7.01 Polar Coordinates Date: ____________

Opener: Plotting Polar Points in Desmos

1. Go to desmos.com/calculator
2. Click the wrench (upper right) and choose the polar grid
3. Put the angle setting in degrees (shocking, right?!?)
4. Equation 1: $r = 5$ from $-6 \le r \le 6$, scale of 1
   Suggestion: Turn off the graph by clicking the colored circle to the left of Equation 1
5. Equation 2: $a = 15$ from $-360 \le a \le 360$, scale of 15
6. Equation 3: $(r \cos a, r \sin a)$
7. Equation 4: (x_1, y_1) *shift-underscore makes subscripts*
8. Equation 5: $x_1 = 1$, with a slider
9. Equation 6: $y_1 = 1$, with a slider

Use the sliders to move the points around.

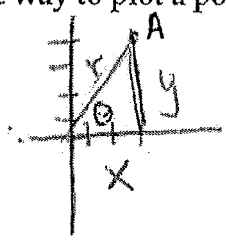
Points (pun definitely intended!) to consider:

- What happens when r is negative?
- What happens when a is negative?

There is more than one way to plot a point:

Rectangular Graph:

A(3, 4)
x, y



Polar Graph:

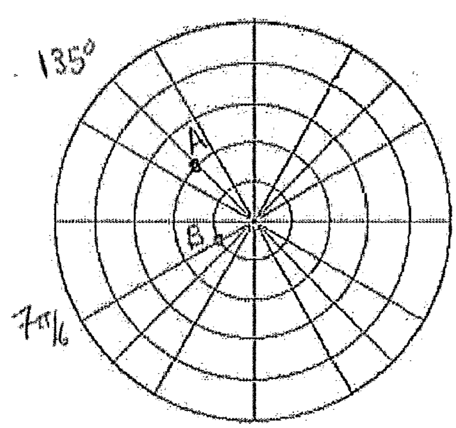
(r, θ)
r = radius
θ = angle

i) +θ is counterclockwise (CCW)
ii) −θ is clockwise (CW)
iii) +r is on the θ angle
iv) −r is the opposite direction along the θ line

Example: Plot the Polar Points: (r, θ)

A (2, 135°)

B $(1, \frac{7\pi}{6})$



Example: Name the location of E in 4 different ways with $-2\pi \leq \theta \leq 2\pi$.

(r, θ)

$E(3, 7\pi/6)$

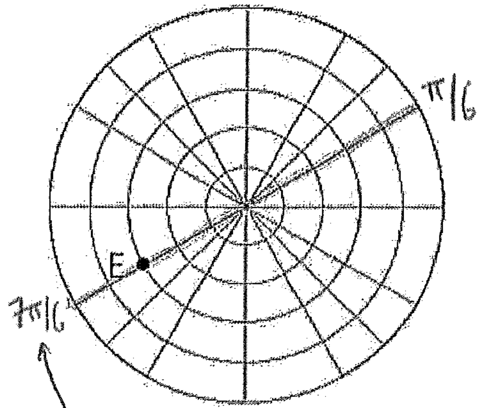
$(3, -\frac{5\pi}{6})$

$(-3, \pi/6)$

$(-3, -\frac{11\pi}{6})$

$\frac{\pi}{6} - 2\pi$

$\frac{\pi}{6} - \frac{12\pi}{6} = \frac{-11\pi}{6}$



$7\pi/6 - 2\pi$

$\frac{7\pi}{6} - \frac{12\pi}{6} = \frac{-5\pi}{6}$

Example: Plot 3 points and determine different pairs of coordinates for them.

$-360° \leq \theta \leq 360°$

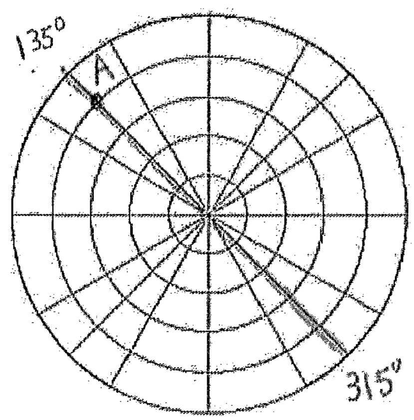
$A(4, 135°)$

$(4, -225°)$

$(-4, 315°)$

$(-4, -45°)$

$135 - 360 \rightarrow \theta = -225°$



$\theta = 315 - 360$

$\theta = -45°$

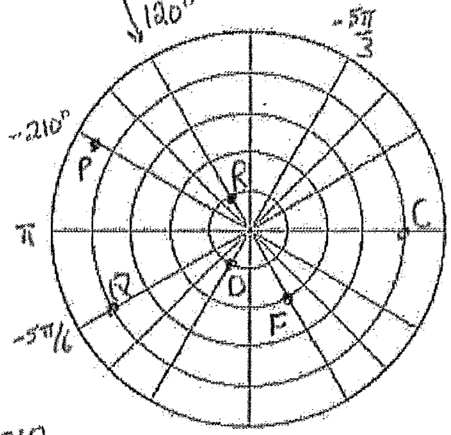
7.01 Practice:

$2\pi/3$

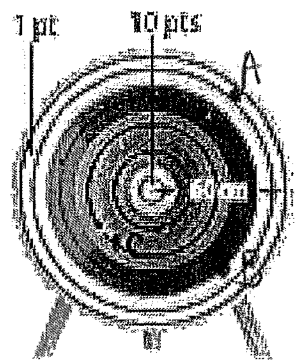
Graph each point on a polar grid. (Examples 1 and 2)

1. $R(1, 120°)$
3. $F\left(-2, \frac{2\pi}{3}\right)$
5. $Q\left(4, -\frac{5\pi}{6}\right)$
7. $D\left(-1, -\frac{5\pi}{3}\right)$ same as $\theta = \pi/3$
9. $C(-4, \pi)$
11. $P(4.5, -210°)$

same as $\theta = -210 + 360$
$\theta = 150°$



13. **ARCHERY** The target in competitive target archery consists of 10 evenly spaced concentric circles with score values from 1 to 10 points from the outer circle to the center. Suppose an archer using a target with a 60-centimeter radius shoots arrows at (57, 45°), (41, 315°), and (15, 240°). (Examples 1 and 2)

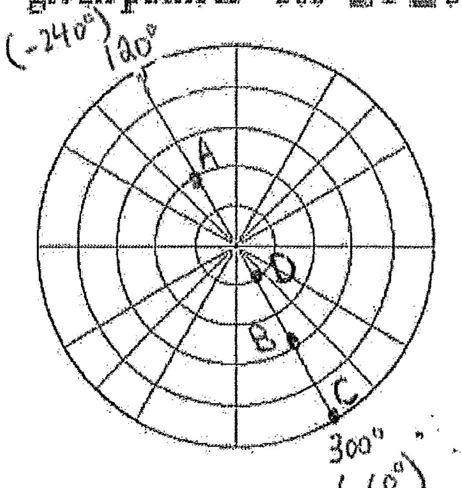


0–6 → 10 pts
6–12 → 9 pts
12–18 → 8 pts
18–24 → 7 pts
24–30 → 6 pts
30–36 → 5 pts
36–42 → 4 pts
42–48 → 3 pts
48–54 → 2 pts
54–60 → 1 pts

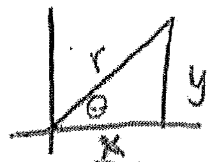
a. Plot the points where the archer's arrows hit the target on a polar grid.

b. How many points did the archer earn? 1 pt + 4pts + 8pts = $\boxed{13\text{pts}}$

Find three different pairs of polar coordinates that name the given point if $-360° \le \theta \le 360°$ or $-2\pi \le \theta \le 2\pi$. (Example 3)

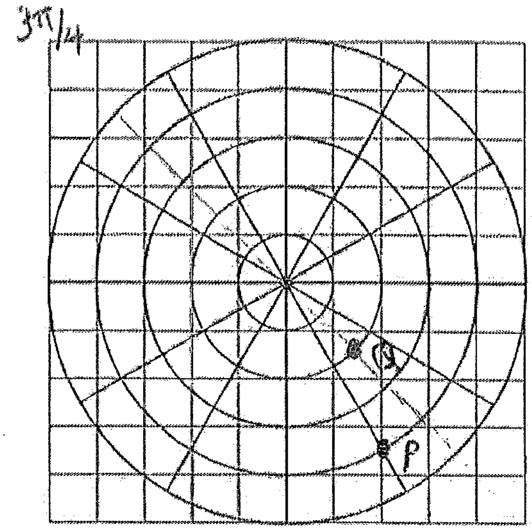


15. $A(-2, 300°)$, $(-2, -60°)$, $(2, 120°)$, $(2, -240°)$
17. $B\left(-3, \frac{2\pi}{3}\right)$, $\left(-3, -\frac{4\pi}{3}\right)$, $\left(3, \frac{5\pi}{3}\right)$, $\left(3, -\frac{\pi}{3}\right)$
19. $C\left(-5, -\frac{4\pi}{3}\right)$, $\left(-5, \frac{2\pi}{3}\right)$, $\left(5, \frac{5\pi}{3}\right)$, $\left(5, -\pi/3\right)$
21. $D(-1, -240°)$, $(-1, 120°)$, $(1, -60°)$, $(1, 300°)$



7.02 Converting Polar Coordinates

Date: ____________



The polar and rectangular grids do overlap so that a location can take on coordinates from either system.

If you know r and θ, how do you calculate x and y?

$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r} \qquad y = r\sin\theta$

If you know x and y, how do you calculate r and θ?

$x^2+y^2=r^2$ | $\tan\theta = \frac{y}{x}$

$r=\sqrt{x^2+y^2}$ | $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Example: Find the rectangular coordinates for each point given in polar coordinates.

1. P (4, -60°)

r θ

$x = 4\cos(-60)$ | $y = 4\sin(-60)$

$x = 4\left(\frac{1}{2}\right) = 2$ | $y = 4\left(-\frac{\sqrt{3}}{2}\right)$

$\boxed{P(2, -2\sqrt{3})} \rightarrow P(2, -3.464)$

2. Q $(-2, \frac{3\pi}{4})$

r θ

$x = -2\cos(3\pi/4)$ | $y = -2\sin\left(\frac{3\pi}{4}\right)$

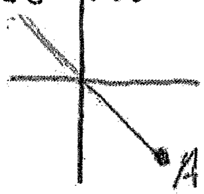
$x = -2\left(-\frac{\sqrt{2}}{2}\right)$ | $y = -2\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

$x = \sqrt{2}$ $\boxed{Q(\sqrt{2}, -\sqrt{2})} \rightarrow (1.41, -1.41)$

Example: For each point given in rectangular coordinates, find four unique polar coordinates with $-2\pi \le \theta \le 2\pi$. $-6.28 < \theta < 6.28$

3. A (2, -5)

x y → Q4



$r = \sqrt{2^2+5^2} = \sqrt{29}$

$\theta = \tan^{-1}\left(\frac{-5}{2}\right) = -1.1902 + 2\pi$

$\theta = 5.093$

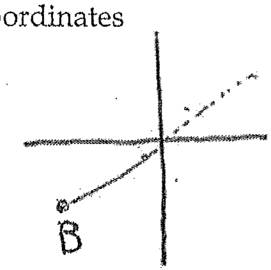
i) $(\sqrt{29}, 5.093)$ -2π

ii) $(\sqrt{29}, -1.190)$ $-\pi$

iii) $-\sqrt{29}, 1.951)$ -2π

iv) $(-\sqrt{29}, -4.331)$

4. B (-9, -4)



$r = \sqrt{9^2+4^2} = \sqrt{97}$

Q3

$\theta = \tan^{-1}\left(\frac{-4}{-9}\right) = 0.418 + \pi$

$\theta = 3.560$

i) $(\sqrt{97}, 3.560)$ -2π $-\pi$

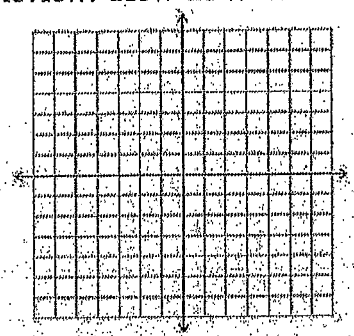
ii) $(\sqrt{97}, -2.723)$

iii) $(-\sqrt{97}, 0.418)$ -2π

iv) $(-\sqrt{97}, -5.865)$

Distance between 2 Points in the Polar Plane

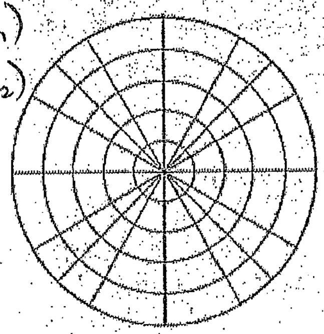
Review: How do we find the distance between two points in the Cartesian Plane?



$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

New method for distance using the Polar Plane:

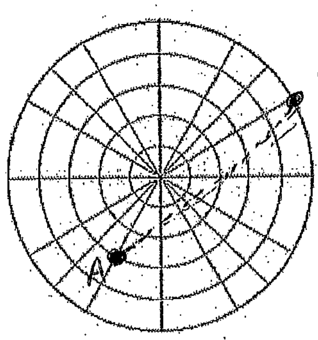
p1: (r_1, θ_1)
p2: (r_2, θ_2)



$$d = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)}$$

Example: Find the distance between the points $\overset{A}{\left(-3, \frac{\pi}{3}\right)}$ *and* $\overset{B}{\left(5, -\frac{11\pi}{6}\right)}$

r_1 θ_1 r_2 θ_2



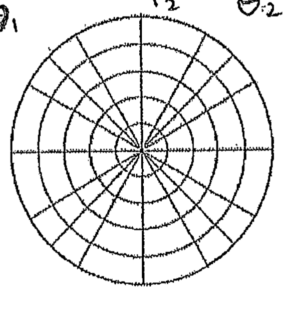
$$d = \sqrt{3^2 + 5^2 - \left[2(-3)(5)\cos\left(-\frac{11\pi}{6} - \frac{\pi}{3}\right)\right]}$$

$$d = \sqrt{34 + 30\left(\frac{\sqrt{3}}{2}\right)}$$

$$d = \sqrt{34 + 15\sqrt{3}} \approx \boxed{7.745 \text{ units}}$$

Example: A radar detects 2 plane s at the same altitude. Their polar coordinates are (5 miles, 310°) and (2 miles, 192°). How far apart are the planes?

r_1 θ_1 r_2 θ_2



$$d = \sqrt{5^2 + 2^2 - \left[2(5)(2)\cos(192 - 310)\right]}$$

$$\boxed{d = 6.196 \text{ mi}}$$

Classwork #1, 3, 5, 13, 15, ~~17~~, 23

7.02 Practice: Complete the odd problems. $(r\cos\theta, r\sin\theta)$

Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest thousandth if necessary. (Example 1)

1. $\left(2, \frac{\pi}{4}\right)$	**2.** $\left(\frac{1}{4}, \frac{\pi}{2}\right)$
3. $(5, 240°)$	**4.** $(2.5, 250°)$
5. $\left(-2, \frac{4\pi}{3}\right)$	**6.** $(-13, -70°)$
7. $\left(3, \frac{\pi}{2}\right)$	**8.** $\left(\frac{1}{2}, \frac{3\pi}{4}\right)$
9. $(-2, 270°)$	**10.** $(4, 210°)$
11. $\left(-1, -\frac{\pi}{6}\right)$	**12.** $\left(5, \frac{\pi}{3}\right)$

1) $\left(2\cos\frac{\pi}{4}, 2\sin\frac{\pi}{4}\right) \rightarrow \left(2\left(\frac{\sqrt{2}}{2}\right), 2\left(\frac{\sqrt{2}}{2}\right)\right)$

