Accel Pre-Calculus

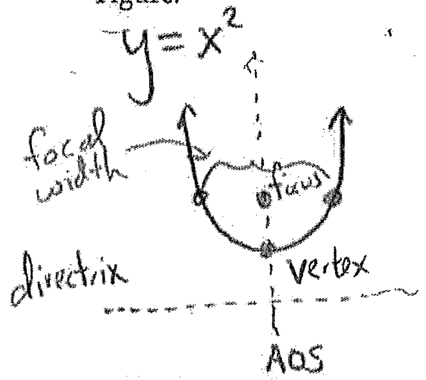
Name: Key

8.01 Parabolas – Day 1

Date:

Parabola: a conic section where the distance from 1 fixed point (focus) and a line (directrix) is equal.

Figure:



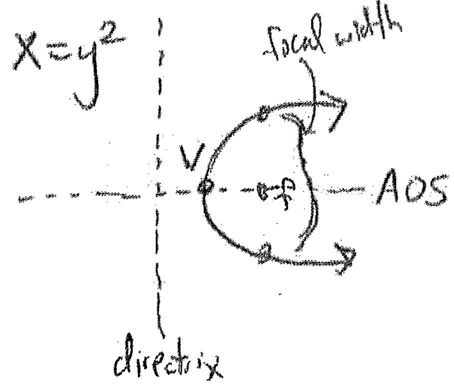
Vertex:

Axis of Symmetry:

Focus:

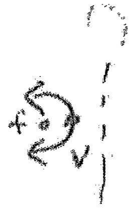
Directrix:

Focal Width:



Vertical Axis of Symmetry	Horizontal Axis of Symmetry
$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$
Vertex: (h,k) Axis of Symmetry: $x=h$ Focus: $(h,\ k+p)$ Directrix: $y=k-p$ Focal width: $\lvert 4p \rvert$	Vertex: (h,k) Axis of Symmetry: $y=k$ Focus: $(h+p,k)$ Directrix: $x=h-p$ Focal width: $\lvert 4p \rvert$

$(y+3)^2 = -4(x-0)$



Examples: Graph the parabola. State the vertex, AOS, focus, directrix, and focal width.

1. $(x-2)^2 = 8(y+1)$

$4p=8$

$p=2$

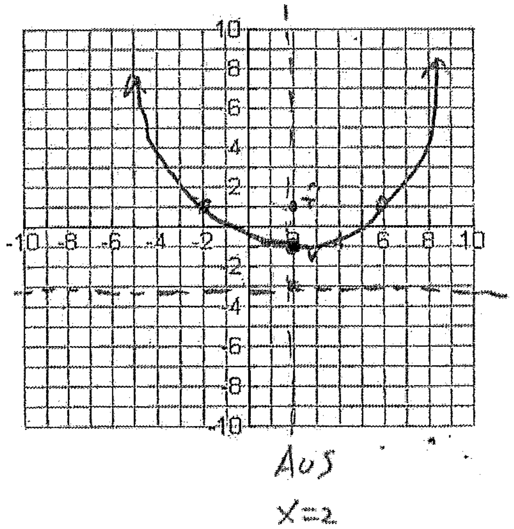
Vertex: V(2,-1)

Axis of Symmetry: x=2

Focus: (2,1)

Directrix: y=-3

Focal Width: 8



2. $(y+3)^2 = -4x$

$4p=-4$

$p=-1$

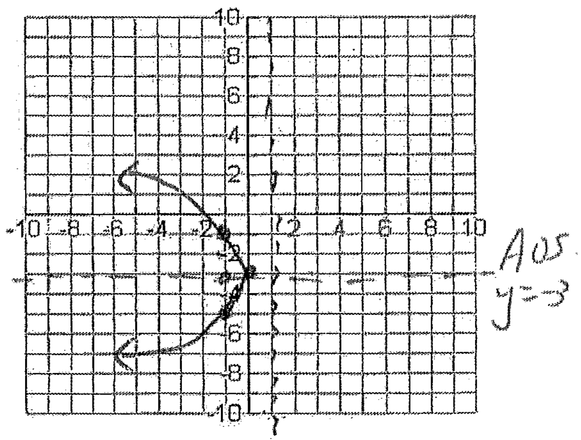
Vertex: (0,-3)

Axis of Symmetry: y=-3

Focus: (-1,-3)

Directrix: x=1

Focal Width: 4



8.01 Practice: Graph the parabola. State the vertex, AOS, focus, directrix, and focal width.

1. $(x+4)^2 = 6(y-2)$ opens up

Vertex: (-4, 2)

Axis of Symmetry: x = -4

Focus: (-4, 3.5)

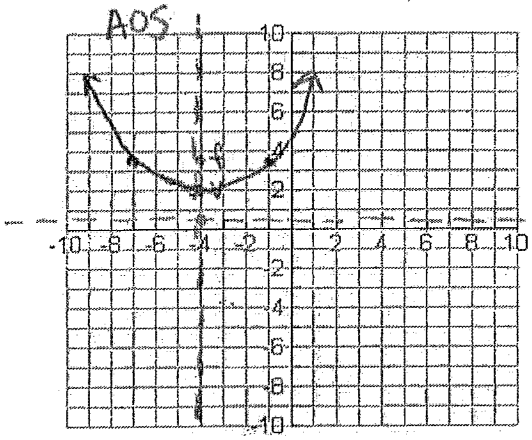
Directrix: y = 0.5

Focal Width: 6

$4p=6$

$p=\frac{6}{4}=\frac{3}{2}$

$p=1.5$



2. $(y-3)^2 = 12(x-1)$ opens right

Vertex: (1, 3)

Axis of Symmetry: y = 3

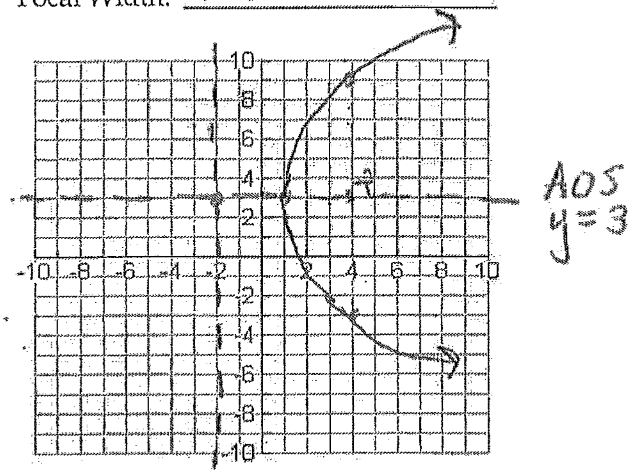
Focus: (4, 3)

Directrix: x = -2

Focal Width: 12

$4p=12$

$p=3$



up/[✓ down] 3. $(x-5)^2 = -4(y+5)$

Vertex: (5, -5)

Axis of Symmetry: x = 5

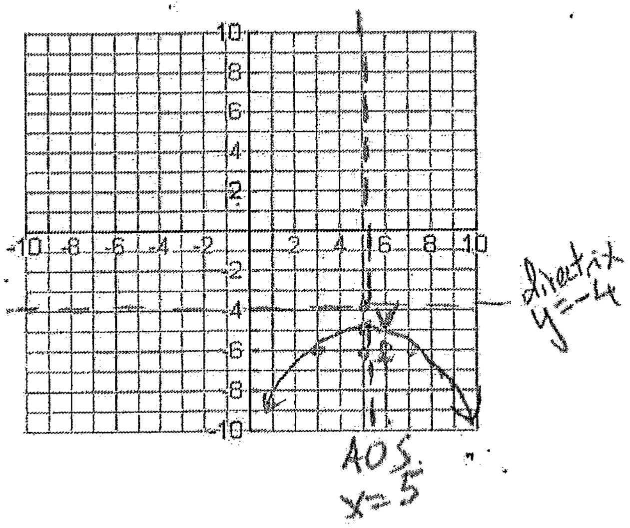
Focus: (5, -6)

Directrix: y = -4

Focal Width: 4

$4p=-4$

$p=-1$



4. $(y+2)^2 = -8(x+2)$

[✓ left]/right

Vertex: (-2, -2)

Axis of Symmetry: y = -2

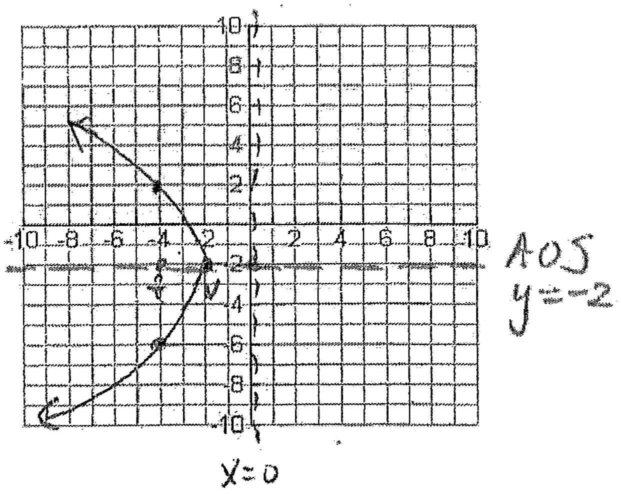
Focus: (-4, -2)

Directrix: x = 0

Focal Width: 8

$4p=-8$

$p=-2$



$(y-k)^2=4p(x-h)$

8.02 Parabolas – Day 2 Date: ______________

Write the standard form of the equation and graph each parabola. Find the vertex, focus, focal width, axis of symmetry, and directrix.

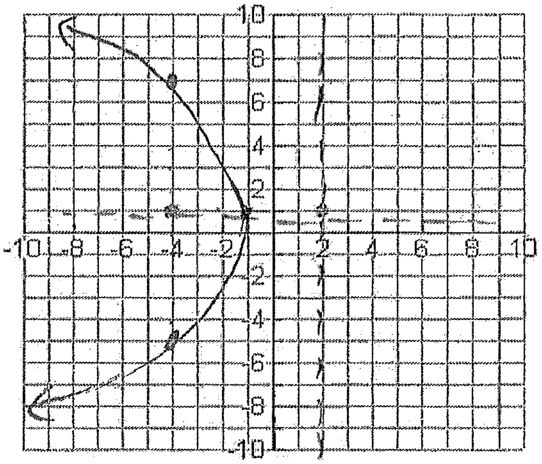
$\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{-2}{2}\right)^2=(-1)^2=1$

1. $y^2 + 12x = 2y - 13$

$y^2-2y+1=-12x-13+1$

$(y-1)(y-1)=-12x-12$

$(y-1)^2=-12(x+1)$



Standard Equation: $(y-1)^2=-12(x+1)$

$4p=-12$

$p=-3$

p= -3 Focus: $(-4,1)$

Vertex: $(-1,1)$ Directrix: $x=2$

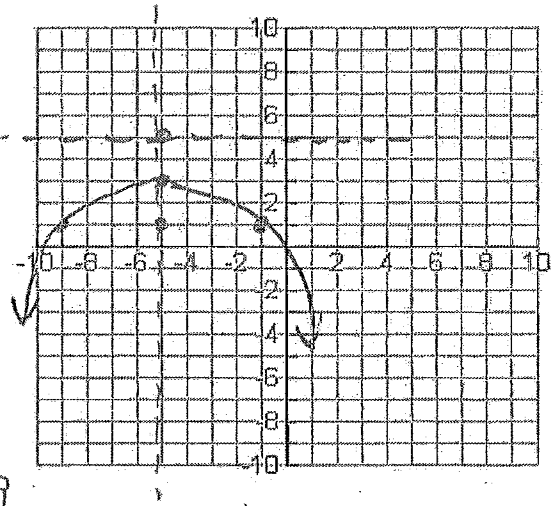
AOS: $y=1$ Focal Width: 12

2. $x^2 + 10x + 25 = -8y + 24$

$(x+5)(x+5)=-8(y-3)$

$(x+5)^2=-8(y-3)$

∩



Standard Equation: $(x+5)^2=-8(y-3)$

$4p=-8$

p= -2 Focus: $(-5,1)$

Vertex: $(-5,3)$ Directrix: $y=5$

AOS: $x=-5$ Focal Width: 8

3. $\dfrac{3x^2 - 30y - 18x + 87 = 0}{3}$

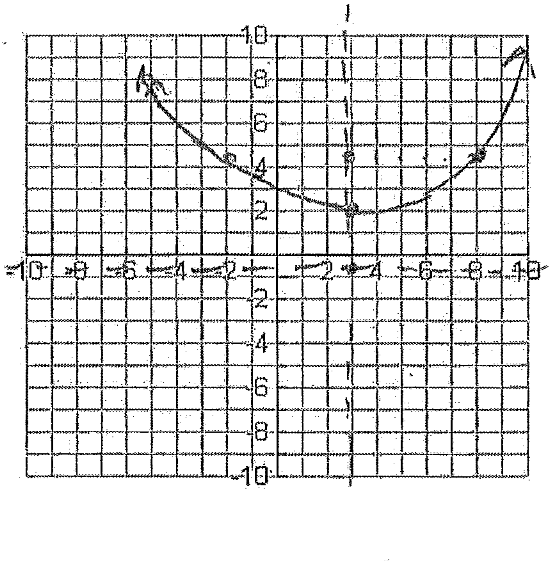
$x^2-10y-6x+29=0$

$\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{-6}{2}\right)^2=(3)^2=9$

∪

$x^2-6x+9=10y-29+9$

$(x-3)(x-3)=10y-20$



Standard Equation: $(x-3)^2=10(y-2)$

$4p=10$

$p=2.5$

p= 2.5 Focus: $(3,4.5)$

Vertex: $(3,2)$ Directrix: $y=-0.5$

AOS: $x=3$ Focal Width: 10

8.02 Practice: Date: ____________

Write the standard form of the equation and graph each parabola. Find the vertex, focus, focal width, axis of symmetry, and directix.

