]^{-1}[B]$" on the home screen. The calculator will return a 3x1 matrix with the solution for (x, y, z).

pg. 21. #2 (elimination method)
p. 26 #3,4,5 (matrix inverse)

5.08 Notes: Solving a 3x3 System of Equations

Solving a 3-variable system of equations: you have 3 equations and can still use the previous methods from 2-variable systems. Mainly, we use the **elimination method** when solving by hand. For this unit, we can use a **matrix equation** and use **technology!**

Solve each system using the elimination method.

1. $$\begin{aligned} -4x + 2y + 4z &= 4 \\ -6x + 2y - 2z &= -30 \\ x - 3y - 6z &= -16 \end{aligned}$$

$-4x + 2y + 4z = 4$
$4(x - 3y - 6z = -16)$

$-\cancel{4x} + 2y + 4z = 4$
$\cancel{4x} - 12y - 24z = -64$

$-10y - 20z = -60$
$-10y - 20(5) = -60$

$-6x + 2y - 2z = -30$
$6(x - 3y - 6z = -16)$

$-\cancel{6x} + 2y - 2z = -30$
$\cancel{6x} - 18y - 36z = -96$

$-16y - 38z = -126$

$-8(-10y - 20z = -60)$
$5(-16y - 38z = -126)$

$\cancel{80y} + 160z = 480$
$-\cancel{80y} - 190z = -630$

$-30z = -150$
$\boxed{z = 5}$

$-10y - 100 = -60$
$-10y = 40$
$\boxed{y = -4}$

$x - 3y - 6z = -16$
$x - 3(-4) - 6(5) = -16$
$x + 12 - 30 = -16$
$x - 18 = -16$
$\boxed{x = 2}$

$\boxed{(2, -4, 5)}$

2. $$\begin{aligned} 3x + 2y - 6z &= 6 \\ 3x + y + 4z &= -20 \\ 6x + 2y - z &= -22 \end{aligned}$$

$3x + 2y - 6z = 6$
$-1(6x + 2y - z = -22)$

$3x + \cancel{2y} - 6z = 6$
$-6x - \cancel{2y} + 1z = 22$

$-3x - 5z = 28$

$-3x - 5(-2) = 28$
$-3x + 10 = 28$
$-3x = 18$
$\boxed{x = -6}$

$3x + 2y - 6z = 6$
$-2(3x + y + 4z = -20)$

$3x + \cancel{2y} - 6z = 6$
$-6x - \cancel{2y} - 8z = 40$

$-3x - 14z = 46$

$-3x - 5z = 28$
$-1(-3x - 14z = 46)$

$-\cancel{3x} - 5z = 28$
$\cancel{3x} + 14z = -46$

$9z = -18$
$\boxed{z = -2}$

$3x + 2y - 6z = 6$
$3(-6) + 2y - 6(-2) = 6$

$-18 + 2y + 12 = 6$
$2y - 6 = 6$
$2y = 12$
$\boxed{y = 6}$

$3x + 2y - 6z = 6$

$\boxed{(-6, 6, -2)}$

Write each system as a matrix equation and then solve with technology.

3. $-x - 9y + 2z = -13$
$6y - 5z = -6$
$6z + 5 = 2y - 3x$

$-x - 9y + 2z = -13$
$0x + 6y - 5z = -6$
$3x - 2y + 6z = -5$

$$\begin{bmatrix} -1 & -9 & 2 \\ 0 & 6 & -5 \\ 3 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ -6 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & -9 & 2 \\ 0 & 6 & -5 \\ 3 & -2 & 6 \end{bmatrix}^{-1}\begin{bmatrix} -13 \\ -6 \\ -5 \end{bmatrix}$$

$\boxed{(-11, 4, 6)}$

4. $2x - 3y + 2z = 16$
$-4x + 6y - 4z = 9$
$8x + z = -11$

Write each system of equations as a matrix equations. Solving using an inverse matrix and technology, if possible.

3. $2x + y - z = -13$
$3x + 2y - 4z = -36$
$x + 6y - 3z = 12$

$2x + 1y - z = -13$
$3x + 2y - 4z = -36$
$x + 6y - 3z = 12$

$$\begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & -4 \\ 1 & 6 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ -36 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & -4 \\ 1 & 6 & -3 \end{bmatrix}^{-1}\begin{bmatrix} -13 \\ -36 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 7 \\ 8 \end{bmatrix}$$

$\boxed{(-6, 7, 8)}$

4. $3x - 2y + 6z = 38 - 2z$
$6x + 3y - 9z = -12$
$4y + 20z = -4x$

$3x - 2y + 8z = 38$
$6x + 3y - 9z = -12$
$4x + 4y + 20z = 0$

$$\begin{bmatrix} 3 & -2 & 8 \\ 6 & 3 & -9 \\ 4 & 4 & 20 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 38 \\ -12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -2 & 8 \\ 6 & 3 & -9 \\ 4 & 4 & 20 \end{bmatrix}^{-1}\begin{bmatrix} 38 \\ -12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -9 \\ 1 \end{bmatrix}$$

$\boxed{(4, -9, 1)}$

5. $-x + 2y - z = 2 - 2x$
$2x + 3z = y + 4$
$3x + y + 2z = 6$

$1x + 2y - 1z = 2$
$2x - 1y + 3z = 4$
$3x + 1y + 2z = 6$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$\boxed{\text{no solution}}$

5.09 Matrix Applications Notes

Date ______________

For each, determine if you need to multiply matrices or write a matrix equation and solve using an inverse.