$\boxed{(\sqrt{2}, \sqrt{2})}$

3) $(5\cos 240, 5\sin 240) \rightarrow \boxed{\left(\frac{-5}{2}, \frac{-5\sqrt{3}}{2}\right)}$

5) $\left(-2\cos\left(\frac{4\pi}{3}\right), -2\sin\left(\frac{4\pi}{3}\right)\right)$

$\left(-2(-\tfrac{1}{2}), -2\left(-\frac{\sqrt{3}}{2}\right)\right)$

$\boxed{(1, \sqrt{3})}$

Find 4 pairs of polar coordinates for each point with the given rectangular coordinates for $[-2\pi, 2\pi]$. Round to the nearest thousandth if necessary. (Example 2)

13. $(7, 10)$	**14.** $(-13, 4)$	**15.** $(-6, -12)$
16. $(4, -12)$	**17.** $(2, -3)$	**18.** $(0, -173)$
19. $(a, 3a), a > 0$	**20.** $(-14, 14)$	**21.** $(52, -31)$
22. $(3b, -4b), b > 0$	**23.** $(1, -1)$	**24.** $(2, \sqrt{2})$

13) $r = \sqrt{7^2+10^2} = \sqrt{149}$

$\theta = \tan^{-1}\left(\frac{10}{7}\right) = 0.96$ (Q1) $+\pi$

Q1 i) $(\sqrt{149}, 0.960)$ -2π

Q1 ii) $(\sqrt{149}, -5.23)$

Q3 iii) $(-\sqrt{149}, 4.102)$ -2π

Q3 iv) $(-\sqrt{149}, -2.182)$

15) $\sqrt{6^2+12^2} = \sqrt{180}$ or $6\sqrt{5}$

$\theta = \tan^{-1}\left(\frac{-12}{-6}\right) = 1.107 + \pi \rightarrow \theta = 4.249$

Q3 $-\pi$

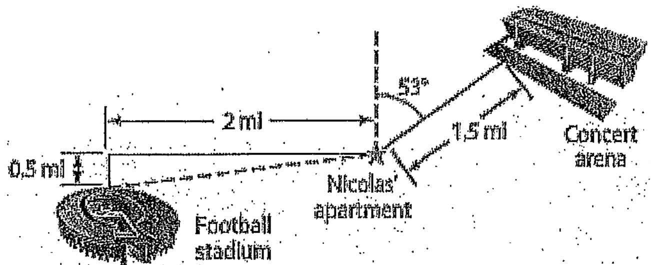
Q3 i) $(6\sqrt{5}, 4.249)$ -2π

Q3 ii) $(6\sqrt{5}, -2.034)$

Q1 iii) $(-6\sqrt{5}), 1.107)$ -2π

Q1 iv) $(-6\sqrt{5}, -5.176)$

25. DISTANCE Standing on top of his apartment building, Nicolas determines that a concert arena is 53° east of north. Suppose the arena is exactly 1.5 miles from Nicolas' apartment. (Example 3)



a. How many miles north and east will Nicolas have to travel to reach the arena?

b. If a football stadium is 2 miles west and 0.5 mile south of Nicolas' apartment, what are the polar coordinates of the stadium if Nicolas' apartment is at the pole?

7.02 Homework: Page 557, #1 – 25 odd

Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest thousandth if necessary. (Example 1)

1. $\left(2, \frac{\pi}{4}\right)$ $(\sqrt{2}, \sqrt{2})$

3. $(5, 240°)$ $\left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$

5. $\left(-2, \frac{4\pi}{3}\right)$ $(1, \sqrt{3})$

7. $\left(3, \frac{\pi}{2}\right)$ $(0, 3)$

9. $(-2, 270°)$ $(0, 2)$

11. $\left(-1, -\frac{\pi}{6}\right)$ $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Find 4 pairs of polar coordinates for each point with the given rectangular coordinates for [-2π, 2π] . Round to the nearest thousandth if necessary. (Example 2)

13. $(7, 10)$

15. $(-6, -12)$

17. $(2, -3)$

19. $(a, 3a), a > 0$

21. $(52, -31)$

23. $(1, -1)$

13. $(\sqrt{149}, 0.960)$
$(\sqrt{149}, -5.323)$
$(-\sqrt{149}, -2.182)$
$(-\sqrt{149}, 4.102)$

15. $(6\sqrt{5}, 4.249)$
$(6\sqrt{5}, -2.034)$
$(-6\sqrt{5}, 1.107)$
$(-6\sqrt{5}, -5.176)$

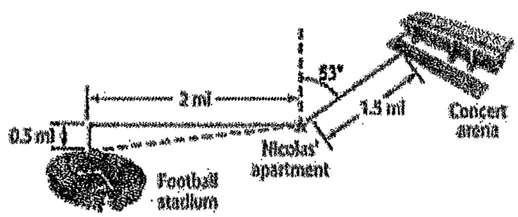
17. $(\sqrt{13}, 5.300)$
$(\sqrt{13}, -0.98)$
$(-\sqrt{13}, 2.159)$
$(-\sqrt{13}, -4.124)$

19. $r = \sqrt{a^2 + (3a)^2} = \sqrt{a^2 + 9a^2}$
$r = \sqrt{10a^2} = a\sqrt{10}$
$(a\sqrt{10}, 1.249)$
$(a\sqrt{10}, -5.034)$
$(-a\sqrt{10}, -1.893)$
$(-a\sqrt{10}, 4.391)$

21. $(\sqrt{3665}, 5.746)$
$(\sqrt{3665}, -0.538)$
$(-\sqrt{3665}, 2.604)$
$(-\sqrt{3665}, -3.679)$

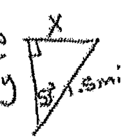
23. $\left(\sqrt{2}, -\frac{\pi}{4}\right)$
$\left(\sqrt{2}, \frac{7\pi}{4}\right)$
$\left(-\sqrt{2}, \frac{3\pi}{4}\right)$
$\left(-\sqrt{2}, -\frac{5\pi}{4}\right)$

25. **DISTANCE** Standing on top of his apartment building, Nicolas determines that a concert arena is 53° east of north. Suppose the arena is exactly 1.5 miles from Nicolas' apartment. (Example 3)



a. How many miles north and east will Nicolas have to travel to reach the arena?

b. If a football stadium is 2 miles west and 0.5 mile south of Nicolas' apartment, what are the polar coordinates of the stadium if Nicolas' apartment is at the pole?

(a) 

$\sin 53° = \frac{x}{1.5}$
$x = 1.5\sin 53° = 1.198$ mi E
$\cos 53° = \frac{y}{1.5}$
$y = 1.5\cos 53° = 0.903$ mi N

(b) $r = \sqrt{2^2 + 0.5^2}$
$r = \sqrt{4.25}$
$\theta = \tan^{-1}\left(\frac{-0.5}{-2}\right) + 180°$
$\theta = 194.036°$
$(\sqrt{4.25}, 194.036°)$

Key

7.03 Quiz Review: Date ____________

Polar Coordinates, Equations, and Distance

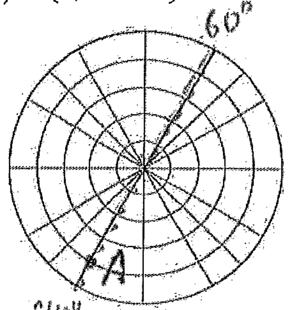
1) Graph each point on the polar grid. Find three other pairs of polar coordinates that name the point

if $-360° \leq \theta \leq 360°$ if $-2\pi \leq \theta \leq 2\pi$

$\rightarrow \theta = -120+360 = 240°$

$\frac{7\pi}{4} - 2\pi \rightarrow \frac{7\pi}{4} - \frac{8\pi}{4} = -\pi/4$

a) $A(4, -120°)$



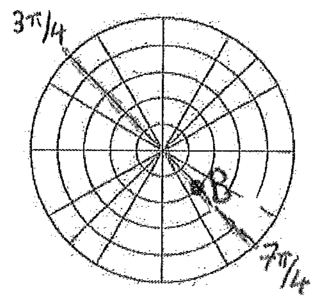
$(4, 240°)$

$(-4, 60°)$

$(-4, -300°)$

$-360°$

b) $B\left(2, \frac{7\pi}{4}\right)$



$(2, -\pi/4)$

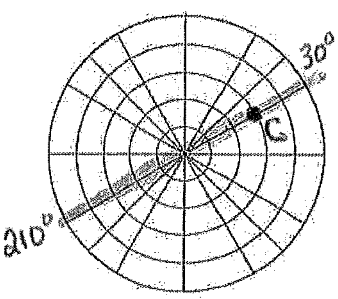
$(-2, 3\pi/4)$

$(-2, -5\pi/4)$

-2π

$\theta = 210 - 360 = -150°$

c) $C(-3, 210°)$



$(-3, -150°)$

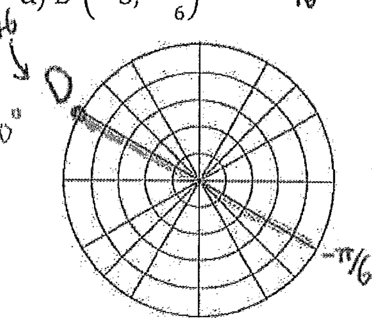
$(3, 30°)$

$(3, -330°)$

$-360°$

$-\pi/6 + 2\pi = 11\pi/6$

d) $D\left(-5, -\frac{\pi}{6}\right)$



$\left(-5, \frac{11\pi}{6}\right)$

$(5, 5\pi/6)$

$(5, -7\pi/6)$

-2π

2) Given the polar distance formula between two points A (r_1, θ_1) and B (r_2, θ_2): $AB = \sqrt{r_1{}^2 + r_2{}^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$, find the distance between A and B.

*degrees mode

a) $A(\overset{r_1}{4}, \overset{\theta_1}{200°})$ $B(\overset{r_2}{-3}, \overset{\theta_2}{60°})$

$= \sqrt{4^2 + 3^2 - 2(4)(-3)\cos(60-200)}$

$= \sqrt{25 + 24\cos(-140)}$

a) AB = 2.572 units

*Radian Mode

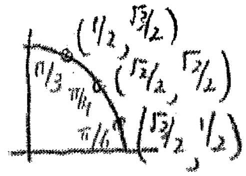
b) $A\left(\overset{r_1}{-7}, \overset{\theta_1}{\frac{5\pi}{6}}\right)$ $B\left(\overset{r_2}{2}, \overset{\theta_2}{-\frac{4\pi}{3}}\right)$

$= \sqrt{7^2 + 2^2 - 2(-7)(2)\cos\left(\frac{-4\pi}{3} - \frac{5\pi}{6}\right)}$

$= \sqrt{53 + 28\cos\left(\frac{-13\pi}{6}\right)}$

b) AB = 8.789 units

$$\left(\overset{x}{r\cos\theta},\ \overset{y}{r\sin\theta}\right)$$



$(\cos\theta, \sin\theta)$

3) Find the rectangular coordinates for each point with the given polar coordinates. Answer in exact form.

a) $\left(\overset{r}{5}, \overset{\theta}{\frac{5\pi}{3}}\right)$ $\left(5\cos\frac{5\pi}{3},\ 5\sin\frac{5\pi}{3}\right)$ $\left(5\left(\frac{1}{2}\right),\ 5\left(\frac{-\sqrt{3}}{2}\right)\right)$

b) $\left(\overset{r}{-6}, \overset{\theta}{\frac{3\pi}{4}}\right)$ $\left(-6\cos\left(\frac{3\pi}{4}\right),\ -6\sin\left(\frac{3\pi}{4}\right)\right)$ $\left(-6\left(\frac{-\sqrt{2}}{2}\right),\ -6\left(\frac{\sqrt{2}}{2}\right)\right)$

a) $\left(\frac{5}{2},\ \frac{-5\sqrt{3}}{2}\right)$ ______

b) $\left(3\sqrt{2},\ -3\sqrt{2}\right)$ ______

c) $\left(\overset{r}{8}, \overset{\theta}{\frac{7\pi}{6}}\right)$ $\left(8\cos\frac{7\pi}{6},\ 8\sin\frac{7\pi}{6}\right)$ $\left(8\left(\frac{-\sqrt{3}}{2}\right),\ 8\left(-\frac{1}{2}\right)\right)$

d) $\left(\overset{r}{-12}, \overset{\theta}{-\frac{3\pi}{2}}\right)$ $\theta = -\frac{3\pi}{2} + 2\pi = \frac{\pi}{2}$

$\left(-12\cos\frac{\pi}{2},\ -12\sin\frac{\pi}{2}\right) \to \left(-12(0),\ -12(1)\right)$

c) $\left(-4\sqrt{3},\ -4\right)$ ______

d) $\left(0,\ -12\right)$ ______

A

4) Find four unique polar coordinates for each point given as rectangular coordinates. Use $-360° \le \theta \le 360°$. Round to the nearest thousandths.

$\boxed{Q2}$ a) $(\overset{x}{-1}, \overset{y}{5})$ $r = \sqrt{1^2 + 5^2} = \sqrt{26}$ $\boxed{Q2}$

$\theta = \tan^{-1}\left(\frac{5}{-1}\right) = -78.69 + 180 = 101.309°$

$\boxed{Q4}$ b) $(3, -7)$ $r = \sqrt{3^2 + 7^2} \doteq \sqrt{58}$ Q4

$\theta = \tan^{-1}\left(\frac{-7}{3}\right) = -66.801° + 360$

$\theta = 293.198°$

a) $\left(\sqrt{26}, 101.309\right)$ $\xrightarrow{-360°}$ $\left(\sqrt{26}, -258.691°\right)$ $+180°$

$\left(-\sqrt{26}, 281.309°\right)$ $\xrightarrow{-360}$ $\left(-\sqrt{26}, -78.691°\right)$

b) $\left(\sqrt{58}, 293.198°\right)$ $\xrightarrow{-360°}$ $\left(\sqrt{58}, -66.802°\right)$ $-180°$

$\left(-\sqrt{58}, 113.198°\right)$ $\xrightarrow{-360}$ $\left(-\sqrt{58}, -246.802°\right)$

Use $-2\pi \le \theta \le 2\pi$. Round to the nearest thousandths.