$\left(\frac{b}{2}\right)^2 \to \left(\frac{2}{2}\right)^2 = 1$

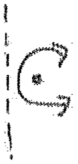
1. $12x - 15 = 3y^2 + 6y$

$\frac{3y^2 + 6y = 12x - 15}{3}$

$y^2 + 2y + 1 = 4x - 5 + 1$

$(y+1)(y+1) = 4x - 4$

$(y+1)^2 = 4(x-1)$



Standard Equation: $(y+1)^2 = 4(x-1)$

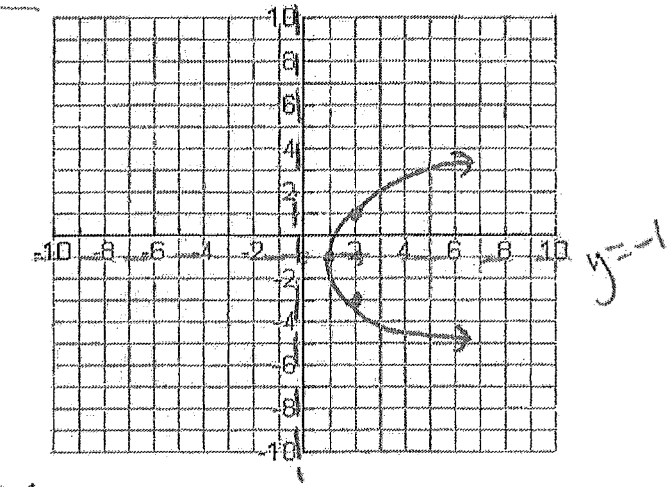
$4p = 4$

$p = 1$

p = 1 Focus: $(2, -1)$

Vertex: $(1, -1)$ Directrix: $x = 0$

AOS: $y = -1$ Focal Width: 4



$\left(\frac{b}{2}\right)^2 \to \left(\frac{8}{2}\right)^2 = 16$

2. $x^2 + 8x + 14y = -44$

$x^2 + 8x = -14y - 44$

$x^2 + 8x + 16 = -14y - 44 + 16$

$(x+4)(x+4) = -14y - 28$

$(x+4)^2 = -14(y+2)$

Standard Equation: $(x+4)^2 = -14(y+2)$

$4p = -14$

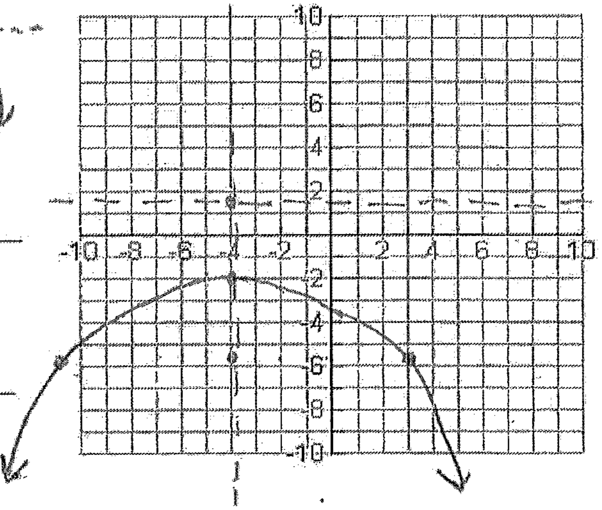
$p = \frac{-14}{4}$

$p = -3.5$

p = -3.5 Focus: $(-4, -5.5)$

Vertex: $(-4, -2)$ Directrix: $y = 1.5$

AOS: $x = -4$ Focal Width: 14



3. $\frac{2y^2 - 4y + 12x + 50}{2} = 0$

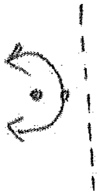
$y^2 - 2y + 6x + 25 = 0$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = 1$

$y^2 - 2y + 1 = -6x - 25 + 1$

$(y-1)(y-1) = -6x - 24$

$(y-1)^2 = -6(x+4)$



Standard Equation: $(y-1)^2 = -6(x+4)$

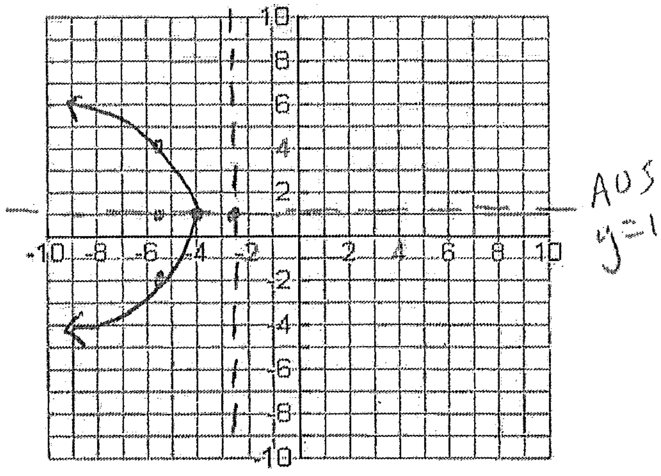
$4p = -6$

$p = -1.5$

p = -1.5 Focus: $(-5.5, 1)$

Vertex: $(-4, 1)$ Directrix: $x = -2.5$

AOS: $y = 1$ Focal Width: 6



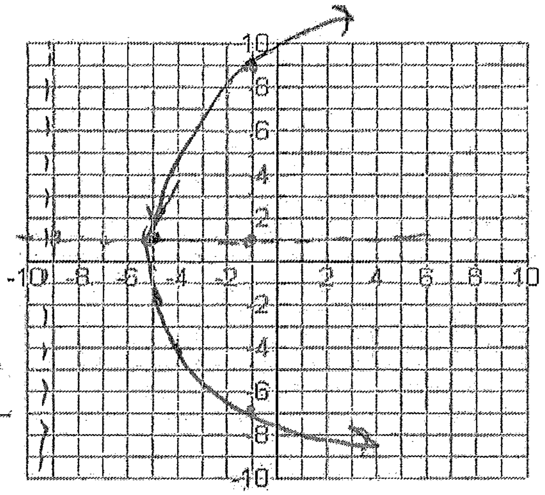
8.03 Parabolas – Day 3 Date: Key

Write the standard form of the equation of each parabola. Sketch the graph and fill in the blanks.

1. The vertex is at $\overset{h}{(-5}, \overset{k}{1})$, and the focus is at (-1, 1).

$(y-k)^2 = 4p(x-h)$

$(y-1)^2 = 4(4)(x+5)$



Equation: $(y-1)^2 = 16(x+5)$

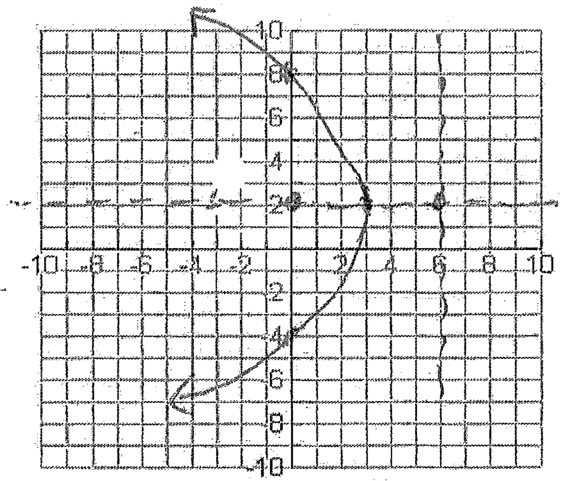
Axis of Symmetry: $y=1$ Directrix: $x=9$

p = 4 Focal Width: 16

2. The equation of the axis of symmetry is y = 2, the focus is at (0, 2), and p = -3.

$(y-k)^2 = 4p(x-h)$

$(y-2)^2 = 4(-3)(x-3)$



Equation: $(y-2)^2 = -12(x-3)$

Vertex: $(\overset{h}{3}, \overset{k}{2})$

Directrix: $x=6$ Focal Width: 12

8.03 Practice

1. The parabola passes through the point at $(\overset{x}{-3}, \overset{y}{1})$, opens to the left and its vertex is at $(\overset{h}{-2}, \overset{k}{-3})$.

$(y-k)^2 = 4p(x-h)$

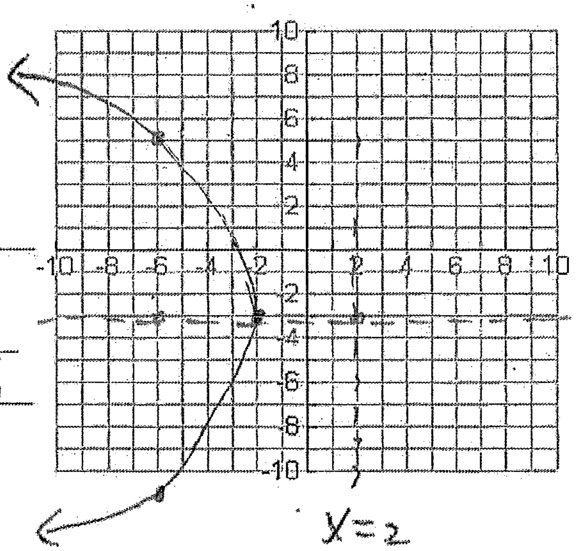
$(1+3)^2 = 4p(-3+2)$

$4^2 = 4p(-1)$

$16 = -4p$

$\frac{16}{-4} = p$

$\boxed{-4 = p}$



Equation: $(y+3)^2 = -16(x+2)$

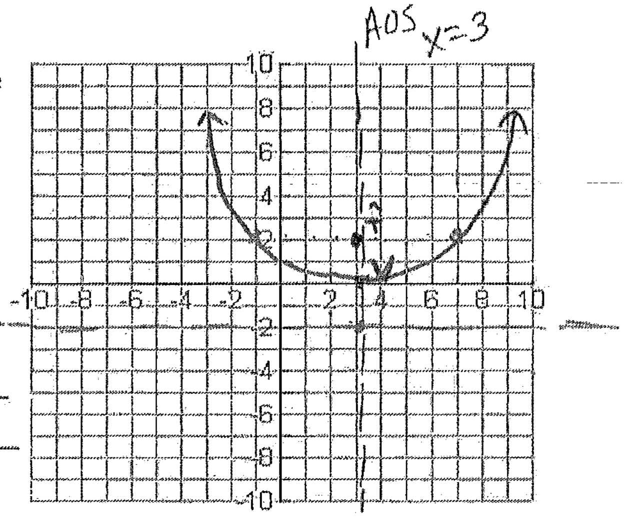
Axis of Symmetry: $y=-3$ Directrix: $x=2$

Focus: $(-6,-2)$ p = -4 Focal Width: 16

2. The focus is at (3, 2), the distance from the focus to the vertex is 2 units, and the function has a minimum.

$(x-h)^2=4p(y-k)$

$(x-3)^2=4(2)(y-0)$

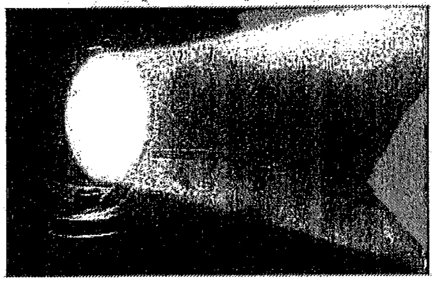


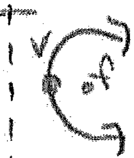
Equation: $(x-3)^2=8(y-0)$

Axis of Symmetry: ______ Directrix: $y=-2$

Vertex: $(3,0)$ p= 2 Focal Width: 8

3. In a searchlight, the bulb is placed at the focus of a parabolic mirror, causing the light rays from the bulb to bounce off the mirror as parallel rays. This provides a concentrated beam of light. Write the equation of the parabola if the focus is 3 feet from the vertex. Assume the vertex is at the origin and the parabola opens to the right.