1. In football a touchdown is 6 points, a field goal is 3 points, a touchdown extra point is 1 point, and a two-point conversion is 2 points. The number of each for the top 3 teams in a high school league are given below. Use this information to determine the team that scored the most points.

Eagles scored the most points

TEAM	TD	FG	EP	2EP
Tigers	21	14	12	9
Rams	24	12	18	3
Eagles	27	7	21	2

$$\begin{array}{c} \\ TD \\ FG \\ EP \\ 2EP \end{array}\begin{array}{c} Pts \\ \begin{bmatrix} 6 \\ 3 \\ 1 \\ 2 \end{bmatrix}\end{array}$$

$\boxed{3} \times \underbrace{4 \quad 4}_{\checkmark} \times \boxed{1}$

$$\begin{matrix} 21 & 14 & 12 & 9 & 21(6)+14(3)+12(1)+9(2) \\ 24 & 12 & 18 & 3 & 24(6)+12(3)+18(1)+3(2) \\ 27 & 7 & 21 & 2 & 27(6)+7(3)+21(1)+2(2) \end{matrix}$$

$$\begin{array}{c} \\ T \\ R \\ E \end{array}\begin{array}{c} \text{Points} \\ \begin{bmatrix} 198 \\ 204 \\ 208 \end{bmatrix}\end{array}$$

2. A restaurant manager wants to purchase 200 sets of dishes. One design costs \$25 per set, while another costs \$45 per set. If she only has \$7400 to spend, how many of each design should she order?

x = # of design 1 dishes
y = # of design 2 dishes

$$x + y = 200$$
$$25x + 45y = 7400$$

$$\begin{bmatrix} 1 & 1 \\ 25 & 45 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 7400 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 25 & 45 \end{bmatrix}^{-1}\begin{bmatrix} 200 \\ 7400 \end{bmatrix}$$

$A^{-1} \cdot B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 80 \\ 120 \end{bmatrix}$$

$$\boxed{x = 80 \quad y = 120}$$

3. A local coffee shop specializes in espresso drinks. The table shows the cups of each drink sold throughout the day. Determine the price of each espresso drink.

Hours	Cappuccino	Latte	Macchiato	Earnings
8-11	103	86	79	1040.25
11-2	48	32	26	406.50
2-5	45	25	18	334.00

x = cost of cappucino z = cost macchiato
y = cost latte

$$103x + 86y + 79z = 1040.25$$
$$48x + 32y + 26z = 406.50$$
$$45x + 25y + 18z = 334.00$$

$$\begin{bmatrix} 103 & 86 & 79 \\ 48 & 32 & 26 \\ 45 & 25 & 18 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1040.25 \\ 406.50 \\ 334.00 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \cdot B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \$3.50 \\ \$4 \\ \$4.25 \end{bmatrix}$$

4. Claire purchased 25 total pounds of dog food, bird seed, and cat food for \$100. The dog food costs \$4.00/lb, the bird seed \$7.00/lb, and the cat food \$3.00/lb. She purchased 10 pounds more dog food than bird seed. Determine the number of pounds of each type of food Claire purchased.

x = # pounds (dog food)
y = # pounds (bird seed)
z = # pounds (cat food)

$$x + y + z = 25$$
$$4x + 7y + 3z = 100$$
$$x - 10 = y$$

$$x + y + z = 25$$
$$4x + 7y + 3z = 100$$
$$x - y + 0z = 10$$

x = 13 lbs
y = 3 lbs
z = 9 lbs

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 7 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 25 \\ 100 \\ 10 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \cdot B$$

5.09 Practice

For each, determine if you need to multiply matrices or write a matrix equation and solve using an inverse.

1. Kelly, Joelle, and Emily are competitive skaters. Their routines are judged on skating skill (SS), choreography (C), and interpretation (I). In a recent competition, they received the following scores shown below. One of two weighted systems is used. System A weights SS 20%, C 50%, and I 30%. System B weights SS 40%, C 30%, and I 30%. Determine which systems favors each skater.

$\boxed{3} \times 3 \quad 3 \times \boxed{2}$

	Skater	SS	C	I
B →	Kelly	6	4	2
A →	Joelle	3	5	1
A →	Emily	2	4	6

$$\begin{bmatrix} 6 & 4 & 2 \\ 3 & 5 & 1 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\begin{array}{cc} \boxed{\text{System A}} & \text{System B} \\ \end{array} \begin{bmatrix} 0.20 & 0.40 \\ 0.50 & 0.30 \\ 0.30 & 0.20 \end{bmatrix}$$

System A	System B
3.8	4.2
3.4	3
4.2	3.8

2. A recently retired couple needs \$8,000 per year to supplement their Social Security. They have \$100,000 to invest to obtain this income. They have decided on two investment options: AA bonds yielding 10% per annum and a Bank Certificate yielding 5%. How much money should they invest in AA bonds and how much in Bank Certificates?

x = AA bonds
y = Bank Certificates

$$x + y = 100{,}000$$
$$0.10x + 0.05y = 8000$$

$$\begin{bmatrix} 1 & 1 \\ 0.10 & 0.05 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 100{,}000 \\ 8000 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \cdot B$$

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \$60{,}000 \\ \$40{,}000 \end{bmatrix}}$$

3. The table shows the number of individuals attending the movies over the weekend at a local theater. Determine the costs for a child, adult, and senior citizen to attend the movies.