$\boxed{Q3}$ c) $(-5\sqrt{3}, -5)$ $r = \sqrt{(5\sqrt{3})^2 + 5^2} = \sqrt{100}$

$r = 10$

$\theta = \tan^{-1}\left(\frac{-5}{-5\sqrt{3}}\right) = 0.524 + \pi = 3.665$

$\boxed{Q1}$ d) $(4, 3)$ $r = \sqrt{4^2 + 3^2} = 5$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 0.643$

c) $\left(10, 3.665\right)$ $\xrightarrow{-2\pi}$ $\left(10, -2.618\right)$ $-\pi$

$\left(-10, 0.524\right)$ $\xrightarrow{-2\pi}$ $\left(-10, -5.760\right)$

d) $\left(5, 0.644\right)$ $\xrightarrow{-2\pi}$ $\left(5, -5.640\right)$ $+\pi$

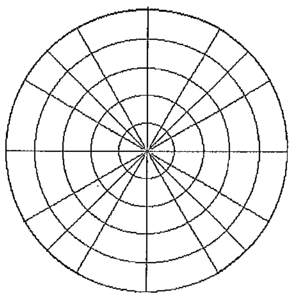
$\left(-5, 3.785\right)$ $\xrightarrow{-2\pi}$ $\left(-5, -2.498\right)$

7.03 Quiz Review WS #2 Date ______________

Polar Coordinates, Equations, and Distance

1) Graph each point on the polar grid. Find three other pairs of polar coordinates that name the point
if $-360° \leq \theta \leq 360°$ if $-2\pi \leq \theta \leq 2\pi$

a) $A(-3, -60°)$



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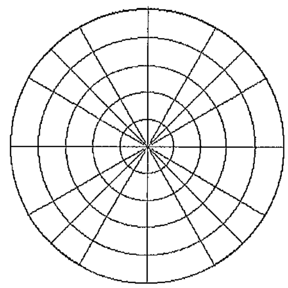
b) $B\left(4, \frac{5\pi}{6}\right)$



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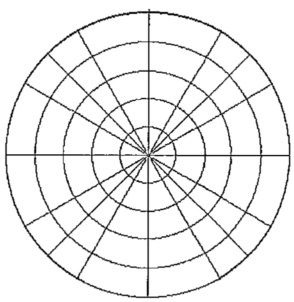
c) $C(2, 135°)$



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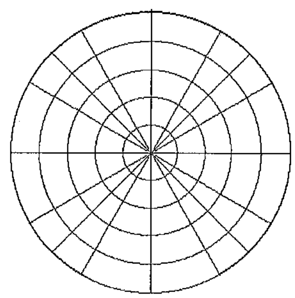
d) $D\left(-2, -\frac{4\pi}{3}\right)$



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2) Given the polar distance formula between two points $A\,(r_1, \theta_1)$ and $B\,(r_2, \theta_2)$:
$AB = \sqrt{r_1{}^2 + r_2{}^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$, find the distance between A and B.

a) $A(2, 120°)$ $B(-4, 45°)$

b) $A\left(3, \frac{5\pi}{3}\right)$ $B\left(-4, -\frac{\pi}{4}\right)$

a) AB = ______________

b) AB = ______________

3) Find the rectangular coordinates for each point with the given polar coordinates. Answer in exact form.

a) $\left(-2, -\frac{4\pi}{3}\right)$

b) $\left(5, \frac{5\pi}{4}\right)$

a) ________________

b) ________________

c) $\left(3, \frac{11\pi}{6}\right)$

d) $\left(-5, -\frac{7\pi}{4}\right)$

c) ________________

d) ________________

4) Find four unique polar coordinates for each point given as rectangular coordinates. Use $-360° \leq \theta \leq 360°$. Round to the nearest thousandths.

a) (5, -3)

b) (-4, 7)

a) ____________ ____________

____________ ____________

b) ____________ ____________

____________ ____________

Use $-2\pi \leq \theta \leq 2\pi$. Round to the nearest thousandths.

c) $(-2, 2\sqrt{3})$

d) (-4, -3)

c) ____________ ____________

____________ ____________

d) ____________ ____________

____________ ____________

7.03 Quiz Review WS #2 Date Key

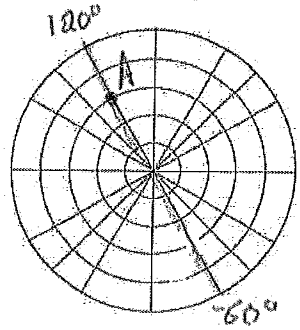
Polar Coordinates, Equations, and Distance

1) Graph each point on the polar grid. Find three other pairs of polar coordinates that name the point

if $-360° \leq \theta \leq 360°$ | if $-2\pi \leq \theta \leq 2\pi$

$\rightarrow \theta = -60+360 = 300°$

a) $A(-3, -60°)$



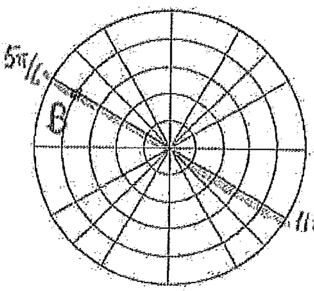
$(-3, 300°)$

$(3, 120°)$ -360

$(3, -240°)$

$\theta = \frac{5\pi}{6} - 2\pi = \frac{-7\pi}{6}$

b) $B\left(4, \frac{5\pi}{6}\right)$

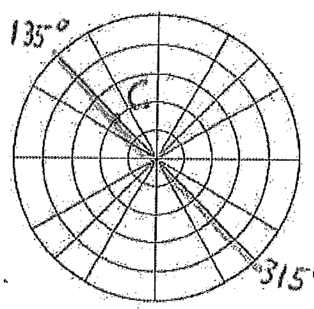


$\left(4, \frac{-7\pi}{6}\right)$

$\left(-4, \frac{11\pi}{6}\right)$ -2π

$\left(-4, -\frac{\pi}{6}\right)$

c) $C(2, 135°)$

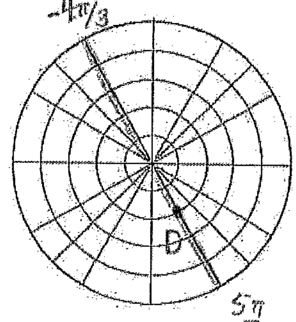


$(2, -225°)$

$(-2, 315°)$ -360

$(-2, -45°)$

d) $D\left(-2, -\frac{4\pi}{3}\right)$ $\theta = -\frac{4\pi}{3} + 2\pi = \frac{2\pi}{3}$



$\left(-2, \frac{2\pi}{3}\right)$

$\left(2, \frac{5\pi}{3}\right)$ -2π

$\left(2, -\frac{\pi}{3}\right)$

2) Given the polar distance formula between two points $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$:

$AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$, find the distance between A and B.

* degree mode | * Radian Mode

a) $A(2, 120°)$ $B(-4, 45°)$ (r_1, θ_1; r_2, θ_2)

$= \sqrt{2^2 + 4^2 - 2(2)(-4)\cos(45-120)}$

$= \sqrt{20 + 16\cos(-75)}$

a) AB = 4.913 units

b) $A\left(3, \frac{5\pi}{3}\right)$ $B\left(-4, -\frac{\pi}{4}\right)$ (r_1, θ_1; r_2, θ_2)

$= \sqrt{3^2 + 4^2 - 2(3)(-4)\cos\left(-\frac{\pi}{4} - \frac{5\pi}{3}\right)}$

$\sqrt{25 + 24\cos\left(-\frac{23\pi}{12}\right)}$

b) AB = 6.941 units

3) Find the rectangular coordinates for each point with the given polar coordinates. Answer in exact form. $(r\cos\theta, r\sin\theta)$

a) $\left(-2, -\frac{4\pi}{3}\right)$ (r_1, θ_1)

$\left(-2\cos\left(\frac{-4\pi}{3}\right), -2\sin\left(\frac{-4\pi}{3}\right)\right)$

$\left(-2\cdot(-\frac{1}{2}), -2(+\frac{\sqrt{3}}{2})\right)$

a) $(1, -\sqrt{3})$

b) $\left(5, \frac{5\pi}{4}\right)$

$\left(5\cos(5\pi/4), 5\sin(5\pi/4)\right)$

$\left(5(-\frac{\sqrt{2}}{2}), 5(-\frac{\sqrt{2}}{2})\right)$

b) $\left(\frac{-5\sqrt{2}}{2}, \frac{-5\sqrt{2}}{2}\right)$

c) $\left(3, \frac{11\pi}{6}\right)$

$3\cos\left(\frac{11\pi}{6}\right), 3\sin\left(\frac{11\pi}{6}\right)$

$3(\frac{\sqrt{3}}{2}), 3(-\frac{1}{2})$

c) $\frac{3\sqrt{3}}{2}, -3/2$

d) $\left(-5, -\frac{7\pi}{4}\right)$

$-5\cos\left(\frac{-7\pi}{4}\right), -5\sin\left(\frac{-7\pi}{4}\right)$

$-5(\frac{\sqrt{2}}{2}), -5(\frac{\sqrt{2}}{2})$

d) $\left(\frac{-5\sqrt{2}}{2}, \frac{-5\sqrt{2}}{2}\right)$

4) Find four unique polar coordinates for each point given as rectangular coordinates. Use $-360° \le \theta \le 360°$. Round to the nearest thousandths.

a) (5, -3) Q4

$r = \sqrt{5^2+3^2} = \sqrt{34}$

$\theta = \tan^{-1}\left(\frac{-3}{5}\right) \rightarrow -30.964 + 360$

$\theta = 329.04°$

b) (-4, 7) Q2

$r = \sqrt{4^2+7^2} = \sqrt{65}$

$\theta = \tan^{-1}\left(\frac{7}{-4}\right) = -60.255° + 180 = 119.745°$

a) $(\sqrt{34}, 329.04°)$ $\xrightarrow{-360}$ $(\sqrt{34}, -30.964°)$

$-180°$: $(-\sqrt{34}, 149.036°)$ $\xrightarrow{-360}$ $(-\sqrt{34}, -210.964°)$

b) $(\sqrt{65}, 119.745°)$ $\xrightarrow{-360°}$ $(\sqrt{65}, -240.255°)$

$+180°$: $(-\sqrt{65}, 299.745°)$ $\xrightarrow{-360}$ $(-\sqrt{65}, -60.255°)$

Use $-2\pi \le \theta \le 2\pi$. Round to the nearest thousandths.

Q2 c) $(-2, 2\sqrt{3})$

$r = \sqrt{2^2+(2\sqrt{3})^2} = 4$

$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right) = -1.047 + \pi = 2.0943$

d) (-4, -3) Q3

$r = \sqrt{4^2+3^2} = 5$

$\theta = \tan^{-1}\left(\frac{-3}{-4}\right) = 0.644 + \pi = 3.785$

c) $(4, 2.094)$ $\xrightarrow{-2\pi}$ $(4, -4.189)$

$+\pi$: $(-4, 5.236)$ $\xrightarrow{-2\pi}$ $(-4, -1.048)$

d) $(5, 3.785)$ $\xrightarrow{-2\pi}$ $(5, -2.498)$

$-\pi$: $(-5, 0.644)$ $\xrightarrow{-2\pi}$ $(-5, -5.639)$

* Divide exponent by 4, find ~~Remainder~~ Remainder

R1 → 0.25 → i
R2 → 0.50 → i^2
R3 → 0.75 → i^3
No Remainder → i^4

7.04 Complex Numbers Review

Recall that the imaginary number i is defined such that $i^2 = -1$.

1. $i = \sqrt{-1}$
2. $i^2 =$ -1 ($i \cdot i$)
3. $i^3 =$ $-i$ ($i^2 \cdot i$)
4. $i^4 =$ $+1$ ($i^2 \cdot i^2$)
5. $i^{11} =$ $-i$ ($11 \div 4 = 2$ R3; $11 - 8 = 3$)
6. $i^{18} =$ -1 ($18 \div 4 = 4$ R2; $18 - 16 = 2$)
7. $i^{67} =$ $-i$ ($67 \div 4 = 16$ R3; $67 - 4 = 27$, $27 - 24 = 3$)
8. $i^{724} =$ 1 ($724 \div 4 = 181$ R0)

A complex number has two parts: the real part and the imaginary part.

9. The standard form for complex number is $a + bi$.

Perform the given operation. Write your answer in standard form of a complex number.

10. $(-4 + 7i) + (2 - 3i) =$ ________

11. $(7 - 12i) - (4 + 9i) =$ $3 - 21i$

$7 - 12i - 4 - 9i$

12. $(5 + 8i) \cdot (2 - 10i) =$ ________

13. $(3 + 4i) \cdot (-8 + 2i) =$ $-32 - 26i$

$-24 + 6i - 32i + 8i^2$

$8(-1)$

$-24 \quad -8$

14. $(3 - 5i) \cdot (3 - 5i) =$ ________

15. $(3 - 5i) \cdot (3 + 5i) =$ $\boxed{34}$

$9 + 15i - 15i - 25i^2$

$9 - 25(-1)$

$9 + 25$

#14: two factors that are the exact same multiplied together (just like a binomial squared). #15: two factors that only have the sign in the middle changed. They are conjugates.

Which product resulted in an entirely real value having no imaginary part? #15

Generalize it as a formula by simplifying: $(a + bi) \cdot (a - bi) = \boxed{a^2 + b^2}$

$a^2 - \cancel{abi} + \cancel{abi} - b^2i^2$

$a^2 - b^2(-1)$

State the conjugate $(a - bi)$ of the given complex number $(a + bi)$.

16. $9 + 4i$ $\underline{9-4i}$ 17. $5 - 2i$ $\underline{5+2i}$ 18. $-3 + 7i$ $\underline{-3-7i}$

Use the conjugate of the denominator to rationalize the following fractions.

19. $\frac{1+i}{5-2i} = \underline{\frac{3}{29} + \frac{7}{29}i}$

$$\frac{(1+i)}{(5-2i)} \cdot \frac{(5+2i)}{(5+2i)} = \frac{5+2i+5i+2i^2}{25+4} = \frac{5+7i+2(-1)}{25+4}$$

$$\frac{3+7i}{29} \rightarrow \boxed{\frac{3}{29} + \frac{7}{29}i}$$

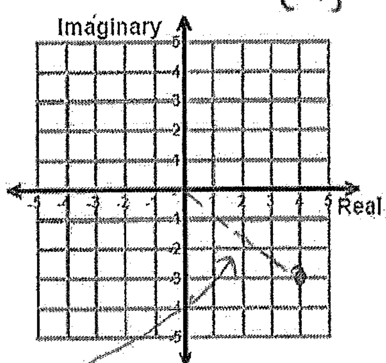
20. $\frac{5-6i}{-3+7i} = \underline{\frac{-57}{58} - \frac{17}{58}i}$

$$\frac{(5-6i)(-3-7i)}{-3+7i \quad -3-7i} \rightarrow \frac{-15-35i+18i+42i^2}{9+49}$$

$$\rightarrow \frac{-57-17i}{58} \rightarrow \boxed{\frac{-57}{58} - \frac{17}{58}i}$$

Graph the number on the complex plane and find its absolute value (distance from zero).

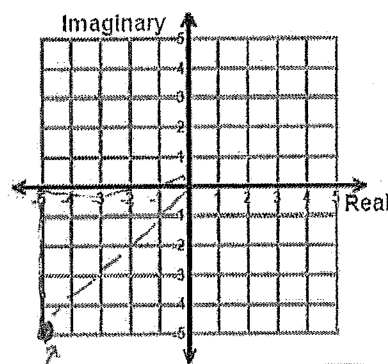
21. $4 - 3i$ $(4, -3)$



$|4 - 3i| = \underline{\sqrt{3^2 + 4^2} = 5}$

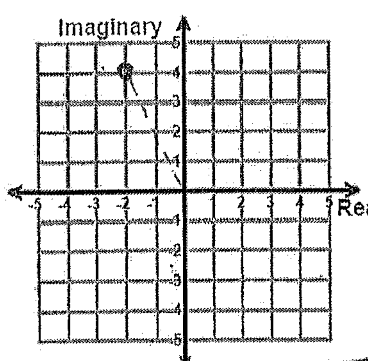
22. $-5 - 5i$

$\sqrt{(-5)^2 + (-5)^2}$



$|-5 - 5i| = \underline{\sqrt{5^2 + 5^2}} = \boxed{5\sqrt{2}}$

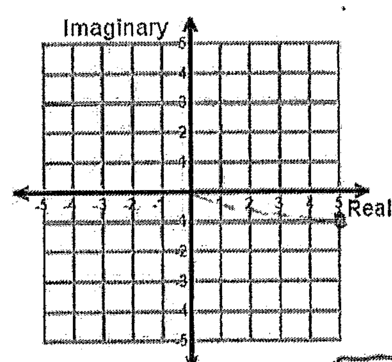
23. $-2 + 4i$



$|-2 + 4i| = \underline{\sqrt{2^2 + 4^2}}$

$\sqrt{20} = \boxed{2\sqrt{5}}$

24. $5 - i$

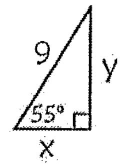


$|5 - i| = \underline{\sqrt{5^2 + 1^2}} = \boxed{\sqrt{26}}$

7.05 Complex Numbers in Rectangular Form Date: ____________

Opener: where we have been this year?