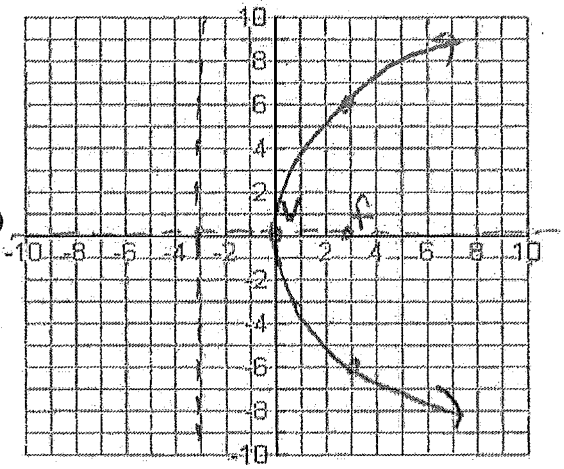




$(y-k)^2=4p(x-h)$

$(y-0)^2=4(3)(x-0)$

$\boxed{y^2=12x}$



Equation: $(y-0)^2=12(x-0)$

Axis of Symmetry: $y=0$ Directrix: $x=-3$

Vertex: $(0,0)$ Focus: $(3,0)$

p= 3 Focal Width: 12

4. The arch in Freedom Park has a parabolic shape. Its height is 25 feet and its base is 30 feet wide. Find an equation, which models this shape, using the *x*-axis to represent the ground.
Hint: Use 1 square = 2 feet and put the origin at the lower left corner of the graph.

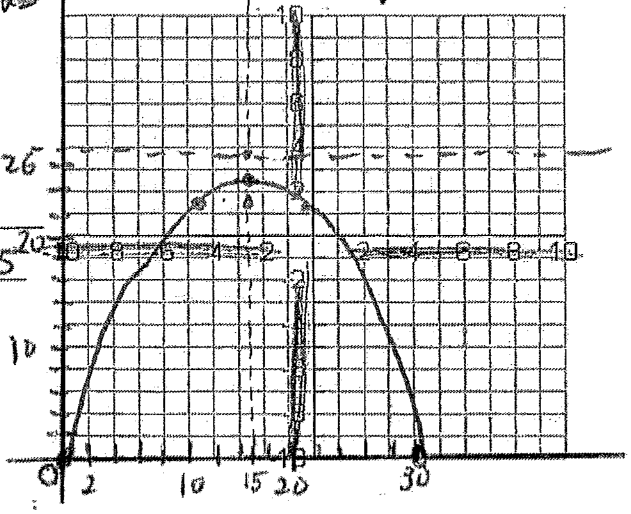
vertex $(15,25)$ h k | point $(0,0)$ x y | $p=\frac{-9}{4}=-2.25$

$(x-h)^2=4p(y-k)$

$(0-15)^2=4p(0-25)$

$225=4p(-25)$

$225=-100p$



Equation: $(x-15)^2=-9(y-25)$

Axis of Symmetry: $x=15$ Directrix: $y=27.25$

Vertex: $(15,25)$ Focus: $(15,22.75)$

p= $-9/4$ Focal Width: 9

$(x-15)^2=4\left(\frac{-9}{4}\right)(y-25)$

8.04 Circles Day 1

Date: ____________

Recall:

The Distance Formula – the distance between the points (x_1, y_1) and (x_2, y_2) is as follows:

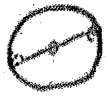
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The Midpoint Formula – the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is as follows:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Apply:

Given the two endpoints of the diameter of a circle, find the diameter's length.



$x_1\ y_1\ x_2\ y_2$

1. $(1, 7), (5, 4)$

$d=\sqrt{(5-1)^2+(4-7)^2}$

$d=\sqrt{4^2+3^2}$

$d=\sqrt{25}=\boxed{5}$

$x_1\ y_1\ x_2\ y_2$

2. $(-4, 7), (0, -1)$

$d=\sqrt{(0--4)^2+(-1-7)^2}$

$d=\sqrt{4^2+8^2}=\sqrt{16+64}$

$\boxed{d=\sqrt{80}}$

Given the two endpoints of the diameter of a circle, find the center of the circle.

3. $(1, 7), (5, 5)$

$M\left(\frac{1+5}{2}, \frac{7+5}{2}\right)$

$M\left(\frac{6}{2}, \frac{12}{2}\right) \rightarrow \boxed{M(3,6)}$

4. $(-8, 9), (2, -4)$

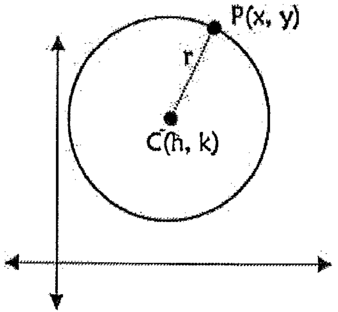
$M\left(\frac{-8+2}{2}, \frac{9-4}{2}\right)$

$M\left(\frac{-6}{2}, \frac{+5}{2}\right)$

$\boxed{M\left(-3, \frac{5}{2}\right)}$

Circle: the set of all points (x, y) that are equidistant from a fixed point, called the center of the circle. The distance r between the center and any point (x, y) on the circle is called the radius.

Suppose the center of this circle is translated from the origin to C(h, k). Let's apply the distance formula to write the equation for the translated circle:



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 1, fill in r, **P** (x, y) and **C** (h, k):

Step 2, square each side:

This is the **standard form of the equation of a circle with radius *r* and center at (*h*, *k*).**

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$

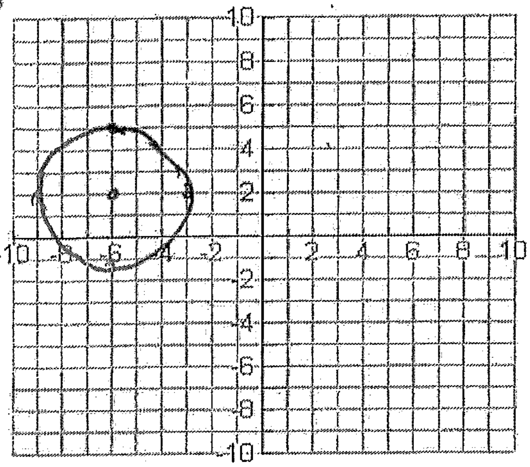
$c(-6,2)$

$r=3$

$(x-h)^2+(y-k)^2=r^2$

Given the standard form for the circle, identify the center and radius. Then graph.

5. $(x+6)^2+(y-2)^2=9$

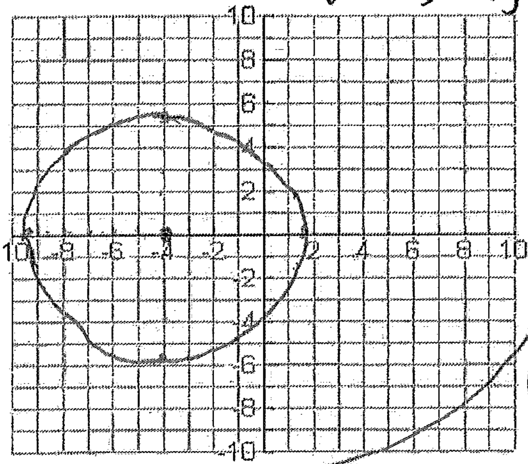


6. $(x+4)^2+y^2=30$

$(x+4)^2+(y-0)^2=30$

$c(-4,0)$

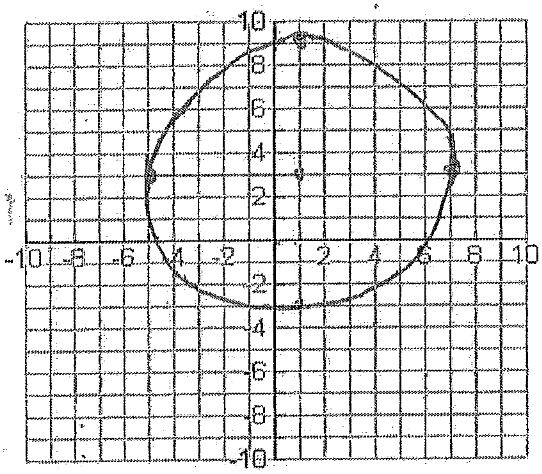
$r=\sqrt{30}$



Write the standard form of the equation of each circle described and graph the equation.

7. Center at $\overset{h\,k}{(1, 3)}$ and radius $\overset{r}{=}$ 6

$(x-1)^2+(y-3)^2=6^2$



8. Center at $\overset{h\,k}{(-2, 6)}$ passing through $\overset{x\,y}{(1, 2)}$

$(x-h)^2+(y-k)^2=r^2$

$(1+2)^2+(2-6)^2=r^2$

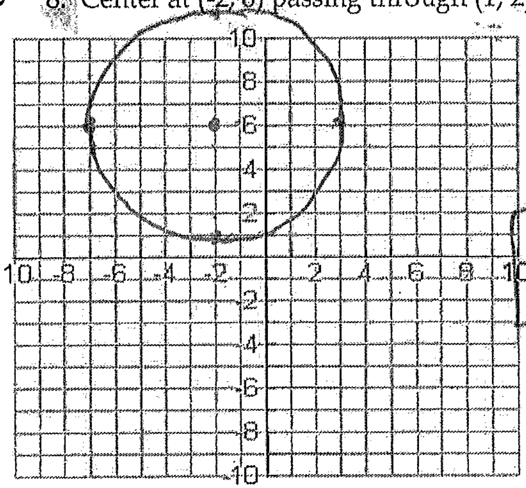
$3^2+(-4)^2=r^2$

$9+16=r^2$

$25=r^2$

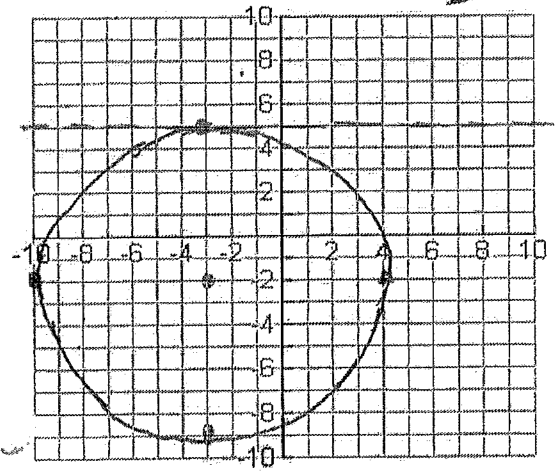
$r=5$

$\boxed{(x+2)^2+(y-6)^2=25}$



9. Center at (-3, -2) and tangent to the line y= 5

$r=7$



10. The endpoints of the diameter are (-2, 1) and (6, 7)

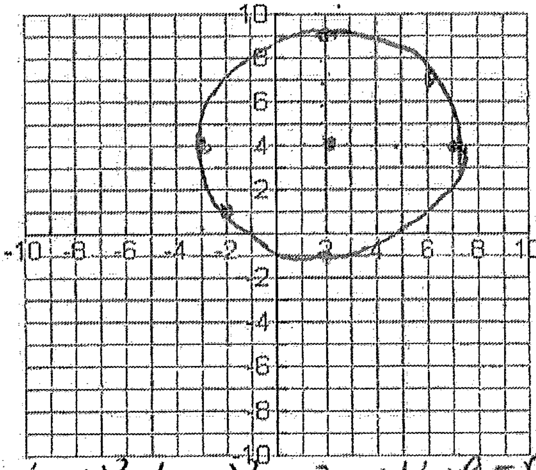
$M\left(\frac{-2+6}{2}, \frac{1+7}{2}\right)$

$M\left(\frac{4}{2}, \frac{8}{2}\right)$

$M(2,4)$

center $\overset{}{(\underset{h}{2}, \underset{k}{4})}$

point $(\underset{x}{-2}, \underset{y}{1})$



$(x-h)^2+(y-k)^2=r^2$

$(-2-2)^2+(1-4)^2=r^2$

$4^2+3^2=r^2$

$16+9=r^2$

$r=5$

$\boxed{(x-2)^2+(y-4)^2=25}$

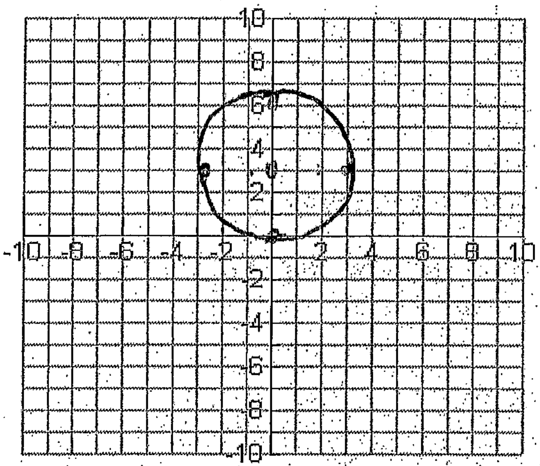
8.04 Practice

Given the standard form for the circle, identify the center and radius. Then graph.