Day	Child	Adult	Senior	Total Paid
Fri	80x +	110y +	25z =	1755
Sat	100x +	175y +	40z =	2685
Sun	45x +	85y +	30z =	1385

$$\begin{bmatrix} 80 & 110 & 25 \\ 100 & 175 & 40 \\ 45 & 85 & 30 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1755 \\ 2685 \\ 1385 \end{bmatrix}$$

x = child cost
y = adult cost
z = senior cost

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \cdot B \qquad \boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \$4 \\ \$11 \\ \$9 \end{bmatrix}}$$

4. Mr. Wiley invested \$5000 in three different accounts at the beginning of last year, yielding him a total of \$182.50 of interest at the end of the year. The three accounts were a simple savings account earning 1%, a certificate of deposit earning 3.5%, and municipal bonds earning 4.3%. His municipal bond investment was 5 times the amount of money invested in the simple savings account? How much did he invest in each account?

x = simple savings
y = CD
z = municipal bonds

$$x + y + z = 5000$$
$$0.01x + 0.035y + 0.043z = 182.50$$
$$z = 5x$$

$$x + y + z = 5000$$
$$0.01x + 0.035y + 0.043z = 182.50$$
$$5x + 0y - 1z = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.01 & 0.035 & 0.043 \\ 5 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 182.50 \\ 0 \end{bmatrix} \qquad X = A^{-1} \cdot B$$

$$\boxed{\begin{aligned} x &= \$500 \\ y &= \$2000 \\ z &= \$2500 \end{aligned}}$$

5.10 More Practice with Matrix Applications

$A^{-1} = \frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Identify the variables, create a system of equations, write the matrix equation, and solve. Show all work for solving 2x2 matrix equations. You may use technology to solve any 3x3 matrix equations.

1. One group of people purchased 10 hot dogs and 5 soft drinks at a cost of $8.75. A second group bought 7 hot dogs and 4 soft drinks at a cost of $6.25. What is the cost of a single hot dog and a single soft drink?

x = cost of hot dog
y = cost of soft drink

$10x + 5y = 8.75$
$7x + 4y = 6.25$

$$\begin{bmatrix} 10 & 5 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8.75 \\ 6.25 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 7 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 8.75 \\ 6.25 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4 & -5 \\ -7 & 10 \end{bmatrix}\begin{bmatrix} 8.75 \\ 6.25 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.75 \\ 0.25 \end{bmatrix}$$

2. Four large cheeseburgers and two chocolate milkshakes cost a total of $7.90. Two milkshakes cost $0.15 more than one cheeseburger. What is the cost of a cheeseburger and what is the cost of a milkshake?

x = cost of cheeseburger
y = cost of milkshake

$4x + 2y = 7.90$
$-1x + 2y = 0.15$

$$\begin{bmatrix} 4 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7.90 \\ 0.15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 7.90 \\ 0.15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 7.90 \\ 0.15 \end{bmatrix}$$

$2y = 1x + 0.15$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \$1.55 \\ 0.85 \end{bmatrix}$$

3. Thompson's Furniture Store borrowed $650,000 to expand its facilities and extend its product line. Some of the money was borrowed at 4%, some at 6.5%, and the rest at 9%. How much was borrowed at each rate if the annual interest was $46,250 and the amount borrowed at 9% was twice the amount borrowed at 4%?

x = Amt borrowed at 4%
y = Amt at 6.5%
z = Amt at 9%

$x + y + z = 650{,}000$
$0.04x + 0.065y + 0.09z = 46250$
$-2x + 0y + 1z = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.04 & 0.065 & 0.09 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 650000 \\ 46250 \\ 0 \end{bmatrix}$$

$z = 2x$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160{,}000 \\ 170{,}000 \\ 320{,}000 \end{bmatrix}$$

4. At the Pittsburgh zoo, children ride a train for 25 cents, adults pay $1.00, and senior citizens 75 cents. On a given day, 1400 passengers paid a total of $740 for the rides. There were 250 more children riders than all other riders. Find the number of children, adult, and senior riders.

x = # of children
y = # of adults
z = # of seniors

$x + y + z = 1400$
$0.25x + 1y + 0.75z = 740$
$1x - 1y - 1z = 250$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.25 & 1 & 0.75 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1400 \\ 740 \\ 250 \end{bmatrix}$$

$x = y + z + 250$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 825 \\ 410 \\ 165 \end{bmatrix} \begin{array}{l} \to \text{children} \\ \to \text{adults} \\ \to \text{seniors} \end{array}$$

5. Grace receives an $80,000 inheritance. She invests part of it in CDs (certificates of deposit) earning 6.7% APY (annual percentage yield), part in bonds earning 9.3% APY, and the remainder in a growth fund earning 15.6% APY. She invests three times as much in the growth fund as in the other two combined. How much does she have in each investment if she receives $10,843 interest the first year?

x = CD's
y = Amt in bonds
z = Amt in Growth Fund

$$x + y + z = 80000$$
$$0.067x + 0.093y + 0.156z = 10843$$
$$-3x - 3y + 1z = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.067 & 0.093 & 0.156 \\ -3 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 80,000 \\ 10843 \\ 0 \end{bmatrix}$$

$$z = 3(x+y)$$
$$z = 3x + 3y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \$14,500 \\ \$5,500 \\ \$10,000 \end{bmatrix}$$

6. Sophia has 74 coins consisting of nickels, dimes, and quarters in her coin box. The total value of the coins is $8.85. If the number of nickels and quarters is four more than the number of dimes, find how many of each coin Sophia has.

7. A company manufactures tables, chairs, and stools. Last week it built a total of 275 items. The number of chairs built was four times the total number of tables and stools built. The total value of these items is $42,125 with a chair selling for $150, a table for $200, and a stool for $75. Determine the number of each item built last week.