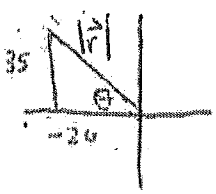
1. From right triangle trigonometry: In the triangle to the right, find x and y.



$\cos(55) = \frac{x}{9}$ | $\sin(55) = \frac{y}{9}$

$\boxed{x = 9\cos 55}$ $\boxed{y = 9\sin(55)}$

2. From vectors: For a bird flying 20m West and 35m North, find the resulting magnitude and direction (measured in standard position) of its flight.



$|\vec{r}| = \sqrt{20^2 + 35^2} = \sqrt{1625} = \boxed{5\sqrt{65}}$

$\theta = \tan^{-1}\left(\frac{35}{-20}\right) = -60.255^\circ + 180 = \boxed{119.745^\circ}$

Q2

3. From polar coordinates: convert (-2, -2) from rectangular form into polar form. Q3

$r = \sqrt{2^2 + 2^2} = \sqrt{8}$ | $\theta = \tan^{-1}\left(\frac{-2}{-2}\right) = 45^\circ + 180$

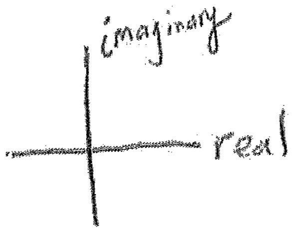
$\boxed{r = 2\sqrt{2}}$ $\boxed{\theta = 225^\circ}$

Complex Numbers:

Rectangular Form, also known as *Standard Form*:

$a + bi$

Graphing a complex number:



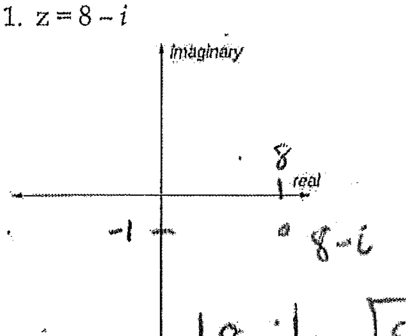
Absolute Value of a complex number, also known as the *modulus*:

$|a + bi| = \sqrt{a^2 + b^2}$

distance (use pythagorean theorem)

Examples: Graph each number in the complex plane and find its modulus.

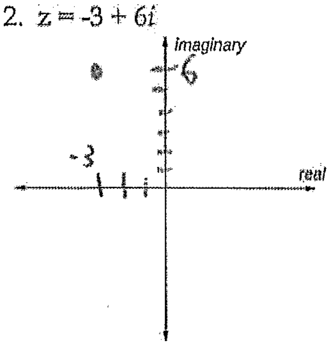
1. $z = 8 - i$



$|8 - i| = \sqrt{8^2 + 1^2}$

$= \boxed{\sqrt{65}}$

2. $z = -3 + 6i$



$|-3 + 6i| = \sqrt{3^2 + 6^2}$

$= \sqrt{45}$

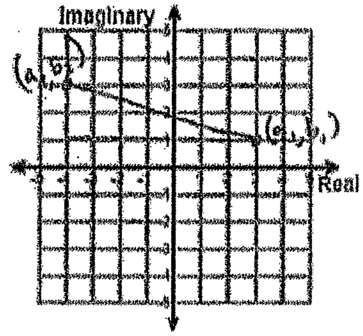
$= \boxed{3\sqrt{5}}$

Distance & Midpoint between Complex Numbers

Investigation: Find the distance between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

First, a visual usually helps, so plot the complex numbers.

How would you find the distance between those two points?



$$d=\sqrt{(a_1-a_2)^2+(b_1-b_2)^2}$$

same

$$z_1-z_2 = a_1-a_2+b_1-b_2$$

$$|z_1-z_2| = \sqrt{(a_1-a_2)^2+(b_1-b_2)^2}$$

Formula: The distance between two complex numbers is

$$d=|z_1-z_2| \quad \text{or} \quad d=\sqrt{(a_1-a_2)^2+(b_1-b_2)^2}$$

Examples: Find the distance between the two complex numbers.

1) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

$$z_1-z_2 = 5-(-1)+(-3i)-(-8i)$$

$$6+5i$$

$$|z_1-z_2| = \sqrt{6^2+5^2} = \sqrt{61}$$

2) $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

$$z_1-z_2 = -9-3i$$

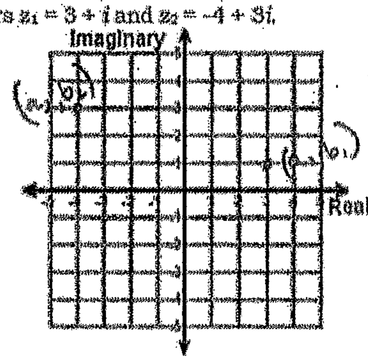
$$|z_1-z_2| = \sqrt{(-9)^2+(-3)^2} = \sqrt{90}$$

$$3\sqrt{10}$$

Investigation: Find the midpoint between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

Again, plot the complex numbers so that you can "see" this.

How would you find the midpoint between the two points?



rectangular points: midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

complex #'s: midpoint: $\frac{a_1+a_2}{2} + \frac{b_1+b_2}{2}i$

$$\frac{z_1+z_2}{2} = \frac{a_1+a_2+(b_1+b_2)i}{2}$$

Formula: The midpoint between two complex numbers is

$$m=\frac{z_1+z_2}{2} \qquad m=\frac{a_1+a_2}{2}+\frac{b_1+b_2}{2}i$$

Example: Find the midpoint between the two complex numbers

3) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

$$m=\frac{z_1+z_2}{2} = \frac{5+(-1)+(-3i)+(-8i)}{2}$$

$$\frac{4-11i}{2} = \boxed{2-\frac{11}{2}i}$$

4. $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

$$\frac{z_1+z_2}{2} = \frac{-7+11i}{2}$$

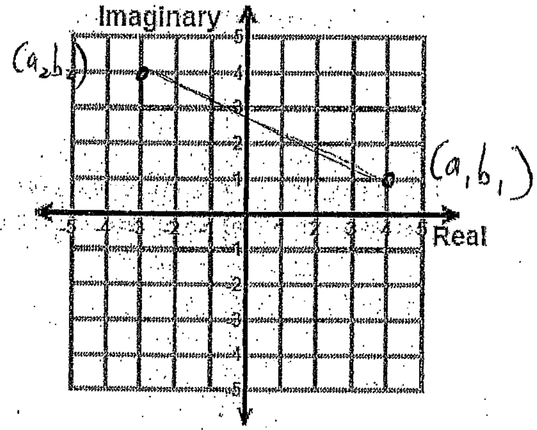
$$= \boxed{-\frac{7}{2}+\frac{11}{2}i}$$

Distance & Midpoint between Complex Numbers

Investigation: Find the distance between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

First, a visual usually helps, so plot the complex numbers.

How would you find the distance between those two points?



$$d = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}$$

Formula: The distance between two complex numbers is

$$d = |z_1 - z_2| \quad \text{or} \quad d = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}$$

Examples: Find the distance between the two complex numbers.

1) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

$(5 - -1) + (-3 - -8)i$

$|6 + 5i| = \sqrt{6^2 + 5^2}$

$= \sqrt{36 + 25} = \boxed{\sqrt{61}}$

2) $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

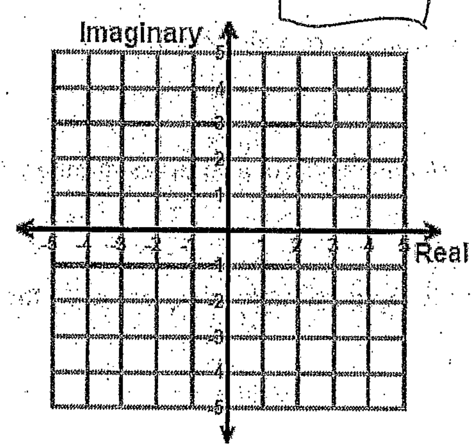
$-8 - 1 + (4 - 7)i$

$|-9 - 3i| = \sqrt{9^2 + 3^2} = \boxed{\sqrt{90} \text{ or } = 3\sqrt{10}}$

Investigation: Find the midpoint between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

Again, plot the complex numbers so that you can "see" this.

How would you find the midpoint between the two points?



Midpoint Formula: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Formula: The midpoint between two complex numbers is

$$m = \frac{z_1 + z_2}{2}$$

Example: Find the midpoint between the two complex numbers

3) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

$\dfrac{5 - 1}{2} + \dfrac{-3 - 8}{2}i \rightarrow \boxed{2 - \dfrac{11}{2}i}$

4. $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

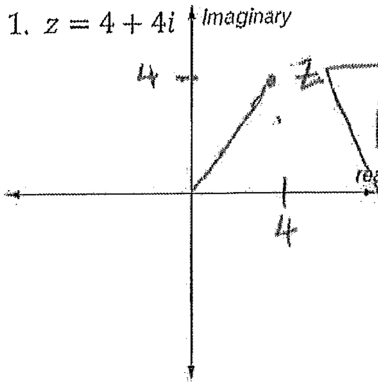
$\dfrac{-8 + 1}{2} + \dfrac{4 + 7}{2}i = \boxed{\dfrac{-7}{2} + \dfrac{11}{2}i}$

Classwork p. 15
1–12 all
p. 17 #1–2

7.05 Homework: Rectangular Form of Complex Numbers

Plot each complex number and find its modulus.

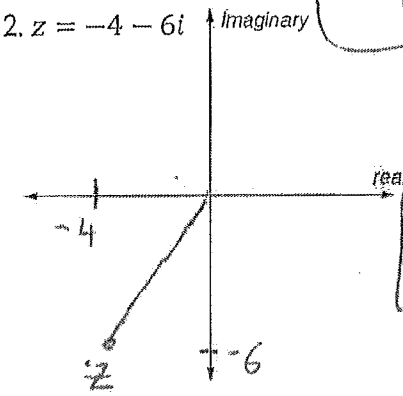
1. $z = 4 + 4i$



$|z| = \sqrt{4^2+4^2}$

$\boxed{|z| = \sqrt{32} \text{ or } 4\sqrt{2}}$

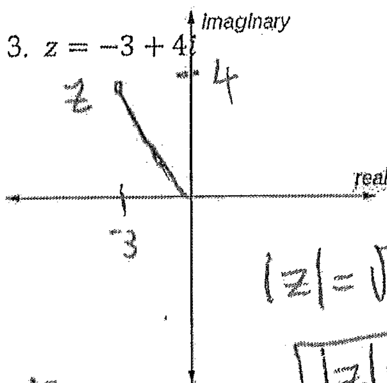
2. $z = -4 - 6i$



$|z| = \sqrt{4^2+6^2}$

$\boxed{|z| = \sqrt{52} = 2\sqrt{13}}$

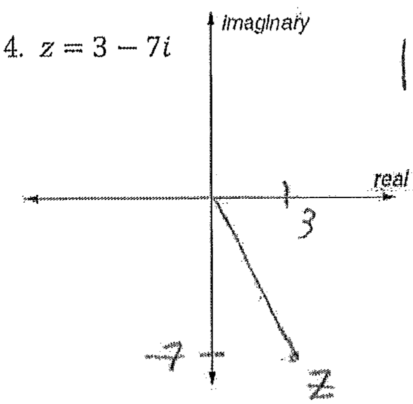
3. $z = -3 + 4i$



$|z| = \sqrt{3^2+4^2}$

$\boxed{|z| = 5}$

4. $z = 3 - 7i$



$|z| = \sqrt{3^2+7^2}$

$\boxed{|z| = \sqrt{58}}$

$|z_1 - z_2|$ Find the distance between the points in the complex plane.

5. $1 + 2i, -1 + 4i$

$1-(-1)+(2-4)i$

$|2 - 2i| = \sqrt{2^2+2^2} = \boxed{\sqrt{8} = 2\sqrt{2}}$

6. $-5 + i, -2 + 5i$

$-5-(-2)+(1-5)i$

$|-3-4i| = \sqrt{3^2+4^2} = \boxed{5}$

7. $6i, 3 - 4i$

$0+6i, 3-4i$

$0-3+(6--4)i$

$|-3+10i|$

$\sqrt{3^2+10^2} = \boxed{\sqrt{109}}$

8. $-7 - 3i, 3 + 5i$

$-7-3+(-3-5)i$

$|-10-8i| = \sqrt{10^2+8^2} = \boxed{\sqrt{164} = 2\sqrt{41}}$

$\frac{z_1+z_2}{2}$ Find the midpoint of the segment connecting the points in the complex plane.

9. $2 + i, 6 + 5i$

$\frac{2+6}{2} + \frac{1+5}{2}i \rightarrow \frac{8}{2} + \frac{6}{2}i$ $\boxed{4+3i}$

10. $-3 + 4i, 1 - 2i$

$\frac{-3+1}{2} + \frac{4-2}{2}i \rightarrow \frac{-2}{2} + \frac{2}{2}i$ $\boxed{-1+1i}$

11. $7i, 9 - 10i$

$0+7i, 9-10i$

$\frac{0+9}{2} + \frac{7-10}{2}i$ $\boxed{\frac{9}{2} - \frac{3}{2}i}$

12. $-1 + \frac{1}{2}i, \frac{1}{2} + \frac{1}{4}i$

$\frac{-1+\frac{1}{2}}{2} + \frac{\frac{1}{2}+\frac{1}{4}}{2}i$ $\boxed{\frac{-1}{4} + \frac{3}{8}i}$

7.06 Adding & Subtracting Complex Numbers Geometrically

Date: ______________________

Recall that complex numbers take the form a + b*i*.
When adding or subtracting complex numbers algebraically, real parts are added together or subtracted then imaginary parts are added together or subtracted - similar to combining like terms.

Complex numbers can also be added or subtracted geometrically/graphically by plotting the points in the complex plane and creating vectors with them. Then using geometric vector addition or subtraction.