1. $x^2 + (y-3)^2 = 9$

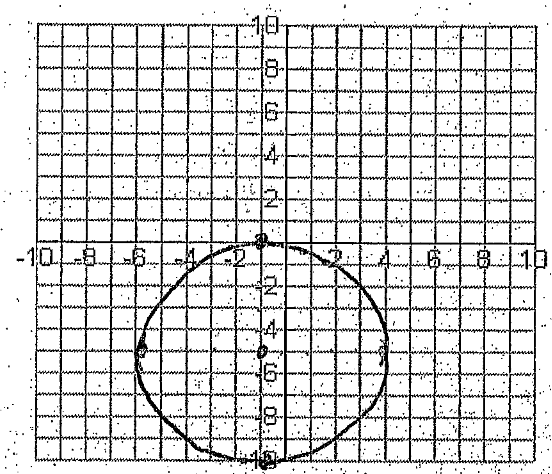
$(x-0)^2+(y-3)^2=9$

$C(0,3)$ $r=3$



2. $(x+1)^2 + (y+5)^2 = 25$

$C(-1,-5)$ $r=5$



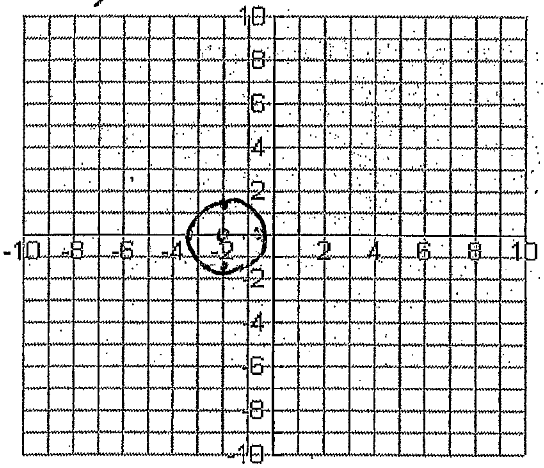
3. $(x+2)^2 + y^2 = \frac{16}{9}$

$(x+2)^2+(y-0)^2=\frac{16}{9}$

$C(-2,0)$

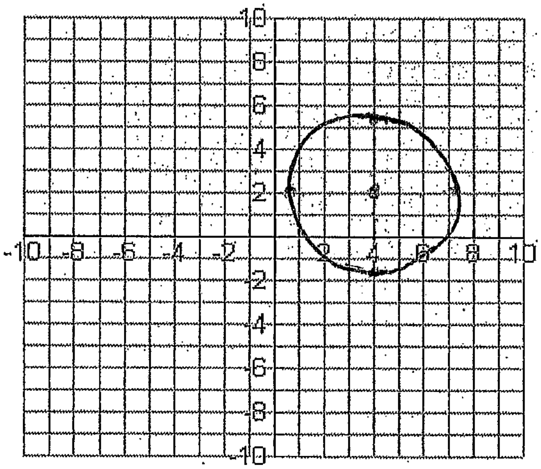
$r=\sqrt{\frac{16}{9}}$

$r=\frac{4}{3}$



4. $(x-4)^2 + (y-2)^2 = 10$

$C(4,2)$ $r=\sqrt{10} \approx 3.1$



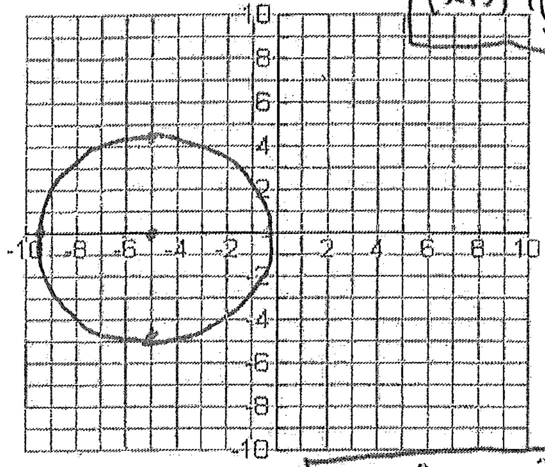
$(x-h)^2+(y-k)^2=r^2$

Write the standard form of the equation of each circle described and graph the equation.

5. Center at (-5, 0), radius $\frac{9}{2}$

$(x+5)^2+(y-0)^2=\left(\frac{9}{2}\right)^2$

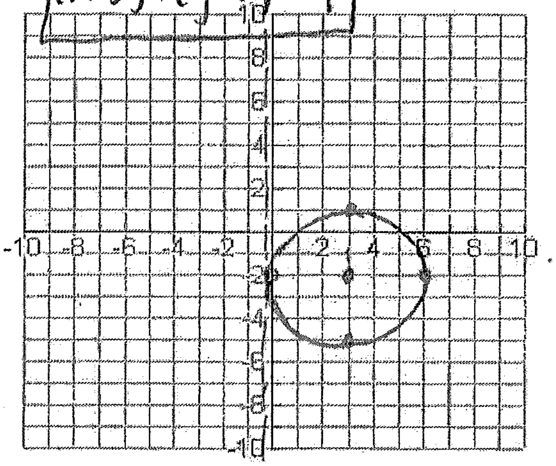
$\boxed{(x+5)^2+(y-0)^2=\frac{81}{4}}$



6. Center at $\overset{h\ k}{(3, -2)}$ and tangent to the y-axis.

$r=3$

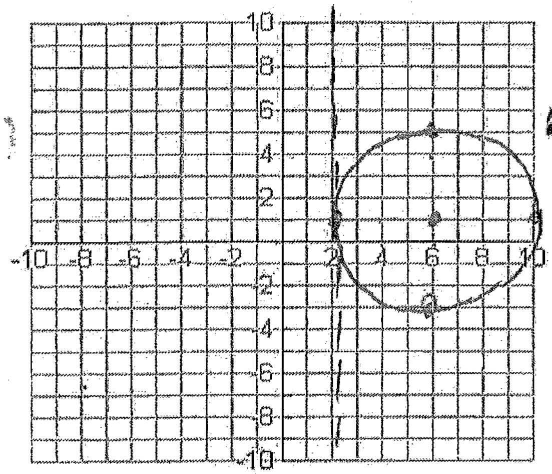
$\boxed{(x-3)^2+(y+2)^2=9}$



7. Center at (6, 1) and tangent to the line x = 2.

$\boxed{(x-6)^2+(y-1)^2=16}$

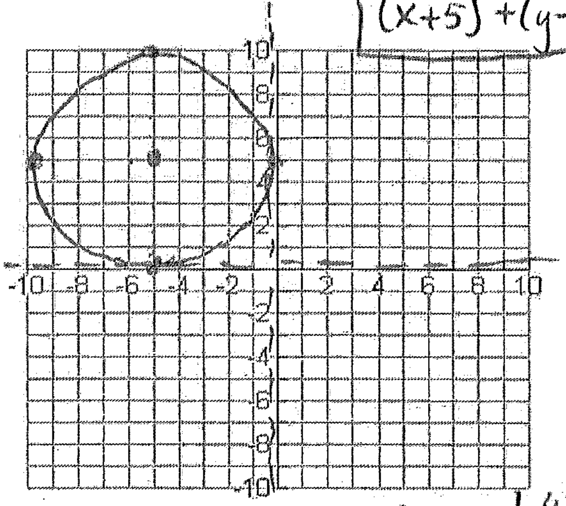
$r=4$



8. Center at (-5, 5) and tangent to both the x and y-axes.

$r=5$

$\boxed{(x+5)^2+(y-5)^2=25}$



Write the equation of the circle that satisfies each set of conditions.

9. The circle passes through the origin and has its center at $\overset{h\ k}{(-4, 3)}$.

$\underset{x\ y}{(0,0)}$

$(x-h)^2+(y-k)^2=r^2$

$(0+4)^2+(0-3)^2=r^2$

$4^2+3^2=r^2$

$16+9=r^2$

$25=r^2,\ r=5$

$\boxed{(x+4)^2+(y-3)^2=25}$

10. The circle passes through the point $\overset{x\ y}{(5, 6)}$ and has its center at $\overset{h\ k}{(2, 3)}$.

$(x-h)^2+(y-k)^2=r^2$

$(5-2)^2+(6-3)^2=r^2$

$3^2+3^2=r^2$

$9+9=r^2$

$18=r^2$

$\boxed{(x-2)^2+(y-3)^2=18}$

11. The endpoints of a diameter are at (2, 3) and at (-6, -5).

$M\left(\frac{2-6}{2},\frac{3-5}{2}\right)$

$M(-2,-1)$

$C(\overset{h}{-2},\overset{k}{-1})$

$\text{point}(\overset{x}{2},\overset{y}{3})$

$(x-h)^2+(y-k)^2=r^2$

$(2--2)^2+(3--1)^2=r^2$

$4^2+4^2=r^2$

$32=r^2$

$\boxed{(x+2)^2+(y+1)^2=32}$

$(x-h)^2+(y-k)^2=r^2$

8.05 Circles Day 2 – Converting to Standard Form

Date: ____________

What if the equation for the circle is not already in standard form? The following are ALL equations of circles. Can you identify the center and radius of each circle?

$x^2+y^2=4$

$(x-0)^2+(y-0)^2=4$

$C(0,0)$ $r=\sqrt{4}=2$

$(x+1)^2+(y+5)^2=20$

$C(-1,-5)$ $r=\sqrt{20}$

$x^2+y^2-6x+8y+9=0$

How can we determine the center and radius for the last equation?

$\left(\frac{6}{2}\right)^2=$ $x^2+y^2-6x+8y+9=0$ $\left(\frac{b}{2}\right)^2$ $\left(\frac{8}{2}\right)^2=4^2=16$

$x^2-6x+\underline{9}+y^2+8y+\underline{16}=-9+\underline{9}+\underline{16}$

$(x-3)(x-3)+(y+4)(y+4)=16$

$(x-3)^2+(y+4)^2=16$

$C(3,-4)$ $r=\sqrt{16}=4$

Examples: Write the standard form of the equation of each circle and then graph the equation. Identify the center and radius.

$\left(\frac{10}{2}\right)^2=25$ $\left(\frac{4}{2}\right)^2=2^2=4$

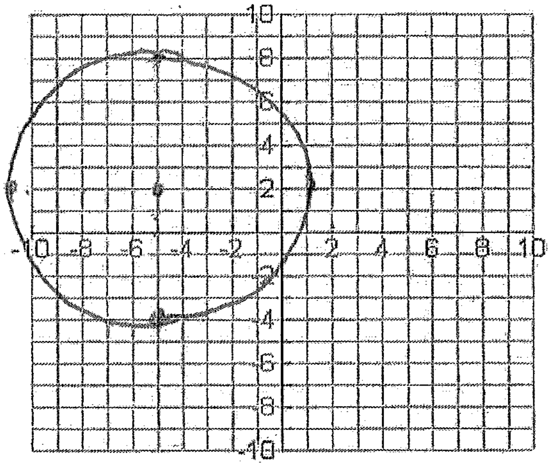
1. $x^2+y^2+10x-4y-7=0$

$x^2+10x+\underline{25}+y^2-4y+\underline{4}=7+\underline{25}+\underline{4}$

$(x+5)(x+5)+(y-2)(y-2)=36$

$(x+5)^2+(y-2)^2=36$

$C(-5,2)$ $r=\sqrt{36}=6$



2. $4x^2+4y^2-8x+32y-170=2$

$4x^2-8x+4y^2+32y=170+2$

$\frac{4x^2-8x+4y^2+32y}{4}=\frac{172}{4}$

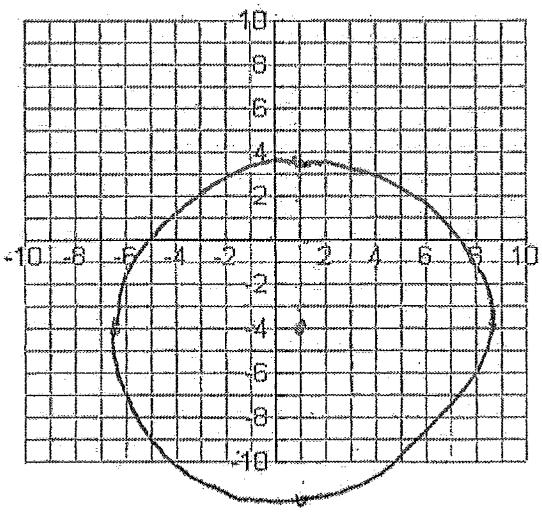
$\left(\frac{2}{2}\right)^2$ $\left(\frac{8}{2}\right)^2$

$x^2-2x+\underline{1}+y^2+8y+\underline{16}=43+\underline{1}+\underline{16}$

$(x-1)(x-1)+(y+4)(y+4)=60$

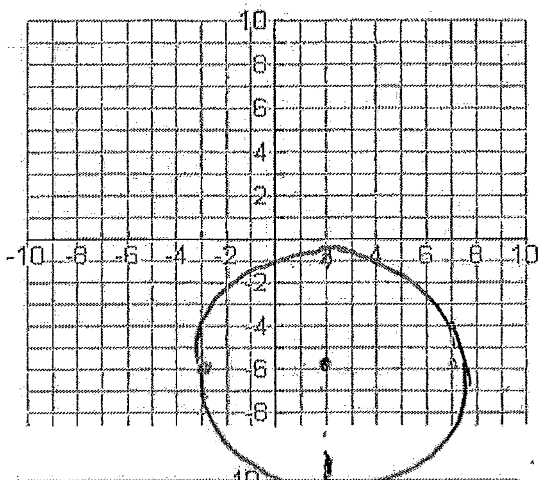
$(x-1)^2+(y+4)^2=60$

$C(1,-4)$ $r=\sqrt{60}$



8.05 Circles Day 2 Practice

Write the standard form of the equation of each circle and then graph the equation. Identify the center and radius.