8. A sports equipment company took out three different loans totaling $350,000 from a bank to buy treadmills. The interest rates for each are: Loan 1 = 6.5%, Loan 2 = 7%, and Loan 3 = 9%. After one year the interest paid was $24,950.00. The amount borrowed in Loan 1 was $50,000 less than the amounts borrowed in the other two Loans combined. How much was borrowed in each loan?

5. Grace receives an $80,000 inheritance. She invests part of it in CDs (certificates of deposit) earning 6.7% APY (annual percentage yield), part in bonds earning 9.3% APY, and the remainder in a growth fund earning 15.6% APY. She invests three times as much in the growth fund as in the other two combined. How much does she have in each investment if she receives $10,843 interest the first year?

C = amt in CDs
B = amt in bonds
G = amt in growth fund

$C + B + G = 80000$
$.067C + .093B + .156G = 10843$
$G = 3(C+B) \rightarrow -3C - 3B + G = 0$

$$\begin{bmatrix} C \\ B \\ G \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ .067 & .093 & .156 \\ -3 & -3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 80000 \\ 10843 \\ 0 \end{bmatrix} = \begin{bmatrix} 14500 \\ 5500 \\ 60000 \end{bmatrix}$$

$14500 in CDs
$5500 in bonds
$60,000 in growth fund

6. Sophia has 74 coins consisting of nickels, dimes, and quarters in her coin box. The total value of the coins is $8.85. If the number of nickels and quarters is four more than the number of dimes, find how many of each coin Sophia has.

N = number of nickels
D = number of dimes
Q = number of quarters

$N + D + Q = 74$
$.05N + .10D + .25Q = 8.85$
$(N+Q) = 4 + D \rightarrow N - D + Q = 4$

$$\begin{bmatrix} N \\ D \\ Q \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ .05 & .10 & .25 \\ 1 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 74 \\ 8.85 \\ 4 \end{bmatrix} = \begin{bmatrix} 22 \\ 35 \\ 17 \end{bmatrix}$$

22 nickels
35 dimes
17 quarters

7. A company manufactures tables, chairs, and stools. Last week it built a total of 275 items. The number of chairs built was four times the total number of tables and stools built. The total value of these items is $42,125 with a chair selling for $150, a table for $200, and a stool for $75. ~~Write and solve a system of equations to~~ determine the number of each item built last week.

T = number of tables
C = number of chairs
S = number of stools

$C + T + S = 275$
$C = 4(T + S) \rightarrow C - 4T - 4S = 0$
$150C + 200T + 75S = 42125$

$$\begin{bmatrix} C \\ T \\ S \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -4 & -4 \\ 150 & 200 & 75 \end{bmatrix}^{-1} \begin{bmatrix} 275 \\ 0 \\ 42125 \end{bmatrix} = \begin{bmatrix} 220 \\ 40 \\ 15 \end{bmatrix}$$

220 chairs
40 tables
15 stools

8. A sports equipment company took out three different loans totaling $350,000 from a bank to buy treadmills. The interest rates for each are: Loan 1 = 6.5%, Loan 2 = 7%, and Loan 3 = 9%. After one year the interest paid was $24,950.00. The amount borrowed ~~at the 6.5%~~ Loan 1 rate was $50,000 less than the amounts borrowed in ~~at the~~ other two loans ~~rates~~ combined. Interpret the solution.

x = amt in Loan 1
y = amt in Loan 2
z = amt in Loan 3

$x + y + z = 350000$
$.065x + .07y + .09z = 24950$
$x = (y+z) - 50000 \rightarrow x - y - z = -50000$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ .065 & .07 & .09 \\ 1 & -1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 350000 \\ 24950 \\ -50000 \end{bmatrix} = \begin{bmatrix} 150000 \\ 140000 \\ 60000 \end{bmatrix}$$

$150000 in Loan 1
$140000 in Loan 2
$60000 in Loan 3

5.11 Matrices Test Review Date: ____________________

Solve for X, given the following matrices. If not possible, state the reason why. ***Show work!!***

$A = \begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 12 \\ 6 \end{bmatrix}$

1. $X - A = D$

$X = A + D$

Not possible, dimensions not alike.

2. $X = C - B$

$$= \begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix} - \begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -3 & -a & 7 \\ -5 & 6 & 4 \end{bmatrix}}$$

3. $-2X = A$

$$X = \frac{-1}{2}[A] = \frac{-1}{2}\begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 1 & 0 \\ -2 & -\frac{1}{2} \end{bmatrix}}$$

4. $3C = X - 2B$

$3C + 2B = X$

$$3\begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix} + 2\begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 6 \\ -9 & 18 & 3 \end{bmatrix} + \begin{bmatrix} 8 & 2a & -10 \\ 4 & 0 & -6 \end{bmatrix} = \boxed{\begin{bmatrix} 11 & 2a & -4 \\ -5 & 18 & -3 \end{bmatrix}}$$

5. $X = AB$

$\underbrace{\boxed{2}\times 2 \text{ and } 2}\times\boxed{3}$

$$\begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$$

$$\begin{matrix} -2(4)+0(2) & -2a+0 & 10+0 \\ 16+2 & 4a+0 & -20-3 \end{matrix}$$

$$= \boxed{\begin{bmatrix} -8 & -2a & 10 \\ 18 & 4a & -23 \end{bmatrix}}$$

6. $X = 2CD$

$\underbrace{2\times 3 \text{ and } 2}_{\text{not equal}}\times 1$

Not possible since inner dimensions are not alike

7. $X = A^{-1}$

$$A^{-1} = \frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$\det(A) = -2(1) - (0)(4) = -2$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 1 & 0 \\ -4 & -2 \end{bmatrix} = \boxed{\begin{bmatrix} -\frac{1}{2} & 0 \\ 2 & 1 \end{bmatrix}}$$