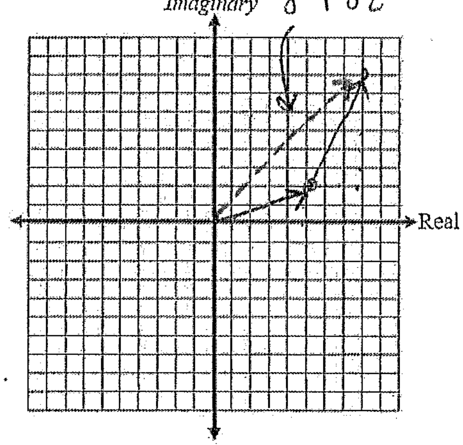
Examples: 1. $(5 + 2i) + (3 + 6i) =$

2. $(-3 + 4i) + (-10i) =$ $-3-6i$

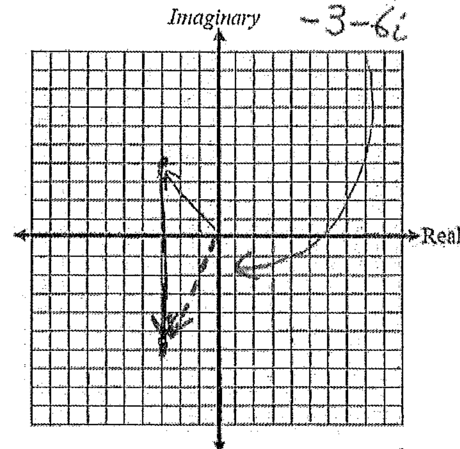
Algebraically: $8+8i$

Geometrically:

8+8i



−3−6i



3. $(5 - i) - (6 - 5i) =$

Algebraically: $5-6 \quad -1i+5i$

$\boxed{-1+4i}$

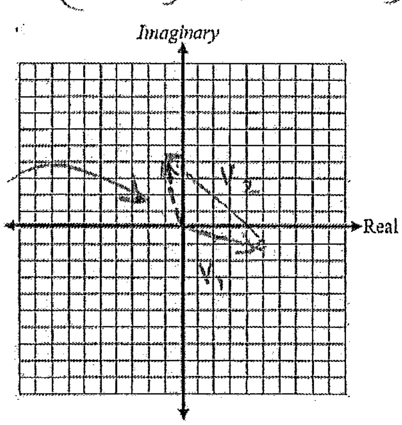
4. $(-8 - 2i) - (-10 - 8i) =$

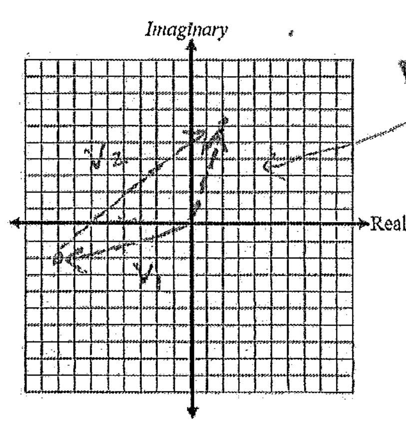
$-8-2i+10+8i$ → $\boxed{2+6i}$

Geometrically: $(5-i)+(-6+5i)$

$-8-2i+(10+8i)$

$\boxed{-1+4i}$

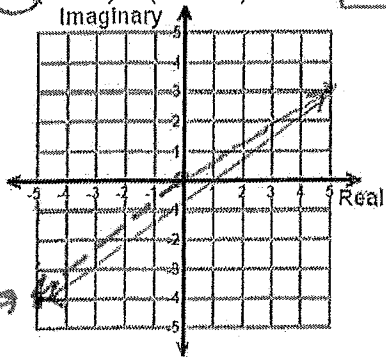




$\boxed{2+6i}$

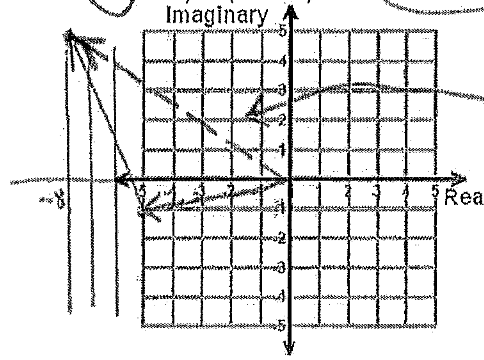
7.06 Practice: Evaluate each sum or difference geometrically, then verify your answer using algebra.

1. $(5 + 3i) + (-10 - 7i)$ $\boxed{-5-4i}$



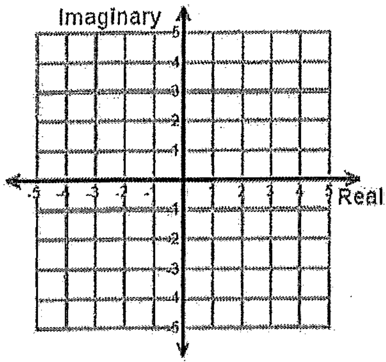
$\boxed{-5-4i}$

2. $(-5 - i) + (-3 + 6i)$ $\boxed{-8+5i}$

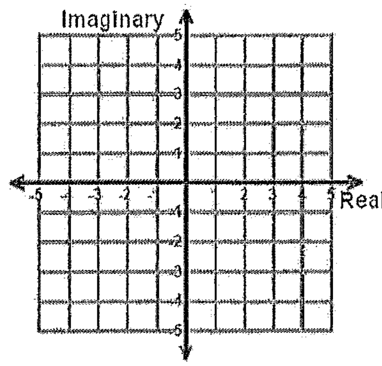


$\boxed{-8+5i}$

3. $(4 - 2i) + (-6 - 2i)$

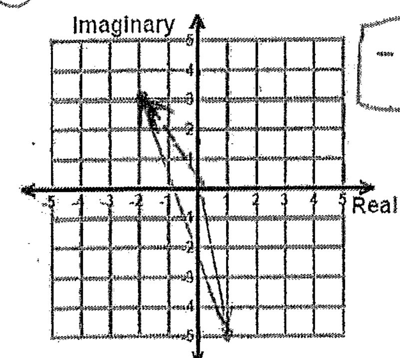


4. $-4i + (3 - i)$

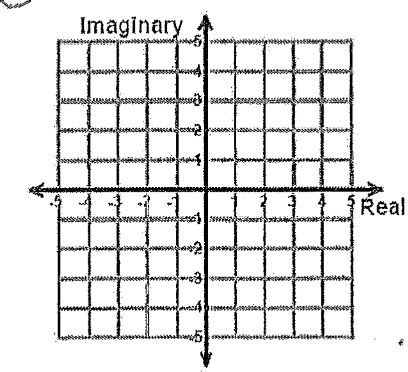


5. $(1 - 5i) - (3 - 8i)$ $(1-5i)+(-3+8i)$

$\boxed{-2+3i}$

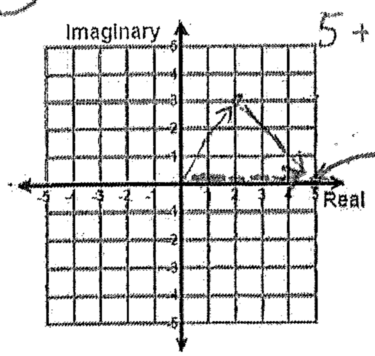


6. $4i - (4 + i)$



7. $(2 + 3i) - (-3 + 3i)$ $(2+3i)+(3-3i)$

$5+0i = \boxed{5}$

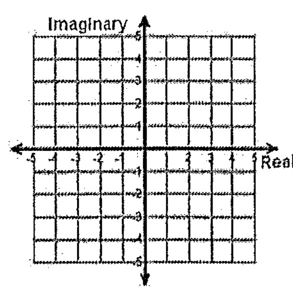


$5+0i$

or

$\boxed{5}$

8. $(-5 - 5i) - (-4 - 2i)$



7.07 Complex Numbers in Polar Form

formal

Date: ______________

Polar Form of Complex Numbers, also know as *Trigonometric Form*:

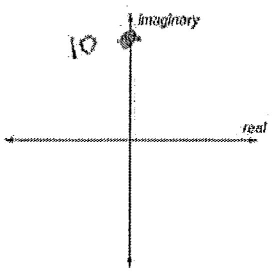
(shorthand)

$$z = r(\cos\theta + i\sin\theta) \to \text{or } \; r\,\text{cis}\,\theta$$

$a = r\cos\theta$	$r = \sqrt{a^2+b^2}$
$b = r\sin\theta$	$\tan\theta = \frac{b}{a}$
$a+bi$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Examples: Graph each complex number on the rectangular plane. Then, find its polar form, where $0 \le \theta \le 2\pi$. Be exact.

1. $z = 10i$ $\quad z = 0 + 10i$

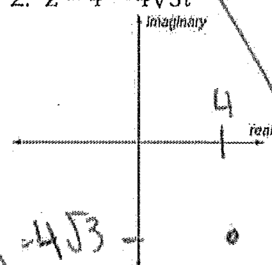


$r = 10$

$\theta = \pi/2$

$z = 10\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \to 10\,\text{cis}\,\frac{\pi}{2}$

2. $z = 4 - 4\sqrt{3}i$



$r = \sqrt{4^2 + (4\sqrt{3})^2} = 8$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

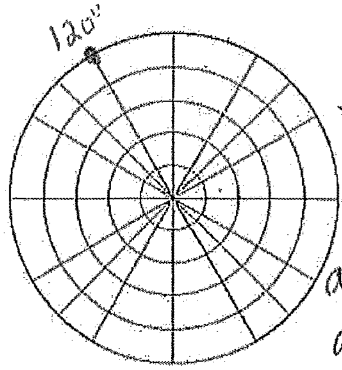
$\theta = \tan^{-1}\left(\frac{-4\sqrt{3}}{4}\right) = -60 + 360$

$\theta = 300° \cdot \frac{\pi}{180}$ $\quad \boxed{\theta = \frac{5\pi}{3}}$

$$\boxed{\begin{array}{c} z = 8\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) \\ z = 8\,\text{cis}\,\frac{5\pi}{3} \end{array}}$$

Examples: Graph each complex number on the polar plane. Then, find its rectangular form. Be exact. $\to a+bi$

3. $5(\cos 120° + i\sin 120°)$



$r = 5$

$\theta = 120$

$*\, a = r\cos\theta$

$b = r\sin\theta$

$a = 5\cos 120$

$a = 5\left(-\frac{1}{2}\right)$

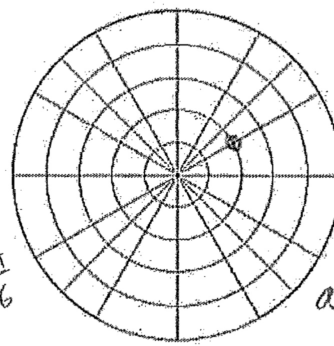
$b = 5\sin 120$

$b = 5\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{-\frac{5}{2} + \frac{5\sqrt{3}}{2}i}$

4. $-2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

$\frac{7\pi}{6}$



$r = -2$

$\theta = \frac{7\pi}{6}$

$a = r\cos\theta$	$b = r\sin\theta$
$a = -2\cos\frac{7\pi}{6}$	$b = -2\sin\left(\frac{7\pi}{6}\right)$
$a = -2\left(\frac{-\sqrt{3}}{2}\right)$	$b = -2\left(\frac{-1}{2}\right)$

$\boxed{\sqrt{3} + 1i}$

7.07 Homework: Directions: Be exact. Work these problems without using a calculator!

Write the polar form of each complex number where $0 \le \theta \le 2\pi$.

1. $z = 2 - 2i$

$r = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$ (Q4)

$\theta = \tan^{-1}\left(\frac{-2}{2}\right) = -45° + 360 = 315°$

$\theta = \frac{7\pi}{4}$ $z = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$\boxed{z = 2\sqrt{2}\ \text{cis}\ \frac{7\pi}{4}}$

2. $z = 3 + 3i$

$r = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{3}{3}\right) = 45° = \frac{\pi}{4}$ (Q1)

$z = 3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\boxed{z = 3\sqrt{2}\ \text{cis}\ \frac{\pi}{4}}$

3. $z = -\sqrt{3} + i$

Q2

$\theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = -30 + 180 = 150°$

$\theta = \frac{5\pi}{6}$

$r = \sqrt{\sqrt{3}^2 + 1}$

$r = 2$

$\boxed{z = 2\,\text{cis}\left(\frac{5\pi}{6}\right)}$

4. $z = -5 - 5\sqrt{3}i$

Q3

$r = \sqrt{5^2 + (5\sqrt{3})^2}$

$r = 10$

$\theta = \tan^{-1}\left(\frac{-5\sqrt{3}}{-5}\right) = 60 + 180$

$\theta = 240° = \frac{4\pi}{3}$

$\boxed{z = 10\ \text{cis}\ \frac{4\pi}{3}}$

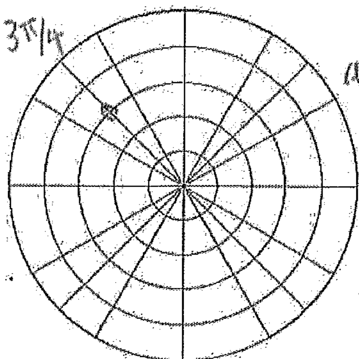
Graph each number on a polar grid. Then express it in rectangular form.

5. $z = 3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$r = 3$

$\theta = \frac{3\pi}{4}$

3π/4



$a = r\cos\theta$

$a = 3\cos\left(\frac{3\pi}{4}\right)$

$a = 3\left(\frac{-\sqrt{2}}{2}\right)$

$b = r\sin\theta$

$b = 3\sin\left(\frac{3\pi}{4}\right)$

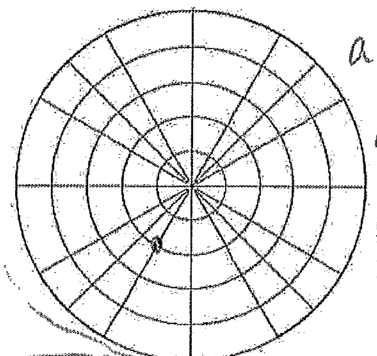
$b = 3\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\frac{-3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i}$

6. $z = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$r = 2$

$\theta = \frac{4\pi}{3}$

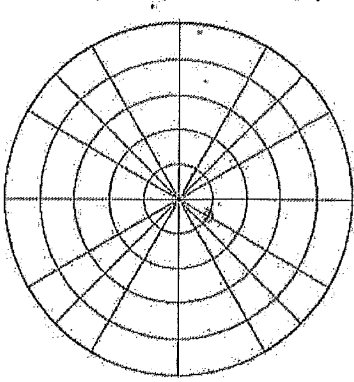


$a = 2\cos\left(\frac{4\pi}{3}\right)$ | $b = 2\sin\left(\frac{4\pi}{3}\right)$

$a = 2\left(\frac{-1}{2}\right)$ | $b = 2\left(\frac{-\sqrt{3}}{2}\right)$

$\boxed{-1 - \sqrt{3}i}$

7. $z = \left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$



$r = 1$

$\theta = 11\pi/6$

$a = 1\cos\left(\frac{11\pi}{6}\right)$ | $b = 1\sin\left(\frac{11\pi}{6}\right)$

$a = 1\left(\frac{\sqrt{3}}{2}\right)$ | $b = 1\left(-\frac{1}{2}\right)$

$\rightarrow \boxed{\frac{\sqrt{3}}{2} - \frac{1}{2}i}$

7.08 Operations with Complex Numbers in Polar Form Date: ____________

Find the *product* of two complex numbers in polar form: derive the formula.

For $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 \cdot z_2 =$ $r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$

$$z_1 \cdot z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$$

Examples: Find the product of the complex numbers in polar form. Answer in both polar form and rectangular form.

1. $z_1 = 4(\cos 225° + i\sin 225°)$ and $z_2 = 3(\cos 90° + i\sin 90°)$

$r_1 \quad \theta_1 \qquad r_2 \quad \theta_2$

$$z_1 \cdot z_2 = 4\cdot 3\left[\cos(225+90) + i\sin(225+90)\right]$$
$$= 12\left[\cos(315) + i\sin(315)\right] \text{ or } \boxed{12\,\text{cis}(315)}$$

Rectangular Form $\left|\begin{array}{l} a = r\cos\theta \\ b = r\sin\theta \end{array}\right|$ $a+bi$ $\left|\begin{array}{l} a = 12\cos 315 \to 12\left(\frac{\sqrt{2}}{2}\right) \\ b = 12\sin 315 \to 12\left(-\frac{\sqrt{2}}{2}\right) \end{array}\right|$

Rectangular Form $\boxed{6\sqrt{2} - 6\sqrt{2}i}$

2. $z_1 = \sqrt{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ and $z_2 = \frac{1}{5}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$$z_1 \cdot z_2 = \sqrt{2}\cdot\frac{1}{5}\left[\cos\left(\frac{2\pi}{3}+\frac{\pi}{6}\right) + i\sin\left(\frac{2\pi}{3}+\frac{\pi}{6}\right)\right]$$

Polar Form

$$= \frac{\sqrt{2}}{5}\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right] \to \boxed{\frac{\sqrt{2}}{5}\,\text{cis}\left(\frac{5\pi}{6}\right)}$$

$$a = \frac{\sqrt{2}}{5}\cos\left(\frac{5\pi}{6}\right) \to \frac{\sqrt{2}}{5}\left(-\frac{\sqrt{3}}{2}\right)$$
$$b = \frac{\sqrt{2}}{5}\sin\left(\frac{5\pi}{6}\right) \to \frac{\sqrt{2}}{5}\left(\frac{1}{2}\right)$$

$$\boxed{\frac{-\sqrt{6}}{10} + \frac{\sqrt{2}}{10}i}$$

Rectangular Form

Division is the opposite operation from Multiplication. How do you think the pattern changes when we divide two complex numbers in polar form?