1. $x^2 + y^2 - 4x + 12y + 15 = 0$

$\left(\frac{4}{2}\right)^2$ $\left(\frac{12}{2}\right)^2=$

$x^2-4x+\underline{4}+y^2+12y+\underline{36}=-15+\underline{4}+\underline{36}$

$(x-2)(x-2)+(y+6)(y+6)=25$

$(x-2)^2+(y+6)^2=25$

$C(2,-6)$ $r=\sqrt{25}=5$

2. $2x^2 + 2y^2 + 4x - 8y + 8 = 6$

2 2

$\left(\frac{2}{2}\right)^2$ $\left(\frac{4}{2}\right)^2$

↓ ↓

$x^2+y^2+2x-4y+4=3$

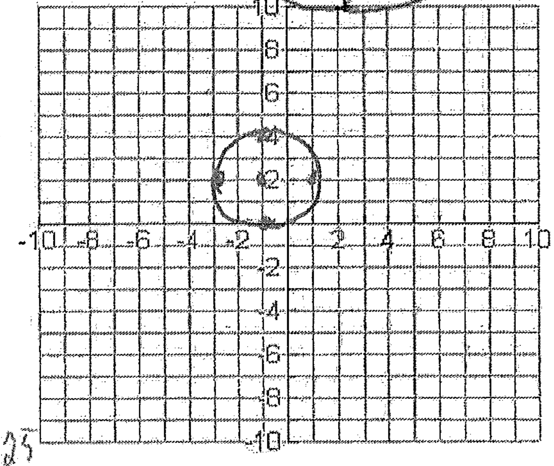
$x^2+2x+\underline{1}+y^2-4y+\underline{4}=-4+3+\underline{1}+4$

$(x+1)(x+1)+(y-2)(y-2)=4$

$(x+1)^2+(y-2)^2=4$

$C(-1,2)$

$r=2$



3. $16x^2 + 16y^2 - 16x - 160y = 172$

16 16

$\left(\frac{1}{2}\right)^2=0.25$ $\left(\frac{10}{2}\right)^2=25$

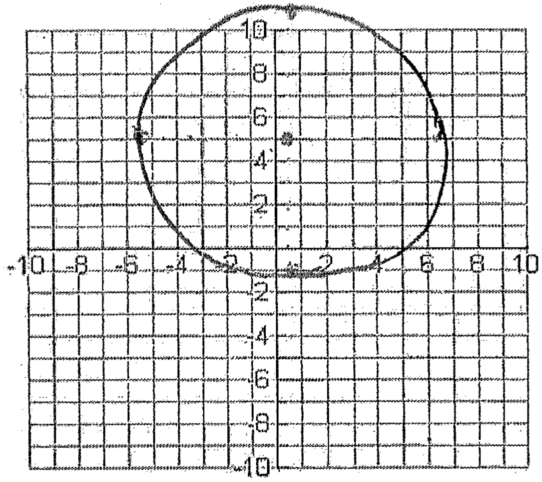
$x^2+y^2-1x-10y=10.75+\underline{\quad}+\underline{\quad}$

$x^2-1x+\underline{0.25}+y^2-10y+\underline{25}=10.75+\underline{0.25}+\underline{25}$

$(x-0.5)^2+(y-5)^2=36$

$C(1/2,5)$

$r=\sqrt{36}=6$



4. $162 - 3x^2 = 3y^2 - 24y$

$162=3x^2+3y^2-24y$

$\frac{3x^2+3y^2-24y}{3}=\frac{162}{3}$

$x^2+y^2-8y=54$

$\left(\frac{8}{2}\right)^2=16$

$x^2+y^2-8y+\underline{16}=54+\underline{16}$

$(x-0)^2+(y-4)(y-4)=70$

$(x-0)^2+(y-4)^2=70$

$C(0,4)$ $r=\sqrt{70}$

$(x-h)^2+(y-k)^2=r^2$

8.05 Circles Day 2 – Converting to Standard Form Date: ____________

What if the equation for the circle is <u>not already in standard form</u>? The following are ALL equations of circles. Can you identify the center and radius of each circle?

$x^2 + y^2 = 4$

$(x-0)^2+(y-0)^2=4$

$C(0,0)$ $r=\sqrt{4}=2$

$(x + 1)^2 + (y + 5)^2 = 20$

$C(-1,-5)$ $r=\sqrt{20}$

$x^2 + y^2 - 6x + 8y + 9 = 0$

How can we determine the center and radius for the last equation?

$\left(\frac{6}{2}\right)^2=$ $x^2+y^2-6x+8y+9=0$ $\left(\frac{6}{2}\right)^2$ $\left(\frac{8}{2}\right)^2=4^2=16$

$x^2-6x+\underline{9}+y^2+8y+\underline{16}=-9+\underline{9}+\underline{16}$

$(x-3)(x-3)+(y+4)(y+4)=16$

$(x-3)^2+(y+4)^2=16$

$C(3,-4)$ $r=\sqrt{16}=4$

Examples: Write the standard form of the equation of each circle and then graph the equation. Identify the center and radius.

$\left(\frac{10}{2}\right)^2=25$ $\left(\frac{4}{2}\right)^2=2^2=4$

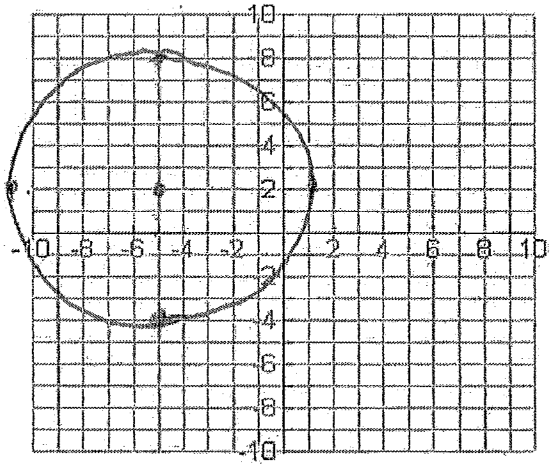
1. $x^2 + y^2 + 10x - 4y - 7 = 0$

$x^2+10x+\underline{25}+y^2-4y+\underline{4}=7+\underline{25}+\underline{4}$

$(x+5)(x+5)+(y-2)(y-2)=36$

$(x+5)^2+(y-2)^2=36$

$C(-5,2)$ $r=\sqrt{36}=6$



2. $4x^2 + 4y^2 - 8x + 32y - 170 = 2$

$4x^2-8x+4y^2+32y=170+2$ $\left(\frac{2}{2}\right)^2$ $\left(\frac{8}{2}\right)^2$

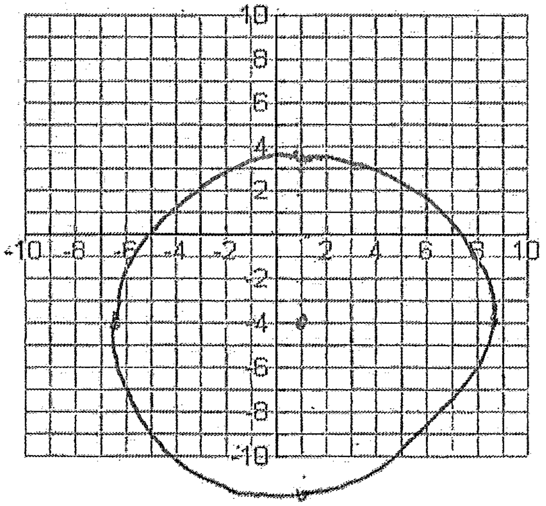
$\frac{4x^2-8x+4y^2+32y}{4}=\frac{172}{4}$

$x^2-2x+\underline{1}+y^2+8y+\underline{16}=43+\underline{1}+\underline{16}$

$(x-1)(x-1)+(y+4)(y+4)=60$

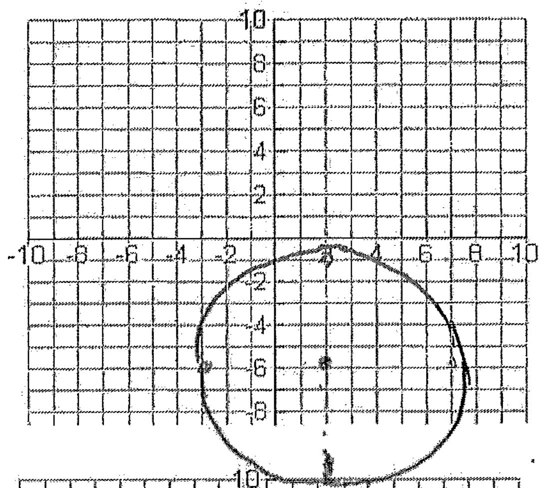
$(x-1)^2+(y+4)^2=60$

$C(1,-4)$ $r=\sqrt{60}$



8.05 Circles Day 2 Practice

Write the standard form of the equation of each circle and then graph the equation. Identify the center and radius.



1. $x^2 + y^2 - 4x + 12y + 15 = 0$

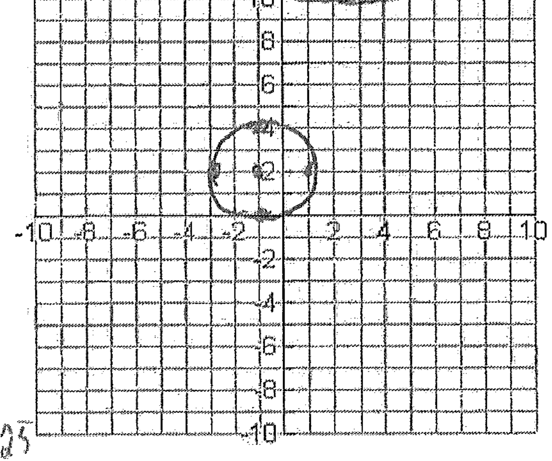
$\left(\frac{4}{2}\right)^2$ $\left(\frac{12}{2}\right)^2 =$

$x^2-4x+\underline{4}+y^2+12y+\underline{36}=-15+\underline{4}+\underline{36}$

$(x-2)(x-2)+(y+6)(y+6)=25$

$(x-2)^2+(y+6)^2=25$

$C(2,-6)$ $r=\sqrt{25}=5$



2. $\frac{2x^2 + 2y^2 + 4x - 8y + 8}{2} = \frac{6}{2}$

$\left(\frac{2}{2}\right)^2$ $\left(\frac{4}{2}\right)^2$

$x^2+y^2+2x-4y+4=3$

$x^2+2x+\underline{1}+y^2-4y+\underline{4}=-4+3+\underline{1}+4$

$(x+1)(x+1)+(y-2)(y-2)=4$

$(x+1)^2+(y-2)^2=4$

$\boxed{\begin{array}{c} C(-1,2) \\ r=2 \end{array}}$

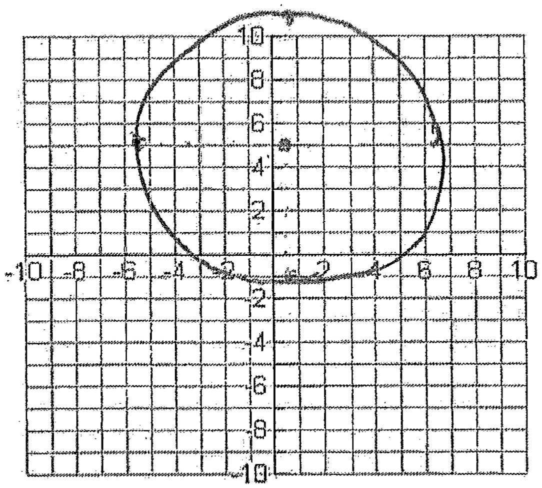
3. $\frac{16x^2 + 16y^2 - 16x - 160y}{16} = \frac{172}{16}$

$\left(\frac{1}{2}\right)^2=0.25$ $\left(\frac{10}{2}\right)^2=25$

$x^2+y^2-1x-10y=10.75+\underline{\quad}+\underline{\quad}$

$x^2-1x+\underline{0.25}+y^2-10y+\underline{25}=10.75+\underline{0.25}+\underline{25}$

$(x-0.5)^2+(y-5)^2=36$ | $C(1/2,5)$ $r=\sqrt{36}=6$



4. $162 - 3x^2 = 3y^2 - 24y$

$162=3x^2+3y^2-24y$

$\frac{3x^2+3y^2-24y}{3}=\frac{162}{3}$

$\left(\frac{8}{2}\right)^2=16$

$x^2+y^2-8y=54$

$x^2+y^2-8y+\underline{16}=54+\underline{16}$

$(x-0)^2+(y-4)(y-4)=70$

$\boxed{\begin{array}{c} (x-0)^2+(y-4)^2=70 \\ C(0,4) \quad r=\sqrt{70} \end{array}}$

8.06 Parabolas and Circles Review Date: ____________

1. Identify the characteristics of the parabola. Graph and label all parts.
$(y-3)^2 = 12(x+1)$

$4p=12$
$p=3$

opens right

p = 3

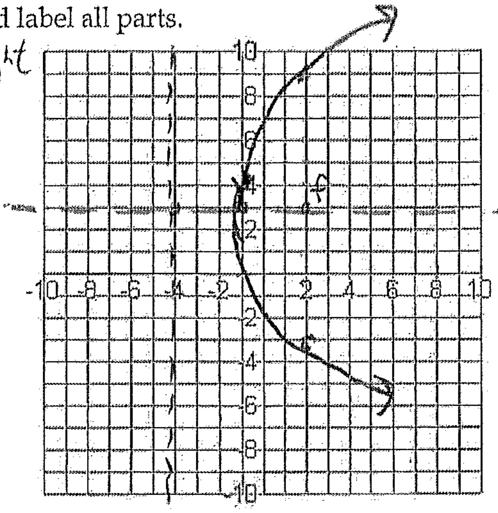
Vertex: (-1,3)

Focus: (2,3)

Directrix: $x=-4$

Axis of Symmetry: $y=3$

Focal Width: 12



AOS
$y=3$

2. Write the equation of the parabola $x^2 - 8y - 4x + 12 = 0$ in standard form. Identify the vertex, focus, directrix, axis of symmetry, and focal width. Graph the parabola and label all parts.