8. $AX = D$

$$X = [A^{-1}][D]$$

$$\begin{bmatrix} -\frac{1}{2} & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 6 \end{bmatrix}$$

$$\begin{matrix} 12(-\frac{1}{2}) + 6(0) \\ 12(2) + 6(1) \end{matrix}$$

$$\boxed{X = \begin{bmatrix} -6 \\ 30 \end{bmatrix}}$$

9. The **determinant** of $\begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & -2 \\ -5 & -6 & x \end{bmatrix}$ is 12. **Solve for x.**

*

$$\begin{matrix} 1 & 1 & 0 & 1 & 1 \\ 3 & 4 & -2 & 3 & 4 \\ -5 & -6 & x & -5 & -6 \end{matrix}$$

$$4x + 10 + 0 - (0 + 12 + 3x) = 12$$

$$4x + 10 - 12 - 3x = 12$$

$$1x - 2 = 12$$

$$\boxed{x = 14}$$

Write each matrix equation, then solve the system of equations using an Inverse Matrix. ***Show work!!***

10. $2x + y = 3$
$5x + 6y = 4$

$\begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\frac{1}{12-5}\begin{bmatrix} 6 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\begin{bmatrix} 6 & -1 \\ -5 & 2 \end{bmatrix}$ $6(3)+4(-1)$ / $-5(3)+2(4)$ $\rightarrow \begin{bmatrix} 14 \\ -7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ -7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\boxed{(2,-1)}$

11. $4x - 18 = 3y$
$8x - 7y = 34$

$4x - 3y = 18$
$8x - 7y = 34$

$\begin{bmatrix} 4 & -3 \\ 8 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 34 \end{bmatrix}$

$\det = 4(-7) - 8(-3) = -4$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 8 & -7 \end{bmatrix}^{-1}\begin{bmatrix} 18 \\ 34 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -7 & 3 \\ -8 & 4 \end{bmatrix}\begin{bmatrix} 18 \\ 34 \end{bmatrix}$

$\begin{bmatrix} 18 \\ 34 \end{bmatrix}$

$\begin{bmatrix} -7 & 3 \\ -8 & 4 \end{bmatrix}$ $-7(18)+3(34)$ / $-8(18)+4(34)$ $\rightarrow \begin{bmatrix} -24 \\ -8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -24 \\ -8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

$\boxed{(6,2)}$

Write each as a matrix equation, then solve the 3 variable system of equations using a calculator.

12. $-x + 2y + 7z = 13$
$2x - y - 2z = -2$
$3x + 5y + 2z = -14$

$\begin{bmatrix} -1 & 2 & 7 \\ 2 & -1 & -2 \\ 3 & 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ -14 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2 & 7 \\ 2 & -1 & -2 \\ 3 & 5 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 13 \\ -2 \\ -14 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$

$\boxed{(0,-4,3)}$

13. A doctor's prescription calls for a daily intake containing 40 mg of vitamin C and 30 mg of vitamin D. Your pharmacy stocks 2 liquids that can be used: one contains 20% vitamin C and 30% vitamin D, the other contains 40% vitamin C and 20% vitamin D. How many milligrams of each compound should be mixed to fill the prescription?

x = amt of compound 1
y = amt of compound 2

vit. C $\rightarrow 0.20x + 0.40y = 40$
vit. D $\rightarrow 0.30x + 0.20y = 30$

$\begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 30 \end{bmatrix}$

$\det = 0.2(0.2) - (0.4)(0.3) = -0.08$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}^{-1}\begin{bmatrix} 40 \\ 30 \end{bmatrix} \rightarrow = \frac{1}{-0.08}\begin{bmatrix} 0.2 & -0.4 \\ -0.3 & 0.2 \end{bmatrix}\begin{bmatrix} 40 \\ 30 \end{bmatrix} = -\frac{1}{0.08}\begin{bmatrix} 8-12 \\ -12+6 \end{bmatrix} \rightarrow \begin{bmatrix} 50 \\ 75 \end{bmatrix}$

14. John has $20,000 to invest. As his financial consultant, you recommend that he invest in Treasury bills that yield 5%, Treasury bonds that yield 7%, and corporate bonds that yield 9%. John wants to have an annual income of $1280, and the amount invested in Treasury bills must be two times the amount invested in corporate bonds. Find the amount in each investment.

x = Amt of Tbills
y = Amt of Tbonds
z = Amt of corp. bonds

$x + y + z = 20000$
$0.05x + 0.07y + 0.09z = 1280$
$1x + 0y - 2z = 0$

$x = 2z$

$\begin{bmatrix} 1 & 1 & 1 \\ 0.05 & 0.07 & 0.09 \\ 1 & 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20000 \\ 1280 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0.05 & 0.07 & 0.09 \\ 1 & 0 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 20000 \\ 1280 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \$12,000 \\ \$2,000 \\ \$6,000 \end{bmatrix}$

$12,000 in Tbills
$2000 in Tbonds
$6000 in C bonds

5.11 Matrices Test Review   Date: ____________________

Solve for X, given the following matrices. If not possible, state the reason why. ***Show work!!***

$A = \begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$ $\quad C = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix}$ $\quad D = \begin{bmatrix} 12 \\ 6 \end{bmatrix}$

1. $X - A = D$ $\quad X = A + D$

Not possible, dimensions not alike.