For $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 \div z_2 = \frac{z_1}{z_2} =$ $\frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$

$\frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$

Example: Find the quotient of the complex numbers in polar form: $\frac{z_1}{z_2}$. Write the answer in both polar form and rectangular form.

3. $z_1 = 2(\cos 210° + i\sin 210°)$ and $z_2 = 8(\cos 60° + i\sin 60°)$

$\frac{z_1}{z_2} = \frac{2}{8}\left[\cos(210-60) + i\sin(210-60)\right]$

$\frac{1}{4}\left[\cos 150 + i\sin 150\right] \rightarrow \boxed{\frac{1}{4}\text{cis}(150)}$ polar

$a = r\cos\theta$ | $a = \frac{1}{4}\cos 150 \rightarrow \frac{1}{4}\left(\frac{-\sqrt{3}}{2}\right)$

$b = r\sin\theta$ | $b = \frac{1}{4}\sin 150 \rightarrow \frac{1}{4}\left(\frac{1}{2}\right)$

$\rightarrow \boxed{-\frac{\sqrt{3}}{8} + \frac{1}{8}i}$ Rectangular

4. $z_1 = \frac{2}{5}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ and $z_2 = \frac{1}{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$\frac{z_1}{z_2} = \frac{\frac{2}{5}}{1/2}\left[\cos\left(\frac{\pi}{2} - \frac{5\pi}{4}\right) + i\sin\left(\frac{\pi}{2} - \frac{5\pi}{4}\right)\right] \rightarrow \frac{2}{5}\cdot\frac{2}{1}\left[\cos\left(\frac{-3}{4}\pi\right) + i\sin\left(\frac{-3\pi}{4}\right)\right]$

$\boxed{\frac{4}{5}\text{cis}\left(\frac{-3\pi}{4}\right)}$ polar

$a = \frac{4}{5}\cos\left(\frac{-3\pi}{4}\right) \rightarrow \frac{4}{5}\cos\left(\frac{5\pi}{4}\right) \rightarrow \frac{4}{5}\left(\frac{-\sqrt{2}}{2}\right)$

$b = \frac{4}{5}\sin\left(\frac{-3\pi}{4}\right) \rightarrow \frac{4}{5}\sin\left(\frac{5\pi}{4}\right) \rightarrow \frac{4}{5}\left(\frac{-\sqrt{2}}{2}\right)$

$\boxed{\frac{-2\sqrt{2}}{5} - \frac{2\sqrt{2}}{5}i}$ rectangular

7.08 Practice: Simplify. Express answers in **both** polar form and in rectangular form. Match angle measurement units to the problem, where $0° \leq \theta \leq 360°$ or $0 \leq \theta \leq 2\pi$.

1. $6\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \cdot 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

1) $6 \cdot 4\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\right]$

$24\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$

$\boxed{24\ cis\left(\frac{3\pi}{4}\right)}$

$a = 24\cos\left(\frac{3\pi}{4}\right) = 24\left(-\frac{\sqrt{2}}{2}\right)$

$b = 24\sin\left(\frac{3\pi}{4}\right) = 24\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{-12\sqrt{2} + 12\sqrt{2}i}$

2. $5(\cos 135° + i\sin 135°) \cdot 2(\cos 45° + i\sin 45°)$

2) $10\left[\cos(135+45) + i\sin(135+45)\right]$

$10\left[\cos 180 + i\sin 180\right] \rightarrow \boxed{10\ cis(180)}$

$a = 10\cos 180 \rightarrow 10(-1)$

$b = 10\sin 180 \rightarrow 10(0)$

$\boxed{-10 + 0i \text{ or } -10}$

Division

3. $3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div \frac{1}{2}(\cos\pi + i\sin\pi)$

3) $\frac{3}{1/2}\left[\cos\left(\frac{3\pi}{4}-\pi\right) + i\sin\left(\frac{3\pi}{4}-\pi\right)\right]$

$3 \cdot \frac{2}{1}\left[\cos\left(\frac{-\pi}{4}\right) + i\sin\left(-\pi/4\right)\right]$

$6\left[\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right]$

$\boxed{6\ cis\left(\frac{7\pi}{4}\right)}$

$a = 6\cos(7\pi/4) \rightarrow 6\left(\frac{\sqrt{2}}{2}\right)$

$b = 6\sin(7\pi/4) \rightarrow 6\left(-\frac{\sqrt{2}}{2}\right)$

$\boxed{3\sqrt{2} - 3\sqrt{2}i}$

4. $2(\cos 90° + i\sin 90°) \cdot 2(\cos 270° + i\sin 270°)$

4) $2 \cdot 2\left[\cos(90+270) + i\sin(90+270)\right]$

$4\left[\cos 360 + i\sin 360\right]$

$\boxed{4\ cis\ 360}$

$a = 4\cos 360 \rightarrow 4(1)$

$b = 4\sin 360 \rightarrow 4(0)$

$\boxed{4 + 0i \text{ or } 4}$

5. $3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \div 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

5) $\frac{3}{4}\left[\cos\left(\frac{\pi}{6}-\frac{2\pi}{3}\right) + i\sin\left(\frac{\pi}{6}-\frac{2\pi}{3}\right)\right]$

$\frac{3}{4}\left[\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right]$

$\frac{3}{4}\left[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right] \rightarrow \boxed{\frac{3}{4}\ cis\left(\frac{3\pi}{2}\right)}$

$a = \frac{3}{4}\cos\left(\frac{3\pi}{2}\right) \rightarrow \frac{3}{4}(0)$

$b = \frac{3}{4}\sin\left(\frac{3\pi}{2}\right) \rightarrow \frac{3}{4}(-1)$

$0 - \frac{3}{4}i$

or $\boxed{-\frac{3}{4}i}$

$\frac{9}{4} - \frac{6}{4} = \frac{3}{4}$

6. $4\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right) \div 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

6) $\frac{4}{2}\left[\cos\left(\frac{9\pi}{4} - \frac{3\pi}{2}\right) + i\sin\left(\frac{9\pi}{4} - \frac{3\pi}{2}\right)\right]$

$2\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right] \rightarrow \boxed{2\,\text{cis}\left(\frac{3\pi}{4}\right)}$

$a = 2\cos\left(\frac{3\pi}{4}\right) \rightarrow 2\left(\frac{-\sqrt{2}}{2}\right)$

$b = 2\sin\left(\frac{3\pi}{4}\right) \rightarrow 2\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{-\sqrt{2} + \sqrt{2}i}$

7. $\frac{1}{2}(\cos 60° + i\sin 60°) \cdot 6(\cos 150° + i\sin 150°)$

8. $6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

9. $5(\cos 180° + i\sin 180°) \cdot 2(\cos 135° + i\sin 135°)$

10. $\frac{1}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \div 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

6. $4\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)\div 2\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$

$\boxed{2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}$

$2\left(-\frac{\sqrt{2}}{2}\right)+2\left(\frac{\sqrt{2}}{2}\right)i$

$\boxed{-\sqrt{2}+\sqrt{2}i}$

7. $\frac{1}{2}(\cos 60^\circ+i\sin 60^\circ)\cdot 6(\cos 150^\circ+i\sin 150^\circ)$

$\boxed{3(\cos 210^\circ+i\sin 210^\circ)}$

$3\left(-\frac{\sqrt{3}}{2}\right)+3\left(-\frac{1}{2}\right)i$

$\boxed{-\frac{3\sqrt{3}}{2}-\frac{3}{2}i}$

8. $6\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)\div 2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$\boxed{3\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)}$

$3(0)+3(1)i$

$\boxed{3i}$

9. $5(\cos 180^\circ+i\sin 180^\circ)\cdot 2(\cos 135^\circ+i\sin 135^\circ)$

$\boxed{10(\cos 315^\circ+i\sin 315^\circ)}$

$10\left(\frac{\sqrt{2}}{2}\right)+10\left(-\frac{\sqrt{2}}{2}\right)i$

$\boxed{5\sqrt{2}-5\sqrt{2}i}$

10. $\frac{1}{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\div 3\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

$\boxed{\frac{1}{6}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)}$

$\frac{1}{6}\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{6}\left(\frac{1}{2}\right)i$

$\boxed{\frac{\sqrt{3}}{12}+\frac{1}{12}i}$

7.09 More Operations with Complex Numbers in Polar Form

Date: ____________

Powers are the shorthand for repeated Multiplication. How do you think the pattern changes when we raise a complex number in polar form to an exponent?

For $z = r(\cos\theta + i\sin\theta)$

$z^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right]$

$3^5(\sqrt{2})^5 \rightarrow 3^5(\sqrt{2})^4\cdot\sqrt{2} = 972\sqrt{2}$

Examples: Find the power of the complex number in polar form. Answer in both polar form and rectangular form.

$n=5$

1. $z^5 = \left[3\sqrt{2}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right]^5$

i) $(3\sqrt{2})^5\left[\cos\left(5\cdot\frac{5\pi}{6}\right) + i\sin\left(5\cdot\frac{5\pi}{6}\right)\right]$

$972\sqrt{2}\left[\cos\left(\frac{25\pi}{6}\right) + i\sin\left(\frac{25\pi}{6}\right)\right]$

or $\boxed{972\sqrt{2}\,\mathrm{cis}\left(\frac{25\pi}{6}\right)}$

$972\sqrt{2}\left(\frac{\sqrt{3}}{2}\right) + 972\sqrt{2}\left(\frac{1}{2}\right)i$

$\boxed{486\sqrt{6} + 486\sqrt{2}i}$

Investigation:

Use multiplication (in rectangular form) or the power rule (in polar form): $(-1+\sqrt{3}i)^3$

$-1+\sqrt{3}i$

$r = \sqrt{1^2+\sqrt{3}^2}$

$r = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) \rightarrow -60 + 180 = 120^\circ$ Q2

$z^3 = \left\{2\left[\cos 120 + i\sin 120\right]\right\}^3$

$2^3\left[\cos(120\cdot 3) + i\sin(120\cdot 3)\right]$

$8\left[\cos 360 + i\sin 360\right] \leftarrow$ polar form

$8(1) + 8(0)i = \boxed{8} \leftarrow$ rectangular form

Again, use either method listed above: $(-1-\sqrt{3}i)^3$

$r = \sqrt{1^2+\sqrt{3}^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = 60^\circ + 180$

$\theta = 240^\circ$

$z = 2\left[\cos 240 + i\sin 240\right]$

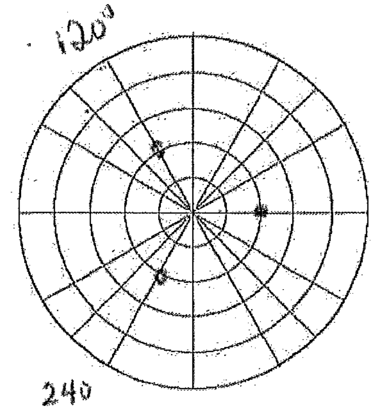
$z = 2^3\left[\cos(240\cdot 3) + i\sin(240\cdot 3)\right]$

$z = 8\left[\cos 720 + i\sin 720\right] \leftarrow$ polar

$8 + 0i = \boxed{8} \leftarrow$ rectangular

What do you notice?

What is $\sqrt[3]{8}$ equivalent to? Plot your answers in the complex plane.



$\sqrt[3]{(-1+\sqrt{3}i)^3} = 8$

$(-1-\sqrt{3}i)^3 = 8$

$\sqrt[3]{8} = 2,\ -1+\sqrt{3}i,\ -1-\sqrt{3}i$

1) $2 + 0i$

2) $2(\cos 120 + i\sin 120)$

3) $2(\cos 240 + i\sin 240)$

Can we use DeMoivre's Theorem (the Power Rule above) to derive a formula for evaluating roots of complex numbers in polar form?

$$\sqrt[n]{z} = z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\tfrac{1}{n}\theta\right) + i\sin\left(\tfrac{1}{n}\theta\right)\right]$$

or

$$r^{\frac{1}{n}}\left[\cos\left(\tfrac{\theta}{n}\right) + i\sin\left(\tfrac{\theta}{n}\right)\right]$$

Example: Find all distinct fourth roots of -5+12*i*.

$r = \sqrt{5^2+12^2} = 13$

$\theta = \tan^{-1}\left(\frac{12}{-5}\right) = -67.38 + 180 = 112.619^\circ$

Q2

① polar form

$13\left[\cos(112.619) + i\sin(112.619)\right]$

② Apply the root $z^{1/4} \rightarrow \sqrt[4]{13}\left[\cos\left(\frac{112.619}{4}\right) + i\sin\left(\frac{112.619}{4}\right)\right]$

i) $\sqrt[4]{13}\left[\cos 28.155^\circ + i\sin 28.155^\circ\right]$

③ Find the other solutions: $\frac{360}{n} \rightarrow n=4 \rightarrow \frac{360}{4} = 90^\circ$ +90

ii) $\sqrt[4]{13}\left[\cos 118.155^\circ + i\sin(118.155^\circ)\right]$ +90

iii) $\sqrt[4]{13}\left[\cos 208.155^\circ + i\sin(208.153)\right]$ +90

iv) $\sqrt[4]{13}\left[\cos 298.155 + i\sin(298.155)\right]$

7. $\left(-1+\sqrt{3}i\right)^{5}$

Convert each complex number into polar form. Then find all distinct roots of the complex number. Answer in polar form where $0 \le \theta \le 2\pi$.

$\boxed{\frac{2\pi}{6} = \frac{\pi}{3}}$ * keep adding $\frac{\pi}{3}$ to get to next solution!

8. Sixth roots of i $0+1i$

$r = 1$

$\theta = \frac{\pi}{2}$

$z = 1\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\pi/2\right)\right]$

$z^{1/6} = \sqrt[6]{1}\left[\cos\left(\frac{1}{6}\cdot\frac{\pi}{2}\right) + i\sin\left(\frac{1}{6}\cdot\frac{\pi}{2}\right)\right]$

i) $z^{1/6} = \sqrt[6]{1}\left[\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)\right]$

ii) $\sqrt[6]{1}\left[\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right]$

iii) $\sqrt[6]{1}\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$

iv) $\sqrt[6]{1}\left[\cos\left(\frac{13\pi}{12}\right) + i\sin\left(\frac{13\pi}{12}\right)\right]$

v) $\sqrt[6]{1}\left[\cos\left(\frac{17\pi}{12}\right) + i\sin\left(\frac{17\pi}{12}\right)\right]$

vi) $\sqrt[6]{1}\left[\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{24}\right)\right]$

9. Fourth roots of $4\sqrt{3} - 4i$

$r = \sqrt{(4\sqrt{3})^2 + (4)^2} = \sqrt{64} = 8$

$\theta = \tan^{-1}\left(\frac{-4}{4\sqrt{3}}\right) = -30 + 360$

$\theta = 330 \cdot \frac{\pi}{180}$

$\theta = \frac{11\pi}{6}$

$z = 8\left[\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right]$

$z^{1/4} = \sqrt[4]{8}\left[\cos\left(\frac{1}{4}\cdot\frac{11\pi}{6}\right) + i\sin\left(\frac{1}{4}\cdot\frac{11\pi}{6}\right)\right]$

i) $\sqrt[4]{8}\left[\cos\left(\frac{11\pi}{24}\right) + i\sin\left(\frac{11\pi}{24}\right)\right]$

$\frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

ii) $\sqrt[4]{8}\left[\cos\frac{23\pi}{24} + i\sin\left(\frac{23\pi}{24}\right)\right]$

iii) $\sqrt[4]{8}\left[\cos\frac{35\pi}{24} + i\sin\left(\frac{35\pi}{24}\right)\right]$

iv) $\sqrt[4]{8}\left[\cos\left(\frac{47\pi}{24}\right) + i\sin\left(\frac{47\pi}{24}\right)\right]$

10. Fifth roots of unity (1)

7. $(-1+\sqrt{3}i)^5$

Convert each complex number into polar form. Then find all distinct roots of the complex number. Answer in polar form where $0 \le \theta \le 2\pi$.