Standard Form: $(x-2)^2=8(y-1)$

$\left(\frac{4}{2}\right)^2=4$

$x^2-4x=8y-12$
$x^2-4x+\underline{4}=8y-12+\underline{4}$
$(x-2)^2=8y-8$
$(x-2)^2=8(y-1)$

$4p=8$
$p=2$

p = 2

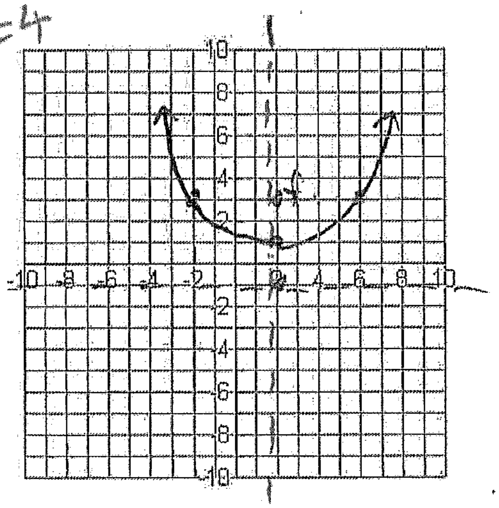
Vertex: (2,1)

Focus: (2,3)

Directrix: $y=-1$

Axis of Symmetry: $x=2$

Focal Width: 8



3. Write the standard form of the equation for the parabola with a focus at (4,2) and directrix at $y=-8$ Identify its characteristics. Graph the parabola and label all parts.

Standard Form: $(x-4)^2=20(y+3)$

$(x-h)^2=4p(y-k)$
$(x-4)^2=4(5)(y+3)$

p = 5

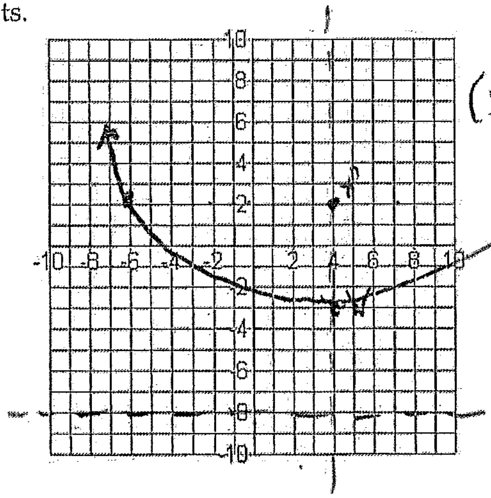
Vertex: (4,-3)

Directrix: $y=-8$

Axis of Symmetry: $x=4$

opens up

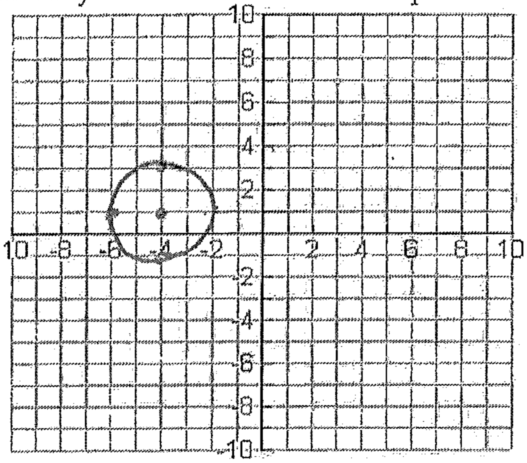
$(x-h)^2=4p(y-k)$



$y=-8$

$(x-h)^2+(y-k)^2=r^2$

4. Use the equation $(x+4)^2+(y-1)^2=4$ and Identify the center and radius. Graph the circle.

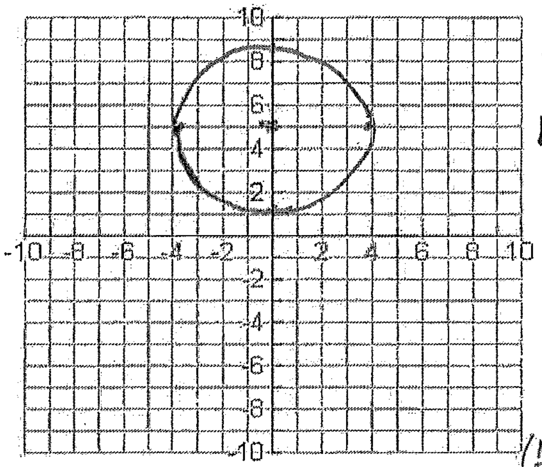


C(-4,1)

r=2

$\rightarrow (x-0)^2+(y-5)^2=\sqrt{15}^2 \rightarrow \boxed{(x-0)^2+(y-5)^2=15}$

5. Write the equation of a circle with center (0, 5) and radius=$\sqrt{15}$. Graph the circle.

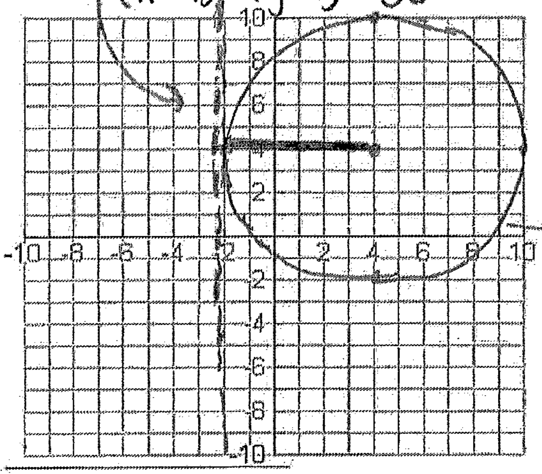


C(0,5)

$r=\sqrt{15}$

6. Given a circle with center (4,4), draw a circle that is tangent to x = -2, then write the equation of the circle. $(x-4)^2+(y-4)^2=36$

r=6



7. Put the equation of the circle into standard form. Identify the center and radius.
$x^2+y^2-10x-16y+88=0$

$\left(\frac{10}{2}\right)^2=25$

$\left(\frac{16}{2}\right)^2=64$

$x^2-10x+25+y^2-16y+64=-88+25+64$

~~$(x-5)(x-5)+(y-8)(y-8)$~~ $=1$

$\boxed{(x-5)^2+(y-8)^2=1}$

$\boxed{\begin{matrix}C=(5,8)\\ r=1\end{matrix}}$

8) $(x-h)^2+(y-k)^2=r^2$

$(-3-5)^2+(1--2)^2=r^2$

$8^2+3^2=r^2$

$r=\sqrt{73}$

9)

$8^2+10^2=r^2$

$r^2=164$

$\boxed{(x-12)^2+(y+1)^2=164}$

8. Find the equation of a circle whose center is at $(\overset{h}{5},\overset{k}{-2})$ and contains the point $(\overset{x}{-3},\overset{y}{1})$.

$\boxed{(x-5)^2+(y+2)^2=73}$

9. Find the equation of a circle whose diameter has endpoints at $(\overset{x}{4},\overset{y}{9})$ and (20, –11).

Midpt: $\left(\frac{4+20}{2},\frac{9-11}{2}\right)\rightarrow M(\overset{h}{12},\overset{k}{-1})$

$(x-h)^2+(y-k)^2=r^2$

$(4-12)^2+(9--1)^2=r^2$

10. A furniture store advertises free delivery up to a 30-mile radius from the store. Which, if any, of these customers qualifies for free delivery?

Hannah lives 7 miles north and 29 miles east of the store. $\sqrt{7^2+29^2}=r$ $r=29.83<30$ ✓

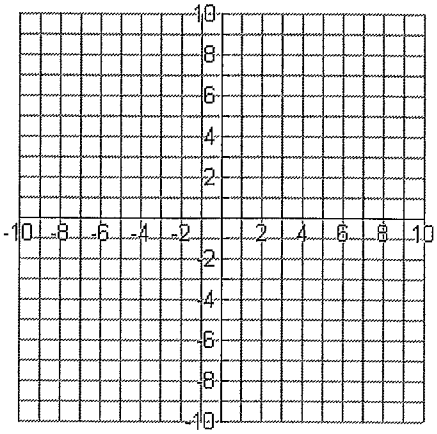
Anna lives 16 miles west and 24 miles north of the store. $r=\sqrt{16^2+24^2}$ $r=28.84<30$ ✓

Nikki lives 28 miles west and 12 miles south of the store. $r=\sqrt{12^2+28^2}$ $r=30.463>30$

no free delivery

8.06b Parabolas and Circles Review WS #2

1. Identify the characteristics of the parabola. Graph and label all parts.
   $(y-2)^2 = -16(x+3)$



p = ______

Vertex: ______________________

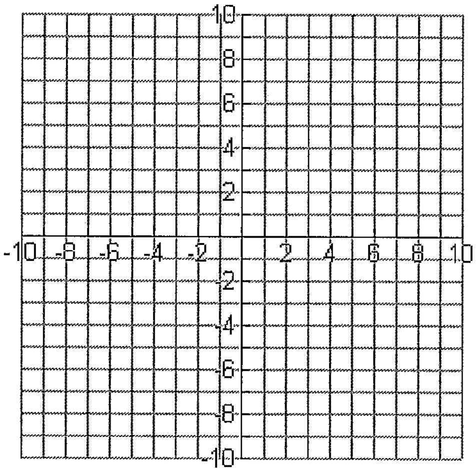
Focus: ______________________

Directrix: ______________________

Axis of Symmetry: ______________________

Focal Width: ______________________

2. Write the equation of the parabola $x^2 - 4x + 8y - 13 = 39$ in standard form. Identify the vertex, focus, directrix, axis of symmetry, and focal width. Graph the parabola and label all parts.



Standard Form: ______________________________

p = ______

Vertex: ______________________

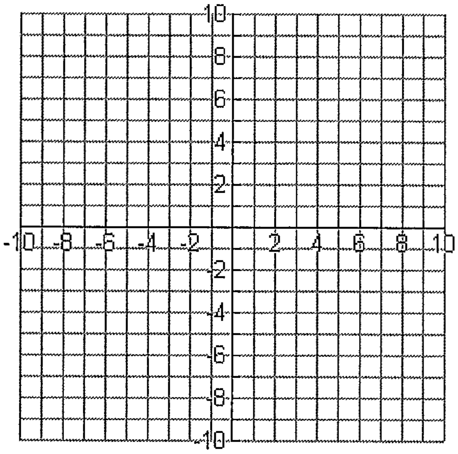
Focus: ______________________

Directrix: ______________________

Axis of Symmetry: ______________________

Focal Width: ______________

3. Write the standard form of the equation for the parabola with a focus at (2,1) and directrix at $x = -2$ Identify its characteristics. Graph the parabola and label all parts.