2. $X = C - B$

$$= \begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix} - \begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -3 & -a & 7 \\ -5 & 6 & 4 \end{bmatrix}}$$

3. $-2X = A$

$$X = -\tfrac{1}{2}[A] = -\tfrac{1}{2}\begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 1 & 0 \\ -2 & -1/2 \end{bmatrix}}$$

4. $3C = X - 2B$ $\quad 3C + 2B = X$

$$3\begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix} + 2\begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 6 \\ -9 & 18 & 3 \end{bmatrix} + \begin{bmatrix} 8 & 2a & -10 \\ 4 & 0 & -6 \end{bmatrix} = \boxed{\begin{bmatrix} 11 & 2a & -4 \\ -5 & 18 & -3 \end{bmatrix}}$$

5. $X = AB$ $\quad \underbrace{2\times 2 \text{ and } 2}_{}\times 3$

$$\begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$$

$$\begin{matrix} -2(4)+0(2) & -2a+0 & 10+0 \\ 16+2 & 4a+0 & -20-3 \end{matrix}$$

$$= \boxed{\begin{bmatrix} -8 & -2a & 10 \\ 18 & 4a & -23 \end{bmatrix}}$$

6. $X = 2CD$

$2\times\underbrace{3 \text{ and } 2}_{\text{not equal}}\times 1$

Not possible since inner dimensions are not alike

7. $X = A^{-1}$ $\quad A^{-1} = \frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\det(A) = -2(1) - (0)(4) = -2$$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 1 & 0 \\ -4 & -2 \end{bmatrix} = \boxed{\begin{bmatrix} -1/2 & 0 \\ 2 & 1 \end{bmatrix}}$$

8. $AX = D$

$$X = [A^{-1}][D]$$

$$\begin{bmatrix} -1/2 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 6 \end{bmatrix}$$

$$\begin{matrix} 12(-1/2)+6(0) \\ 12(2)+6(1) \end{matrix}$$

$$\boxed{X = \begin{bmatrix} -6 \\ 30 \end{bmatrix}}$$

9. The **determinant** of $\begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & -2 \\ -5 & -6 & x \end{bmatrix}$ is 12. **Solve for x.**

*

$$\begin{matrix} 1 & 1 & 0 & 1 & 1 \\ 3 & 4 & -2 & 3 & 4 \\ -5 & -6 & x & -5 & -6 \end{matrix}$$

$$4x + 10 + 0 - (0 + 12 + 3x) = 12$$

$$4x + 10 - 12 - 3x = 12$$

$$1x - 2 = 12$$

$$\boxed{x = 14}$$

Write each matrix equation, then solve the system of equations using an Inverse Matrix. ***Show work!!***

10. $2x + y = 3$
$5x + 6y = 4$

$$\begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\frac{1}{12-5}\begin{bmatrix} 6 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -1 \\ -5 & 2 \end{bmatrix} \begin{matrix} 6(3)+4(-1) \\ -5(3)+2(4) \end{matrix} \rightarrow \begin{bmatrix} 14 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$\boxed{(2,-1)}$

11. $4x - 18 = 3y$
$8x - 7y = 34$

$4x - 3y = 18$
$8x - 7y = 34$

$$\begin{bmatrix} 4 & -3 \\ 8 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 34 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 8 & -7 \end{bmatrix}^{-1}\begin{bmatrix} 18 \\ 34 \end{bmatrix}$$

$det = 4(-7) - 8(-3) = -4$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -7 & 3 \\ -8 & 4 \end{bmatrix}\begin{bmatrix} 18 \\ 34 \end{bmatrix}$$

$$\begin{bmatrix} -7 & 3 \\ -8 & 4 \end{bmatrix} \begin{bmatrix} 18 \\ 34 \end{bmatrix} \begin{matrix} -7(18)+3(34) \\ -8(18)+4(34) \end{matrix} \rightarrow \begin{bmatrix} -24 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -24 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$\boxed{(6,2)}$

Write each as a matrix equation, then solve the 3 variable system of equations using a calculator.

12. $-x + 2y + 7z = 13$
$2x - y - 2z = -2$
$3x + 5y + 2z = -14$

$$\begin{bmatrix} -1 & 2 & 7 \\ 2 & -1 & -2 \\ 3 & 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2 & 7 \\ 2 & -1 & -2 \\ 3 & 5 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 13 \\ -2 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$$

$\boxed{(0,-4,3)}$

13. A doctor's prescription calls for a daily intake containing 40 mg of vitamin C and 30 mg of vitamin D. Your pharmacy stocks 2 liquids that can be used: one contains 20% vitamin C and 30% vitamin D, the other contains 40% vitamin C and 20% vitamin D. How many milligrams of each compound should be mixed to fill the prescription?

x = amt of compound 1
y = amt of compound 2

vit. C → $0.20x + 0.40y = 40$
vit D → $0.30x + 0.20y = 30$

$$\begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 30 \end{bmatrix}$$

$det = 0.2(0.2) - (0.4)(0.3) = -0.08$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}^{-1}\begin{bmatrix} 40 \\ 30 \end{bmatrix} \rightarrow = \frac{1}{-0.08}\begin{bmatrix} 0.2 & -0.4 \\ -0.3 & 0.2 \end{bmatrix}\begin{bmatrix} 40 \\ 30 \end{bmatrix} = \frac{-1}{0.08}\begin{bmatrix} 8-12 \\ -12+6 \end{bmatrix} \rightarrow \begin{bmatrix} 50 \\ 75 \end{bmatrix}$$