$\boxed{\frac{2\pi}{6} = \frac{\pi}{3}}$ * keep adding $\frac{\pi}{3}$ to get to next solution!

8. Sixth roots of i $0+1i$

$r = 1$

$\theta = \frac{\pi}{2}$

$z = 1\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\pi/2\right)\right]$

$z^{1/6} = \sqrt[6]{1}\left[\cos\left(\frac{1}{6}\cdot\frac{\pi}{2}\right) + i\sin\left(\frac{1}{6}\cdot\frac{\pi}{2}\right)\right]$

i) $z^{1/6} = \sqrt[6]{1}\left[\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)\right]$

ii) $\sqrt[6]{1}\left[\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right]$

iii) $\sqrt[6]{1}\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$

iv) $\sqrt[6]{1}\left[\cos\left(\frac{13\pi}{12}\right) + i\sin\left(\frac{13\pi}{12}\right)\right]$

v) $\sqrt[6]{1}\left[\cos\left(\frac{17\pi}{12}\right) + i\sin\left(\frac{17\pi}{12}\right)\right]$

vi) $\sqrt[6]{1}\left[\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right]$

9. Fourth roots of $4\sqrt{3} - 4i$

$r = \sqrt{(4\sqrt{3})^2 + (4)^2} = \sqrt{64} = 8$

$\theta = \tan^{-1}\left(\frac{-4}{4\sqrt{3}}\right) = -30 + 360$

$\theta = 330 \cdot \frac{\pi}{180}$

$\theta = \frac{11\pi}{6}$

$z = 8\left[\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right]$

$z^{1/4} = \sqrt[4]{8}\left[\cos\left(\frac{1}{4}\cdot\frac{11\pi}{6}\right) + i\sin\left(\frac{1}{4}\cdot\frac{11\pi}{6}\right)\right]$

i) $\sqrt[4]{8}\left[\cos\left(\frac{11\pi}{24}\right) + i\sin\left(\frac{11\pi}{24}\right)\right]$

$\frac{\cancel{2\pi}}{4} = \boxed{\frac{\pi}{2}}$

ii) $\sqrt[4]{8}\left[\cos\frac{23\pi}{24} + i\sin\left(\frac{23\pi}{24}\right)\right]$

iii) $\sqrt[4]{8}\left[\cos\frac{35\pi}{24} + i\sin\left(\frac{35\pi}{24}\right)\right]$

iv) $\sqrt[4]{8}\left[\cos\left(\frac{47\pi}{24}\right) + i\sin\left(\frac{47\pi}{24}\right)\right]$

$n=5$ 10. Fifth roots of unity (1) $1+0i$

$\frac{2\pi}{5}$

$r = \sqrt{1^2+0^2} = 1$

$\theta = 0$

$z = 1\left[\cos 0 + i\sin 0\right]$

$z^{1/5} = \sqrt[5]{1}\left[\cos\left(\frac{0}{5}\right) + i\sin\left(\frac{0}{5}\right)\right]$

i) $\sqrt[5]{1}\left[\cos 0 + i\sin 0\right]$

ii) $\sqrt[5]{1}\left[\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right]$ $+\frac{2\pi}{5}$

iii) $\sqrt[5]{1}\ \text{cis}\left(\frac{4\pi}{5}\right)$

iv) $\sqrt[5]{1}\ \text{cis}\left(\frac{6\pi}{5}\right)$

v) $\sqrt[5]{1}\ \text{cis}\left(\frac{8\pi}{5}\right)$

7.10 More Practice with Operations of Complex Numbers

Find the product $z_1 \cdot z_2$ and the quotient $\frac{z_1}{z_2}$. Express answers in both polar and rectangular form. Match angle measurement units to the problem, where $0° \le \theta \le 360°$ or $0 \le \theta \le 2\pi$.

product $z_1 \cdot z_2$

1. Let $z_1 = 7\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$ and $z_2 = 2\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

$z_1 \cdot z_2 = 7\cdot 2\left[\cos\left(\frac{9\pi}{8}+\frac{\pi}{8}\right) + i\sin\left(\frac{9\pi}{8}+\frac{\pi}{8}\right)\right]$

$= \boxed{14\left[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right]}$ polar form

$14\cos\left(\frac{5\pi}{4}\right) + 14\sin\left(\frac{5\pi}{4}\right)i$

$14\left(-\frac{\sqrt{2}}{2}\right) + 14\left(-\frac{\sqrt{2}}{2}\right)i$

$\boxed{-7\sqrt{2} - 7\sqrt{2}i}$ Rectangular Form

quotient $\frac{z_1}{z_2}$

2. Let $z_1 = 4(\cos 200° + i\sin 200°)$ and $z_2 = 25(\cos 150° + i\sin 150°)$

$\frac{z_1}{z_2} = \frac{4}{25}\left[\cos(200-150) + i\sin(200-150)\right]$

$\boxed{\frac{4}{25}\left[\cos 50 + i\sin 50\right]}$ polar Form

$\boxed{0.103 + 0.123i}$ Rectangular Form

Convert each complex number into polar form. Then find each power. Answer in polar form where $0 \le \theta \le 2\pi$.

3. $(1-\sqrt{3}i)^4$

$r = \sqrt{1^2 + \sqrt{3}^2}$

$r = 2$ Q4

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -60 + 360$

$\theta = 300 \cdot \frac{\pi}{180} = \frac{5\pi}{3}$

$z = 2\left[\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right]$

$z^4 = 2^4\left[\cos\left(4\cdot\frac{5\pi}{3}\right) + i\sin\left(4\cdot\frac{5\pi}{3}\right)\right]$

$z^4 = 16\left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right]$

4. $(-\sqrt{2}+\sqrt{2}i)^5$

$r = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$

$r = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right)$

$\theta = -45 + 180 = 135 = \frac{3\pi}{4}$

$z = 2\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$

$z^5 = 2^5\left[\cos\left(5\cdot\frac{3\pi}{4}\right) + i\sin\left(5\cdot\frac{3\pi}{4}\right)\right]$

$z^5 = 32\left[\cos\frac{15\pi}{4} + i\sin\frac{15\pi}{4}\right]$

$\boxed{32\,\mathrm{cis}\,\frac{7\pi}{4}}$

Convert each complex number into polar form. Then find all distinct roots of the complex number. Answer in polar form where $0 \le \theta \le 2\pi$.

5. Fifth roots of 32

$32 + 0i$

$r = \sqrt{32^2 + 0^2} = 32$

$\theta = 0$

$z = 32\left[\cos 0 + i\sin 0\right]$

$z^{1/5} = 32^{1/5}\left[\cos\left(\frac{0}{5}\right) + i\sin\left(\frac{0}{5}\right)\right]$

(1) $z^{1/5} = 2\left[\cos 0 + i\sin 0\right]$

* $\frac{2\pi}{5}$

(2) $2\,\mathrm{cis}\left(\frac{2\pi}{5}\right)$

(3) $2\,\mathrm{cis}\left(\frac{4\pi}{5}\right)$

(4) $2\,\mathrm{cis}\left(\frac{6\pi}{5}\right)$

(5) $2\,\mathrm{cis}\left(\frac{8\pi}{5}\right)$

6. Fourth roots of $-81i$

$0 - 81i$

$r = \sqrt{81^2 + 0^2} = 81$

$\theta = 270$ or $\frac{3\pi}{2}$

$z = 81\left[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right]$

$z^{1/4} = \sqrt[4]{81}\left[\cos\left(\frac{1}{4}\cdot\frac{3\pi}{2}\right) + i\sin\left(\frac{1}{4}\cdot\frac{3\pi}{2}\right)\right]$

(1) $= 3\left[\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}\right]$ $+\pi/2$ Add $\frac{2\pi}{n} \to \frac{2\pi}{4} \to \frac{\pi}{2}$

(2) $3\left[\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}\right]$

(3) $3\,\mathrm{cis}\,\frac{11\pi}{8}$

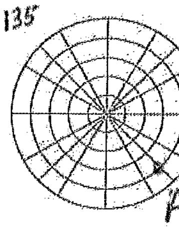
(4) $3\,\mathrm{cis}\,\frac{15\pi}{8}$

7.11: Test Review

Date Key

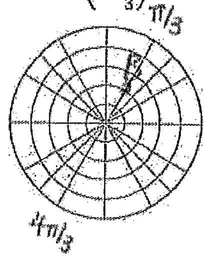
First plot each point, given as polar coordinates. Then, determine 3 other coordinates for the same point. Use -360° ≤ θ ≤ 360° if in degrees, or use $-2\pi \le \theta \le 2\pi$ if in radians. *No calculator

1. A = (-4, 135°)



$(4, 315°)$
$(4, -45°)$
$(-4, -225°)$

2. B = $\left(2, \frac{\pi}{3}\right)$



$\left(2, \frac{-5\pi}{3}\right)$
$\left(-2, \frac{4\pi}{3}\right)$
$\left(-2, -\frac{2\pi}{3}\right)$

Find the distance between the polar points. Use the polar method: $\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

3. (-6, 210°) and (4, -45°) (r_1, θ_1, r_2, θ_2)

$\sqrt{6^2+4^2-(2\cdot6\cdot4\cos(-45-210))}$

$\sqrt{36+16+48\cos(-255)} = \sqrt{39.57668}$

$\boxed{\text{distance} = 6.291}$

4. $\left(1, \frac{2\pi}{3}\right)$ and $\left(-5, -\frac{7\pi}{6}\right)$

$\sqrt{1^2+5^2-\left(2\cdot1\cdot-5\cos\left(\frac{-7\pi}{6}-\frac{2\pi}{3}\right)\right)}$

$\sqrt{1+25+10\cos\left(\frac{-11\pi}{6}\right)} = \sqrt{34.66025}$

$\boxed{\text{distance} = 5.887}$

Convert the given rectangular coordinates into polar coordinates, where $0 \le \theta \le 2\pi$. (r, θ)

5. (-3, 3) *No calculator

$r = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{3}{-3}\right) = -45° + 180 = 135°$ Q2

$\theta = 135 \cdot \frac{\pi}{180} = \frac{3\pi}{4}$ $\quad (r, \theta)$ $\left(3\sqrt{2}, \frac{3\pi}{4}\right)$

6. $(-4\sqrt{5}, -2)$

$r = \sqrt{(4\sqrt{5})^2+2^2} = \sqrt{84} = 2\sqrt{21}$

$\theta = \tan^{-1}\left(\frac{-2}{-4\sqrt{5}}\right) \to 0.219 + \pi = 3.362$

Q3 $\boxed{(2\sqrt{21}, 3.362 \text{ rad})}$

Convert the given polar coordinates into rectangular coordinates.

*$(r\cos\theta, r\sin\theta)$

Q3 7. (14, 210°) *No calculator

$x = 14\cos 210 \to 14\left(\frac{-\sqrt{3}}{2}\right)$

$y = 14\sin 210 \to 14\left(\frac{-1}{2}\right)$

$\boxed{(-7\sqrt{3}, -7)}$

8. $\left(2\sqrt{3}, \frac{11\pi}{7}\right)$

$x = 2\sqrt{3}\cos\left(\frac{11\pi}{7}\right)$

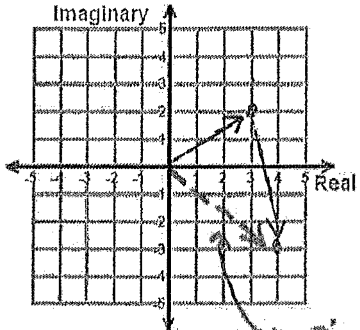
$y = 2\sqrt{3}\sin\left(\frac{11\pi}{7}\right)$

Radian Mode

$\boxed{(0.771, -3.377)}$

Simply each expression using geometric methods. ***No calculator**

9. $(3+2i)+(1-5i)$ $\rightarrow 4-3i$

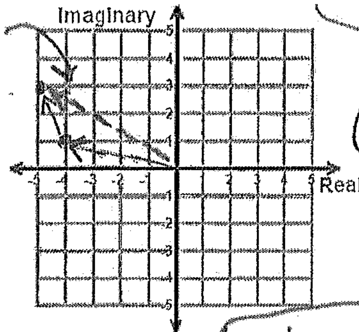


$4-3i$

10. $(-4+i)-(1-2i)$ $= -5+3i$

$(-4+i)+(-1+2i)$

$-5+3i$



$\boxed{* \; |z_1 - z_2|}$

Find the distance between the complex numbers. ***No calculator**

11. $(13+2i)$ and $(9-5i)$

$|4+7i| = \sqrt{4^2+7^2}$

$= \sqrt{16+49} = \boxed{\sqrt{65}}$

12. $(-8+5i)$ and $(-2-i)$

$|-6+6i| = \sqrt{6^2+6^2} = \sqrt{72} = \boxed{6\sqrt{2}}$

$\boxed{\dfrac{z_1+z_2}{2}}$

Find the midpoint between the complex numbers. ***No calculator**

13. $(13+2i)$ and $(9-5i)$

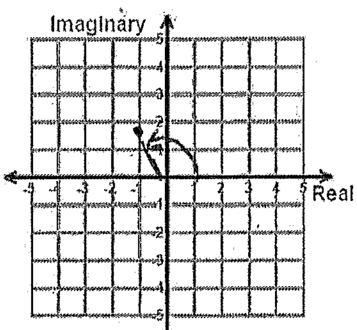
$\dfrac{13+9}{2} + \dfrac{2-5}{2}i \rightarrow \boxed{11-\dfrac{3}{2}i}$

14. $(-8+5i)$ and $(-2-i)$

$\dfrac{-8-2}{2} + \dfrac{5-1}{2}i \rightarrow \boxed{-5+2i}$

Graph each complex number, find its modulus (absolute value) and argument (direction), and then write in polar form, where $\boxed{0 \le \theta \le 2\pi.}$ ***No calculator**

15. $z = -1+\sqrt{3}i$



Angle(θ)

Modulus: $\sqrt{1^2+\sqrt{3}^2} = \boxed{2}$

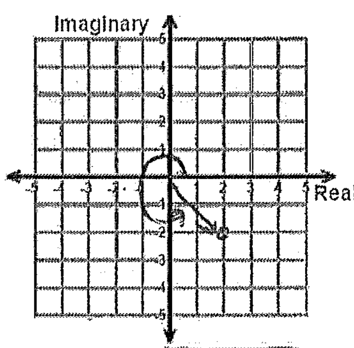
Argument: $\theta = \tan^{-1}\left(\dfrac{\sqrt{3}}{-1}\right)$