Standard Form: ______________________________

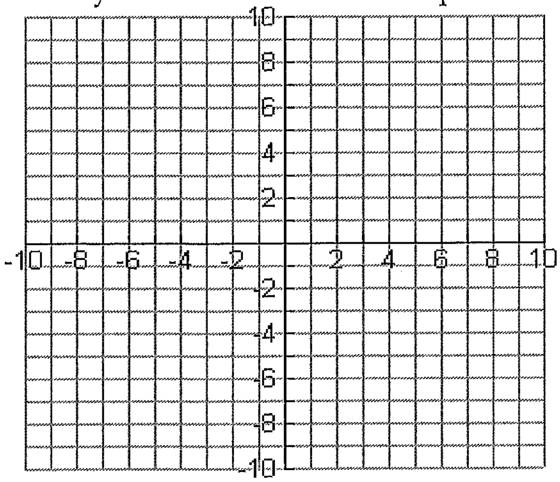
p = ______

Vertex: ______________________

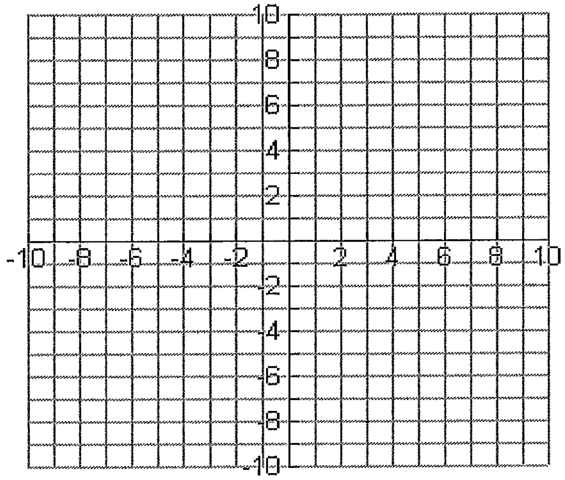
Directrix: ______________________

Axis of Symmetry: ______________________

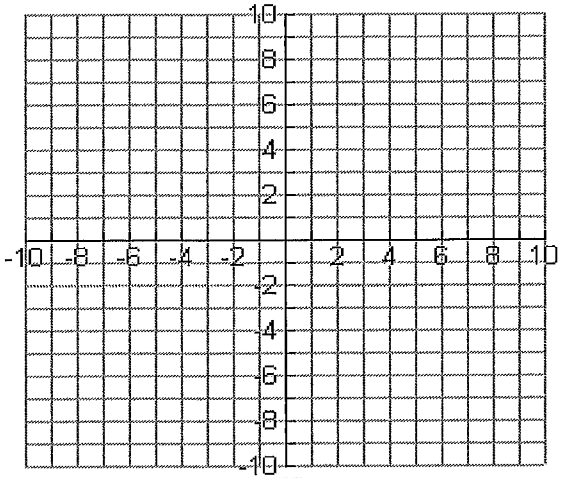
4. Use the equation $x^2 + (y - 1)^2 = 4$ and Identify the center and radius. Graph the circle.



5. Write the equation of a circle with center (1, 3) and radius=$\sqrt{12}$. Graph the circle.



6. Given a circle with center (3,6), draw a circle that is tangent to y = 2, then write the equation of the circle.



7. Put the equation of the circle into standard form. Identify the center and radius.
$x^2 + y^2 + 14x = 2y - 41$

8. Find the equation of a circle whose center is at (0, –8) and contains the point (2, -5).

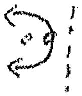
9. Find the equation of a circle whose diameter has endpoints at (-13,-7) and (11, 11).

Key

8.06b Parabolas and Circles Review WS #2

1. Identify the characteristics of the parabola. Graph and label all parts.
$(y-2)^2 = -16(x+3)$

opens left



$4p=-16$
$p=-4$

p = -4

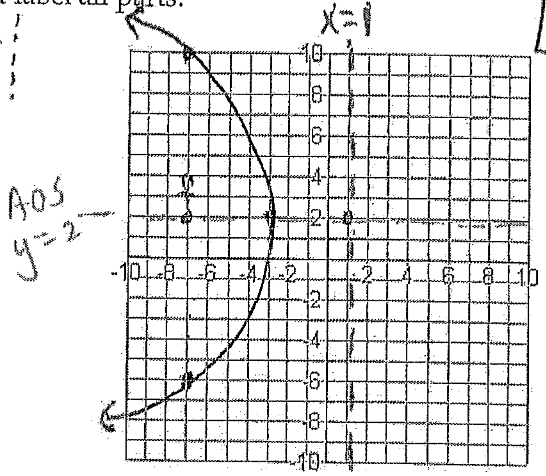
Vertex: $(-3,2)$

Focus: $(-7,2)$

Directrix: $x=1$

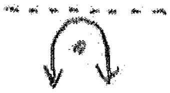
Axis of Symmetry: $y=2$

Focal Width: 16



2. Write the equation of the parabola $x^2-4x+8y-13=39$ in standard form. Identify the vertex, focus, directrix, axis of symmetry, and focal width. Graph the parabola and label all parts.

opens down



Standard Form: $(x-2)^2=-8(y-7)$

$\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{4}{2}\right)^2=4$

$x^2-4x+4=-8y+13+39+4$

$(x-2)(x-2)=-8y+56$

$(x-2)^2=-8(y-7)$

$4p=-8$
$p=-2$

p = -2

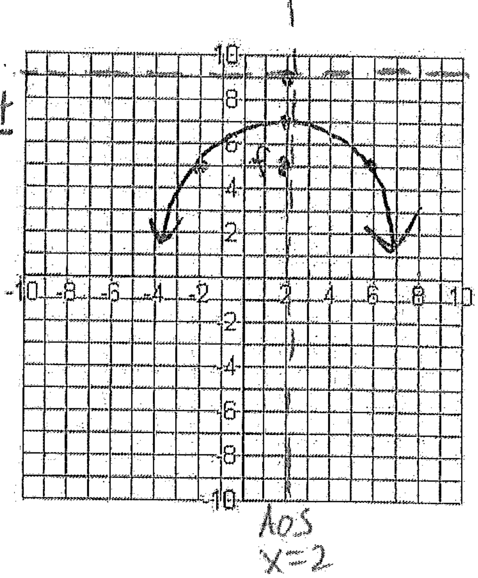
Vertex: $(2,7)$

Focus: $(2,5)$

Directrix: $y=9$

Axis of Symmetry: $x=2$

Focal Width: 8



3. Write the standard form of the equation for the parabola with a focus at (2,1) and directrix at $x=-2$ Identify its characteristics. Graph the parabola and label all parts.

Standard Form: $(y-1)^2=8(x-0)$

$(y-k)^2=4p(x-h)$

$(y-1)^2=4(2)(x-0)$

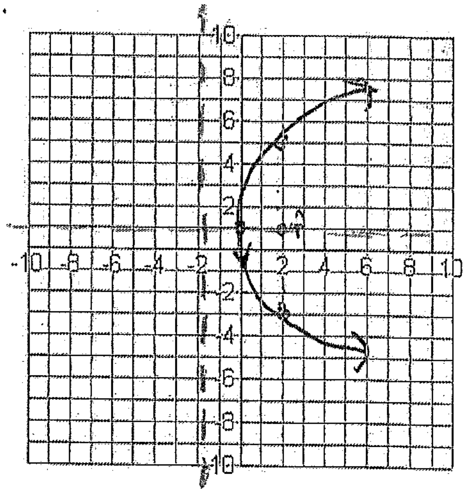
p = 2

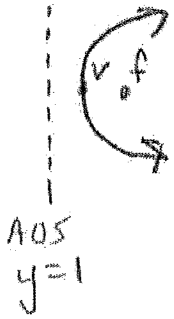
Vertex: $(0,1)$

Directrix: $x=-2$

xis of Symmetry: $y=1$

Focal width: 8





$(x-h)^2+(y-1)^2=4$

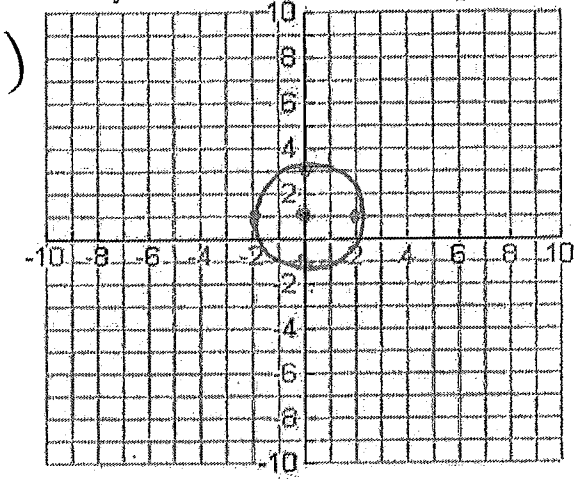
$(x-1)^2+(y-3)^2=\sqrt{12}^2 \rightarrow \boxed{(x-1)^2+(y-3)^2=12}$

4. Use the equation $x^2 + (y-1)^2 = 4$ and Identify the center and radius. Graph the circle.

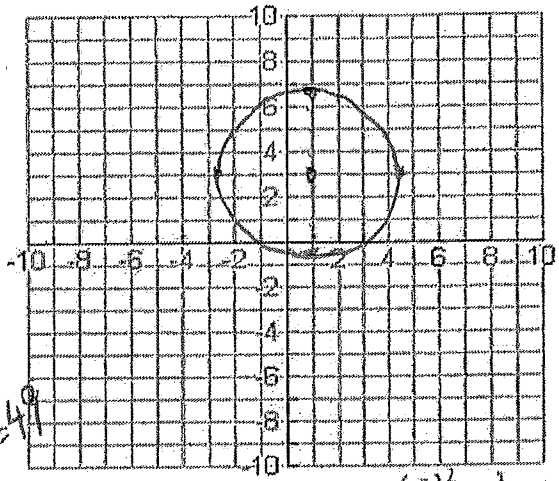
C(0,1)

$r=\sqrt{4}$

$r=2$

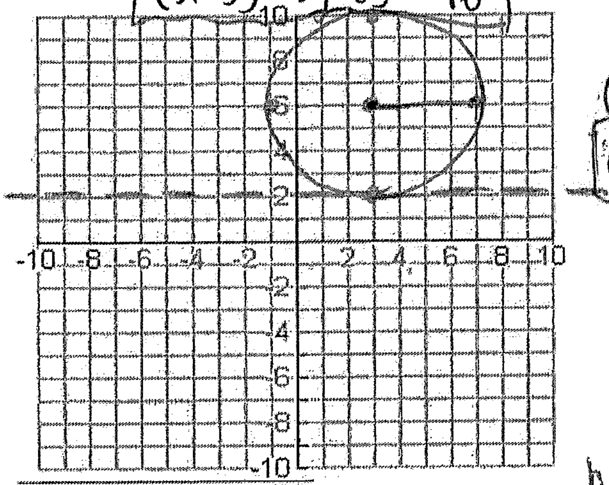


5. Write the equation of a circle with center (1, 3) and radius=$\sqrt{12}$. Graph the circle.



6. Given a circle with center (3,6), draw a circle that is tangent to y = 2, then write the equation of the circle. $\boxed{(x-3)^2+(y-6)^2=16}$

r=4



$\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{14}{2}\right)^2 = 49$

7. Put the equation of the circle into standard form. Identify the center and radius. $\left(\frac{-2}{2}\right)^2 = 1^2 = 1$

$x^2 + y^2 + 14x = 2y - 41$

$x^2+14x+\underline{49}+y^2-2y+\underline{1} = -41+\underline{49}+\underline{1}$

$(x+7)(x+7)+(y-1)(y-1) = 9$

$\boxed{(x+7)^2+(y-1)^2=9}$

$\boxed{\begin{matrix} C(-7,1) \\ r=3 \end{matrix}}$

$r^2=9$

$r=\sqrt{9}=3$

8. Find the equation of a circle whose center is at $(\overset{h}{0}, \overset{k}{-8})$ and contains the point $(\overset{x}{2}, \overset{y}{-5})$.

$(x-h)^2+(y-k)^2=r^2$

$(2-0)^2+(-5+8)^2=r^2$

$2^2+3^2=r^2$

$4+9=r^2$

$r^2=13$

$\boxed{(x-0)^2+(y+8)^2=13}$

9. Find the equation of a circle whose diameter has endpoints at (-13,-7) and $(\overset{x}{11}, \overset{y}{11})$.

$\text{Midpt}\left(\frac{-13+11}{2}, \frac{-7+11}{2}\right) \rightarrow \text{Center}(\overset{h}{-1}, \overset{k}{2})$

$\boxed{(x+1)^2+(y-2)^2=225}$

$(x-h)^2+(y-k)^2=r^2$

$(11+1)^2+(11-2)^2=r^2$

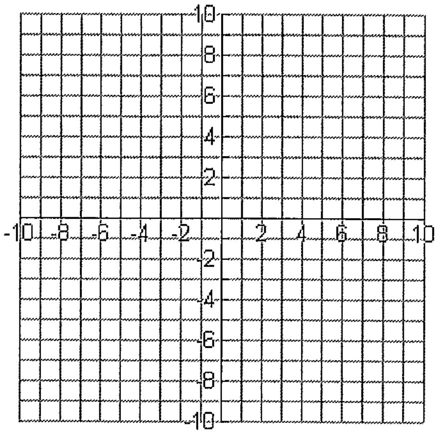
$12^2+9^2=r^2$

$144+81=r^2$

$225=r^2$

8.06c Parabolas and Circles Review WS #3

1. Identify the characteristics of the parabola. Graph and label all parts.
   $(y-1)^2 = -8x$



p = ________

Vertex: ____________________

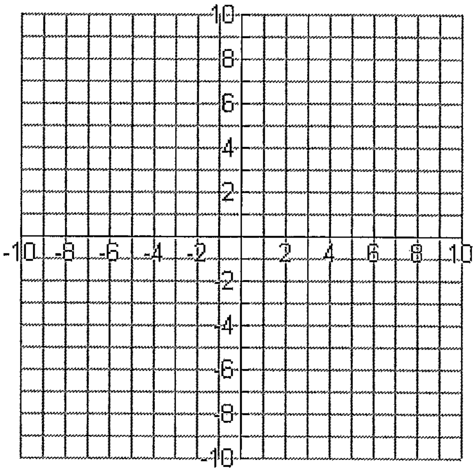
Focus: ____________________

Directrix: ____________________

Axis of Symmetry: ____________________

Focal Width: ____________________

2. Write the equation of the parabola $y^2 - 8y + 4x = 12$ in standard form. Identify the vertex, focus, directrix, axis of symmetry, and focal width. Graph the parabola and label all parts.