14. John has $20,000 to invest. As his financial consultant, you recommend that he invest in Treasury bills that yield 5%, Treasury bonds that yield 7%, and corporate bonds that yield 9%. John wants to have an annual income of $1280, and the amount invested in Treasury bills must be two times the amount invested in corporate bonds. Find the amount in each investment.

x = Amt of Tbills
y = Amt of Tbonds
z = Amt of corp. bonds

$x + y + z = 20000$
$0.05x + 0.07y + 0.09z = 1280$
$1x + 0y - 2z = 0$

$x = 2z$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.05 & 0.07 & 0.09 \\ 1 & 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20000 \\ 1280 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0.05 & 0.07 & 0.09 \\ 1 & 0 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 20000 \\ 1280 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \$12,000 \\ \$2,000 \\ \$6,000 \end{bmatrix}$$

$12,000 in Tbills
$2000 in Tbonds
$6000 in C bonds

Unit 5 Matrices Help Session Test Review Worksheet

Solve for X, given the following matrices. If not possible, state the reason why. Show work!

Given the Following Matrices, complete the indicated operations.

$$A = \begin{bmatrix} 2 & -5 \\ 4 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 6 & -8 \\ -2 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 8 & -4 \\ 2 & -4 & 10 \end{bmatrix} \quad D = \begin{bmatrix} -2 & -4 & -4 & 2 \\ 6 & -8 & 12 & 4 \\ 4 & 6 & 8 & 14 \end{bmatrix}$$

$$E = \begin{bmatrix} -5 & 2 & -4 & 1 \\ 3 & 6 & 7 & -13 \\ -5 & 8 & -7 & -2 \end{bmatrix} \quad F = \begin{bmatrix} -3 & -1 & 5 & 7 \\ 5 & 3 & 11 & -3 \\ -1 & -9 & 7 & 15 \end{bmatrix} \quad G = \begin{bmatrix} -7 & 3 & -5 \\ -3 & -9 & 1 \end{bmatrix}$$

1) $2X = 3C - G$

2) $2X = AG$

3) $X = A^{-1}$

4) $AX = B$

5) Find inverse of matrix A if $A = \begin{bmatrix} -2 & b \\ 3 & 7 \end{bmatrix}$

Change only the value of the element in G_{12} that would make matrix G singular.

6. $G = \begin{bmatrix} -3 & 7 \\ -8 & 4 \end{bmatrix}$

7) The **determinant** of $\begin{bmatrix} 3 & 1 & 0 \\ 3 & x & -2 \\ -5 & -1 & 2 \end{bmatrix}$ is 6. **Solve for x.**

Write each matrix equation, then solve the system of equations using an Inverse Matrix. ***Show work!!***

8. $2y + 17 = x$

$3x + 5 = -y$

For #9 and #10: Set up using matrix equations. Then solve using calculators.

9) Patrick has stashed away $16.60 in nickels, dimes, and quarters in his sock drawer. The sum of the nickels and dimes is four less than three times the number of quarters. The total number of coins is 136. Find the number of each kind of coin.

10) Jasper picked strawberries on three days. He picked a total of 87 quarts. On Tuesday he picked 15 quarts more than on Monday. On Wednesday he picked 3 quarts fewer than on Tuesday. How many quarts did he pick each day?

Key

Unit 5 Matrices Help Session Test Review Worksheet

Solve for X, given the following matrices. If not possible, state the reason why. Show work!

Given the Following Matrices, complete the indicated operations.

$$A=\begin{bmatrix}2 & -5\\4 & 3\end{bmatrix}\quad B=\begin{bmatrix}6 & -8\\-2 & 4\end{bmatrix}\quad C=\begin{bmatrix}6 & 8 & -4\\2 & -4 & 10\end{bmatrix}\quad D=\begin{bmatrix}-2 & -4 & -4 & 2\\6 & -8 & 12 & 4\\4 & 6 & 8 & 14\end{bmatrix}$$

$$E=\begin{bmatrix}-5 & 2 & -4 & 1\\3 & 6 & 7 & -13\\-5 & 8 & -7 & -2\end{bmatrix}\quad F=\begin{bmatrix}-3 & -1 & 5 & 7\\5 & 3 & 11 & -3\\-1 & -9 & 7 & 15\end{bmatrix}\quad G=\begin{bmatrix}-7 & 3 & -5\\-3 & -9 & 1\end{bmatrix}$$

1) 2X = 3C − G

$$2X=3\begin{bmatrix}6 & 8 & -4\\2 & -4 & 10\end{bmatrix}-\begin{bmatrix}-7 & 3 & -5\\-3 & -9 & 1\end{bmatrix}$$

$$2X=\begin{bmatrix}18 & 24 & -12\\6 & -12 & 30\end{bmatrix}-\begin{bmatrix}-7 & 3 & -5\\-3 & -9 & 1\end{bmatrix}$$

$$2X=\begin{bmatrix}25 & 21 & -7\\9 & -3 & 29\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}25 & 21 & -7\\9 & -3 & 29\end{bmatrix}$$

$$X=\begin{bmatrix}25/2 & 21/2 & -7/2\\9/2 & -3/2 & 29/2\end{bmatrix}$$

2) 2X = AG

$\boxed{2}\times 2\quad 2\times\boxed{3}$

$$\begin{bmatrix}2 & -5\\4 & 3\end{bmatrix}\begin{bmatrix}-7 & 3 & -5\\-3 & -9 & 1\end{bmatrix}$$