$\theta = -60+180 = 120°$

Q2 $\rightarrow$ $\boxed{\theta = \dfrac{2\pi}{3}}$

Polar: $2 \operatorname{cis}\left(\dfrac{2\pi}{3}\right)$

16. $z = 2-2i$



Modulus: $\sqrt{2^2+2^2} = \boxed{2\sqrt{2}}$

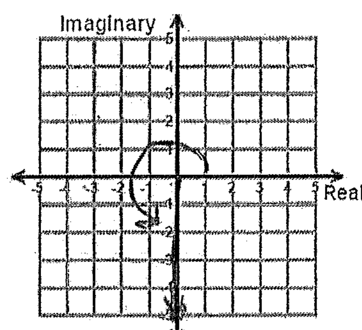
Argument: $\theta = \tan^{-1}\left(\dfrac{-2}{2}\right)$

$\theta = -45+360 = 315°$

$\boxed{\theta = 7\pi/4}$

Polar: $2\sqrt{2} \operatorname{cis}\left(\dfrac{7\pi}{4}\right)$

17. $z = -5i$ $\rightarrow 0-5i$



Modulus: $\sqrt{0^2+5^2} = \boxed{5}$

Argument: $\boxed{\dfrac{3\pi}{2}}$

Polar: $5 \operatorname{cis}\left(\dfrac{3\pi}{2}\right)$

#15) Polar: $2\,cis\left(\frac{2\pi}{3}\right)$

#16) Polar: $2\sqrt{2}\,cis\left(\frac{7\pi}{4}\right)$

#17) Polar: $5\,cis\left(\frac{3\pi}{2}\right)$

18. Convert z = -5 + 12i to polar form, where $0 \le \Theta \le 2\pi$.

$r = \sqrt{5^2+12^2} = 13$

Q2 → $\Theta = \tan^{-1}\left(\frac{12}{-5}\right) = -1.176 + \pi = 1.967$

$\boxed{13\,cis\,1.967 \text{ rad}}$

19. Convert z = $4\sqrt{3}(\cos 30° + i \sin 30°)$ to rectangular form. ***No calculator**

* distribute $4\sqrt{3}$ through parentheses

$4\sqrt{3}\cos 30 \to 4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) \to 2\sqrt{9} \to 2(3) = 6$

$4\sqrt{3}\sin 30 \to 4\sqrt{3}\left(\frac{1}{2}\right) \to 2\sqrt{3}$

$\boxed{6 + 2\sqrt{3}i}$

Simplify each expression using polar methods. Answer in polar form, where $0 \le \Theta \le 2\pi$.
***No calculator**

Given: $z_1 = 3\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$, $z_2 = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$, $z_3 = 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

*multiply r, add Θ

20. $z_1 \cdot z_2$ $\quad 3 \cdot 4\,cis\left(\frac{4\pi}{3} + \frac{5\pi}{6}\right)$

$12\left[\cos\frac{13\pi}{6} + i\sin\frac{13\pi}{6}\right]$

$\boxed{12\,cis\,\pi/6}$ ← subtract 2π

21. $z_2 \cdot z_3$ $\quad 4 \cdot 2\,cis\left[\frac{5\pi}{6} + \frac{3\pi}{4}\right]$

$\boxed{8\,cis\left(\frac{19\pi}{12}\right)}$

*divide r, subtract Θ

22. $\frac{z_1}{z_2}$ $\quad \frac{3}{4}\,cis\left(\frac{4\pi}{3} - \frac{5\pi}{6}\right)$

$\frac{3}{4}\,cis\left(\frac{3\pi}{6}\right) \to \boxed{\frac{3}{4}\,cis\,\frac{\pi}{2}}$

23. $\frac{z_3}{z_2}$ $\quad \frac{2}{4}\,cis\left(\frac{3\pi}{4} - \frac{5\pi}{6}\right)$ → $\frac{9\pi}{12} - \frac{10\pi}{12} = \frac{-\pi}{12}$

$\frac{1}{2}\,cis\left(\frac{-\pi}{12}\right) \to \boxed{\frac{1}{2}\,cis\left(\frac{23\pi}{12}\right)}$

Add 2π

* raise r to exponent multiply Θ

24. $(z_2)^4$

$4^4\left(\cos\left(4 \cdot \frac{5\pi}{6}\right) + i\sin\left(4 \cdot \frac{5\pi}{6}\right)\right)$

$4^4\,cis\left(\frac{10\pi}{3}\right) \to \boxed{4^4\,cis\left(\frac{4\pi}{3}\right)}$

subtract 2π

25. $(z_3)^3$

$2^3\left[\cos\left(3 \cdot \frac{3\pi}{4}\right) + i\sin\left(3 \cdot \frac{3\pi}{4}\right)\right]$

$8\,cis\left(\frac{9\pi}{4}\right)$ $\quad \boxed{8\,cis\left(\frac{\pi}{4}\right)}$

Subtract 2π

26. Find the cube roots of z_2.

$z_2 = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$z^{1/3} = 4^{1/3}\left[\cos\left(\frac{1}{3} \cdot \frac{5\pi}{6}\right) + i\sin\left(\frac{1}{3} \cdot \frac{5\pi}{6}\right)\right]$

1 $\sqrt[3]{4}\left[\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}\right]$

2 $\sqrt[3]{4}\,cis\,\frac{17\pi}{18}$

3 $\sqrt[3]{4}\,cis\,\frac{29\pi}{18}$

$+\frac{2\pi}{3}$

* Add $\frac{2\pi}{n} \to \frac{2\pi}{3}$

27. Find the fourth roots of z_1

$z_1 = 3\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

1 $z^{1/4} = \sqrt[4]{3}\left[\cos\left(\frac{1}{4} \cdot \frac{4\pi}{3}\right) + i\sin\left(\frac{1}{4} \cdot \frac{4\pi}{3}\right)\right] = \boxed{\sqrt[4]{3}\,cis\left(\frac{\pi}{3}\right)}$

2 $\sqrt[4]{3}\,cis\,\frac{5\pi}{6}$

3 $\sqrt[4]{3}\,cis\,8\pi/6$

4 $\sqrt[4]{3}\,cis\,\frac{11\pi}{6}$

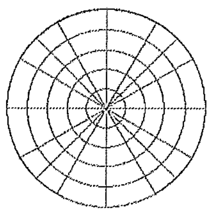
$+\frac{\pi}{2}$ or $\frac{3\pi}{6}$

* Add $\frac{2\pi}{n} \to \frac{2\pi}{4}$ $+\pi/2$

7.12b: Test Review WS #2 ***No calculator***

First plot each point, given as polar coordinates. Then, determine 3 other coordinates for the same point. Use $-360° \le \theta \le 360°$ if in degrees, or use $-2\pi \le \theta \le 2\pi$ if in radians.

1. $A = (-3, \frac{7\pi}{6})$



Convert the given rectangular coordinates into polar coordinates, where $0 \le \theta \le 2\pi$.

2. $(-2, 2\sqrt{3})$

Convert the given polar coordinates into rectangular coordinates.

3. $\left(2\sqrt{3}, \frac{11\pi}{6}\right)$

Find the distance between the complex numbers. ***No calculator**

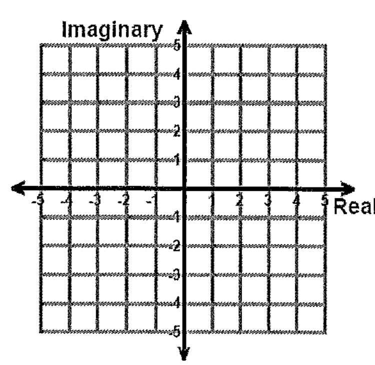
4. $(-4 + 6i)$ and $(1 + 7i)$

Find the midpoint between the complex numbers. ***No calculator**

5. $(14 - 3i)$ and $(3 - 5i)$

Graph each complex number, find its modulus (absolute value) and argument (direction), and then write in polar form, where $0 \le \theta \le 2\pi$. ***No calculator**

6. $z = -\sqrt{2} - \sqrt{2}i$



Modulus:

Argument:

Polar:

7. Convert $z = -2 + 2\sqrt{3}i$ to polar form, where $0 \le \theta \le 2\pi$.

8. Convert $z = 4\sqrt{3}(\cos 240° + i \sin 240°)$ to rectangular form. ***No calculator**

Simplify each expression using polar methods. Answer in polar form, where $0 \le \theta \le 2\pi$.
***No calculator**

Given: $z_1 = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$, $z_2 = 3\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$, $z_3 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

9. $z_1 \cdot z_2$

10. $\frac{z_1}{z_3}$

11. $(z_3)^3$

12. Find the cube roots of $\sqrt{3} - i$.

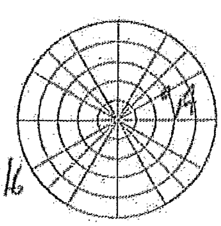
13. Find the fourth roots of z_3

Updated Key

7.12b: Test Review **WS #2** ***No calculator***

First plot each point, given as polar coordinates. Then, determine 3 other coordinates for the same point. Use $-360° \le \theta \le 360°$ if in degrees, or use $-2\pi \le \theta \le 2\pi$ if in radians.

1. $A = (-3, \frac{7\pi}{6})$



$(3, \pi/6)$

$(3, -11\pi/6)$

$(-3, -5\pi/6)$

$\frac{7\pi}{6} - \frac{12\pi}{6} = -\frac{5\pi}{6}$

Convert the given rectangular coordinates into polar coordinates, where $0 \le \theta \le 2\pi$.

2. $(-2, 2\sqrt{3})$
   x y

$r = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right)$

$\theta = \tan^{-1}(-\sqrt{3})$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) \to \frac{\pi}{3}$ in Q2 is $\frac{2\pi}{3}$

$\boxed{\left(4, \frac{2\pi}{3}\right)}$

Convert the given polar coordinates into rectangular coordinates.

3. $(2\sqrt{3}, \frac{11\pi}{6})$ (r, θ)

*$(r\cos\theta, r\sin\theta)$

$x = 2\sqrt{3}\cos\left(\frac{11\pi}{6}\right) \to 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} \to 3$

$y = 2\sqrt{3}\sin\left(\frac{11\pi}{6}\right) \to 2\sqrt{3} \cdot \frac{-1}{2} \to -\sqrt{3}$

$\boxed{(3, -\sqrt{3})}$

Find the distance between the complex numbers. ***No calculator*** * $|z_1 - z_2|$

4. $(-4+6)$ and $(1+7i)$

$-4-1 \quad 6-7$

$|-5-1i| = \sqrt{5^2+1^2} = \boxed{\sqrt{26}}$

Find the midpoint between the complex numbers. ***No calculator*** * $\frac{z_1+z_2}{2}$

5. $(14-3i)$ and $(3-5i)$

$\frac{14+3}{2} \quad \frac{-3-5}{2}$

$\boxed{\frac{17}{2} - 4i}$

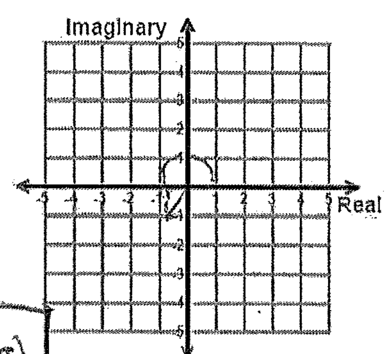
Graph each complex number, find its modulus (absolute value) and argument (direction), and then write in polar form, where $0 \le \theta \le 2\pi$. ***No calculator***

6. $z = -\sqrt{2} - \sqrt{2}i$ Q3

Modulus: $\sqrt{\sqrt{2}^2 + \sqrt{2}^2} = \sqrt{2+2} = \sqrt{4} = 2$ 180

Argument: $\theta = \tan^{-1}\left(\frac{-\sqrt{2}}{-\sqrt{2}}\right) \to \tan^{-1}(1) \to {}^*45°$

$\theta = 225$ or $\frac{5\pi}{4}$



Polar:

$2\left[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right]$ or $\boxed{2\,\text{cis}\left(\frac{5\pi}{4}\right)}$

polar form

7. Convert $z = -2 + 2\sqrt{3}i$ to polar form, where $0 \le \theta \le 2\pi$.

$r = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right) \to \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) \to 60° \to Q2 \to 180 - 60 = 120 \to \frac{2\pi}{3}$

$\boxed{\left(4, \frac{2\pi}{3}\right)} \to \boxed{4 \text{cis} \frac{2\pi}{3}}$

8. Convert $z = 4\sqrt{3}(\cos 240° + i \sin 240°)$ to rectangular form. ***No calculator**

$x = 4\sqrt{3}\cos 240 \to 4\sqrt{3} \cdot \left(\frac{-1}{2}\right) \to -2\sqrt{3}$

$y = 4\sqrt{3}\sin 240 \to 4\sqrt{3}\left(\frac{-\sqrt{3}}{2}\right) \to -6$

$\boxed{-2\sqrt{3} - 6i}$

Simplify each expression using polar methods. Answer in polar form, where $0 \le \theta \le 2\pi$.
***No calculator**

Given: $z_1 = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$, $z_2 = 3\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$, $z_3 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

subtract 2π

9. $z_1 \cdot z_2$ $\quad 6\text{cis}\left(\frac{5\pi}{3} + \frac{7\pi}{6}\right) \to \frac{10\pi}{6} + \frac{7\pi}{6} \to \frac{17\pi}{6} - \frac{12\pi}{6} \to \frac{5\pi}{6}$

$\boxed{6\text{cis}\left(\frac{5\pi}{6}\right)}$

10. $\frac{z_1}{z_3}$ $\quad \frac{2}{2}\text{cis}\left(\frac{5\pi}{3} - \frac{\pi}{4}\right) \to \frac{20\pi}{12} - \frac{3\pi}{12} \to \frac{17\pi}{12}$ $\quad \boxed{1\text{cis}\left(\frac{17\pi}{12}\right)}$

11. $(z_3)^3$ $\quad 2^3\left[\cos\left(3 \cdot \frac{\pi}{4}\right) + i\sin\left(3 \cdot \frac{\pi}{4}\right)\right]$ $\quad \boxed{8\text{cis}\left(\frac{3\pi}{4}\right)}$

12. Find the cube roots of $\sqrt{3} - i$.

$* r = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) \to 30° \to 360 - 30 = 330° \to 11\pi/6$

$z = 2\text{cis}\left(\frac{11\pi}{6}\right)$

$z^{1/3} = \sqrt[3]{2}\text{cis}\left[\frac{1}{3} \cdot \frac{11\pi}{6}\right] \to$ ① $\boxed{\sqrt[3]{2}\text{cis}\frac{11\pi}{18}}$

*interval is $\frac{2\pi}{3} \to$ Add $\frac{12\pi}{18}$

② $\sqrt[3]{2}\text{cis}\frac{23\pi}{18}$

③ $\sqrt[3]{2}\text{cis}\frac{35\pi}{18}$

13. Find the fourth roots of z_3

$z^{1/4} = \sqrt[4]{2}\left[\cos\left(\frac{1}{4} \cdot \frac{\pi}{4}\right) + i\sin\left(\frac{1}{4} \cdot \frac{\pi}{4}\right)\right]$

*interval is $\frac{2\pi}{4} \to \frac{\pi}{2} \to$ Add $\frac{8\pi}{16}$

① $\sqrt[4]{2}\text{cis}\left(\frac{\pi}{16}\right)$

② $\sqrt[4]{2}\text{cis}\left(\frac{9\pi}{16}\right)$

③ $\sqrt[4]{2}\text{cis}\left(\frac{17\pi}{16}\right)$

④ $\sqrt[4]{2}\text{cis}\left(\frac{25\pi}{16}\right)$