$$\begin{matrix}2(-7)+5(3) & 2(3)+5(9) & 2(-5)-5(1)\\4(-7)+3(-3) & 4(3)+3(-9) & 4(-5)+3(1)\end{matrix}$$

$$2X=\begin{bmatrix}1 & 51 & -15\\-37 & -15 & -17\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}1 & 51 & -15\\-37 & -15 & -17\end{bmatrix}$$

$$X=\begin{bmatrix}1/2 & 51/2 & -15/2\\-37/2 & -15/2 & -17/2\end{bmatrix}$$

3) $X = A^{-1}$

$\det A = 2(3)-4(-5)$

$\det A = 26$

$$A^{-1}=\frac{1}{26}\begin{bmatrix}3 & 5\\-4 & 2\end{bmatrix}=\begin{bmatrix}3/26 & 5/26\\-4/26 & 2/26\end{bmatrix}$$

4) AX = B

$$X=[A^{-1}][B]$$

$$\begin{bmatrix}3 & 5\\-4 & 2\end{bmatrix}\begin{bmatrix}6 & -8\\-2 & 4\end{bmatrix}$$

$$\begin{matrix}6(3)+5(-2) & 3(-8)+5(4)\\6(-4)+2(-2) & -8(-4)+2(4)\end{matrix}$$

$$X=\frac{1}{26}\begin{bmatrix}8 & -4\\-28 & 40\end{bmatrix}$$

$$X=\begin{bmatrix}8/26 & -4/26\\-28/26 & 40/26\end{bmatrix}=\begin{bmatrix}4/13 & -2/13\\-14/13 & 20/13\end{bmatrix}$$

5) Find inverse of matrix A if $A=\begin{bmatrix}-2 & b\\3 & 7\end{bmatrix}$

$\det(A)=-2(7)-3(b)$

$=-14-3b$

$$A^{-1}=\frac{1}{-14-3b}\begin{bmatrix}7 & -b\\-3 & -2\end{bmatrix}$$

Change only the value of the element in G_{12} that would make matrix G singular.

6. $G = \begin{bmatrix} -3 & 7 \\ -8 & 4 \end{bmatrix}$

$$\begin{bmatrix} -3 & x \\ -8 & 4 \end{bmatrix} \rightarrow -3(4) - (-8x) = 0$$

$$-12 + 8x = 0$$
$$8x = +12$$
$$x = +12/8$$
$$\boxed{x = +3/2}$$

7) The **determinant** of $\begin{bmatrix} 3 & 1 & 0 \\ 3 & x & -2 \\ -5 & -1 & 2 \end{bmatrix}$ is 6. **Solve for x.**

$$\begin{matrix} 3 & 1 & 0 & 3 & 1 \\ 3 & x & -2 & 3 & x \\ -5 & -1 & 2 & -5 & -1 \end{matrix}$$

$$6x + 10 + 0 - (0 + 6 + 6) = 6$$
$$6x + 10 - 12 = 6$$
$$6x = 8$$
$$x = 8/6 = \boxed{4/3}$$

Write each matrix equation, then solve the system of equations using an Inverse Matrix. ***Show work!!***

8. $2y + 17 = x$
   $3x + 5 = -y$

$$-1x + 2y = 17$$
$$3x + 1y = -5$$

$$\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -17 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}^{-1}\begin{bmatrix} -17 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1-6}\begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} -17 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} -17 \\ -5 \end{bmatrix} \quad \frac{1(-17)+2(5)}{-3(-17)+1(5)} \rightarrow \begin{bmatrix} -7 \\ 56 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} -7 \\ 56 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}$$

<u>For #9 and #10: Set up using matrix equations. Then solve using calculators.</u>

9) Patrick has stashed away \$16.60 in nickels, dimes, and quarters in his sock drawer. The sum of the nickels and dimes is four less than three times the number of quarters. The total number of coins is 136. Find the number of each kind of coin.

$n + d = 3q - 4$

n = # of nickels
d = # of dimes
q = # of quarters

$$n + d + q = 136$$
$$0.05n + 0.10d + 0.25q = 16.60$$
$$1n + 1d - 3q = -4$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.05 & 0.10 & 0.25 \\ 1 & 1 & -3 \end{bmatrix}\begin{bmatrix} n \\ d \\ q \end{bmatrix} = \begin{bmatrix} 136 \\ 16.60 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} n \\ d \\ q \end{bmatrix} = \begin{bmatrix} 45 \\ 56 \\ 35 \end{bmatrix} \begin{matrix} \text{nickels} \\ \text{dimes} \\ \text{quarters} \end{matrix}$$

10) Jasper picked strawberries on three days. He picked a total of 87 quarts. On Tuesday he picked 15 quarts more than on Monday. On Wednesday he picked 3 quarts fewer than on Tuesday. How many quarts did he pick each day?

$t = m + 15$
$w = t - 3$

m = # of quarts picked monday
t = # picked tuesday
w = # picked wednesday

$$m + t + w = 87$$
$$-1m + t + 0w = 15$$
$$0m - 1t + 1w = -3$$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} m \\ t \\ w \end{bmatrix} = \begin{bmatrix} 87 \\ 15 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} m \\ t \\ w \end{bmatrix} = \begin{bmatrix} 20 \\ 35 \\ 32 \end{bmatrix} \begin{matrix} \rightarrow \text{\# quarts (mon)} \\ \rightarrow \text{\# quarts (tues)} \\ \rightarrow \text{\# quarts (wed)} \end{matrix}$$