Standard Form: ______________________________

p = ________

Vertex: ____________________

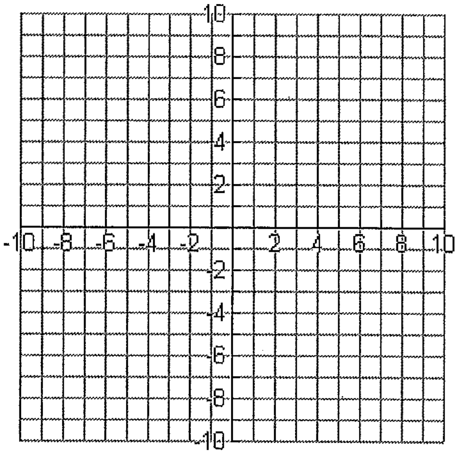
Focus: ____________________

Directrix: ____________________

Axis of Symmetry: ____________________

Focal Width: ______________

3. Write the standard form of the equation for the parabola with a vertex at (3,1) and directrix at $x = 5$ Identify its characteristics. Graph the parabola and label all parts.



Standard Form: ______________________________

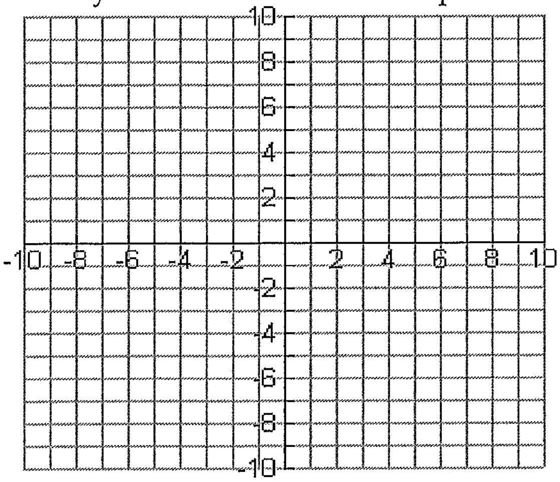
p = ________

Vertex: ____________________

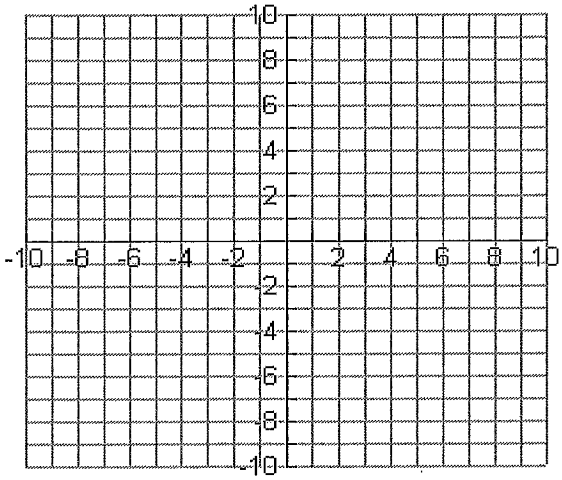
Directrix: ____________________

Axis of Symmetry: ____________________

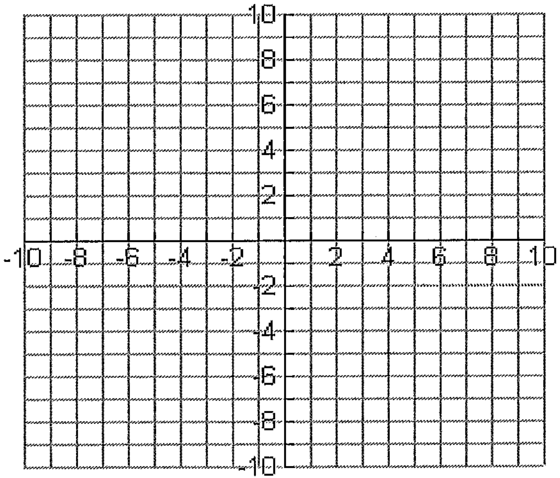
4. Use the equation $(x-3)^2+(y-5)^2=26$ and Identify the center and radius. Graph the circle.



5. Write the equation of a circle with center (-2, -5) and radius=$\sqrt{17}$. Graph the circle.



6. Given a circle with center (3,6), draw a circle that is tangent to x = 8, then write the equation of the circle.



7. Put the equation of the circle into standard form. Identify the center and radius.
$x^2+y^2-8x-4y-5=0$

8. Find the equation of a circle whose center is at (5, 9) and contains the point (7, 8).

9. Find the equation of a circle whose diameter has endpoints at (-5,-8) and (7, 2).

Key

8.06c Parabolas and Circles Review WS #3

1. Identify the characteristics of the parabola. Graph and label all parts.
$(y-1)^2 = -8x$ $\quad (y-1)^2 = -8(x-0)$ opens left

p = -2 $\quad 4p=-8$ $\quad p=-2$

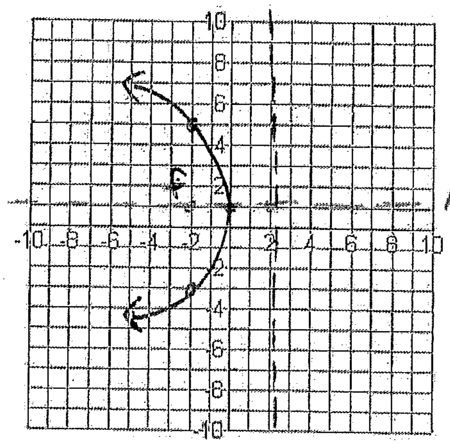
Vertex: $(0,1)$

Focus: $(-2,1)$

Directrix: $x=2$

Axis of Symmetry: $y=1$

Focal Width: 8



2. Write the equation of the parabola $y^2 - 8y + 4x = 12$ in standard form. Identify the vertex, focus, directrix, axis of symmetry, and focal width. Graph the parabola and label all parts. opens left

Standard Form: $(y-4)^2 = -4(x-7)$

$\left(\frac{8}{2}\right)^2 = 4^2 = 16$

$y^2 - 8y + 16 = -4x + 12 + 16$

$(y-4)(y-4) = -4x + 28$

$(y-4)^2 = -4(x-7)$

$4p=-4$ $\quad p=-1$

p = -1

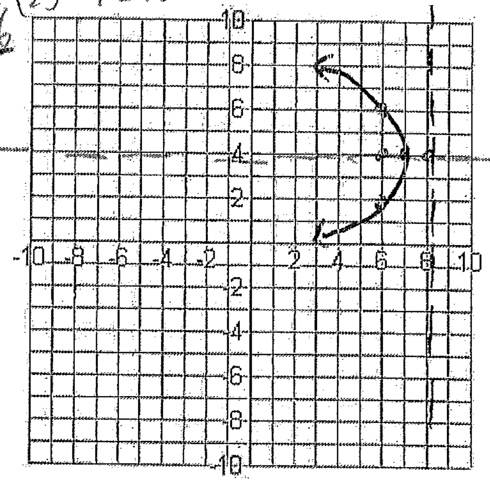
Vertex: $(7,4)$

Focus: $(6,4)$

Directrix: $x=8$

Axis of Symmetry: $y=4$

Focal Width: 4



3. Write the standard form of the equation for the parabola with a vertex at $\overset{h\,k}{(3,1)}$ and directrix at $x = 5$ Identify its characteristics. Graph the parabola and label all parts. opens left

Standard Form: $(y-1)^2 = -8(x-3)$

$(y-k)^2 = 4p(x-h)$

$(y-1)^2 = 4(-2)(x-3)$

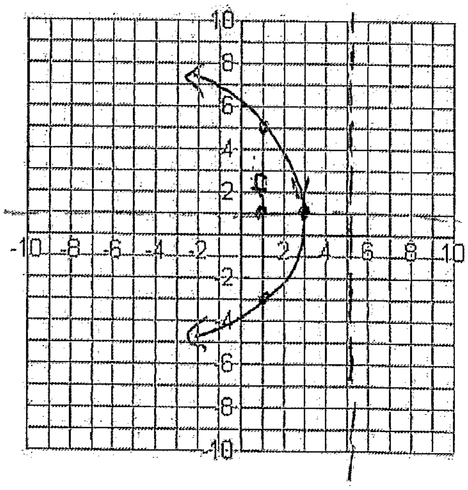
p = -2

Vertex: $(3,1)$

Directrix: $x=5$

xis of Symmetry: $y=1$

Focal Width: 8



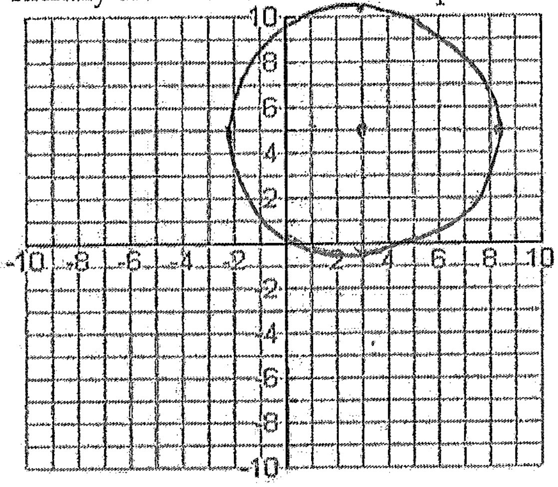
$(x-h)^2+(y-k)^2=r^2$

4. Use the equation $(x-3)^2+(y-5)^2=26$ and Identify the center and radius. Graph the circle.

C(3, 5)

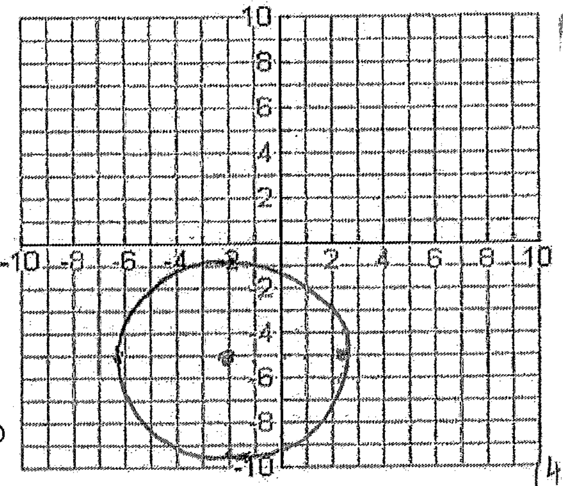
$r=\sqrt{26}$

$r\approx 5.1$



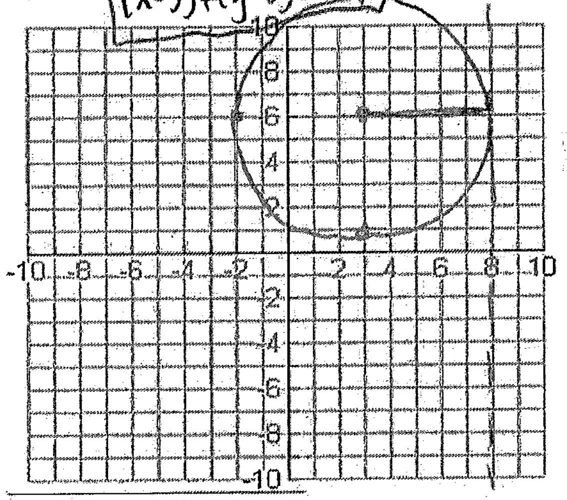
$(x+2)^2+(y+5)^2=\sqrt{17}^2$ $\boxed{(x+2)^2+(y+5)^2=17}$

5. Write the equation of a circle with center (-2, -5) and radius=$\sqrt{17}$. Graph the circle.



6. Given a circle with center (3,6), draw a circle that is tangent to x = 8, then write the equation of the circle $\boxed{(x-3)^2+(y-6)^2=25}$

r=5



$\left(\frac{b}{2}\right)^2 \left(\frac{8}{2}\right)^2=16$

$\left(\frac{4}{2}\right)^2=4$

7. Put the equation of the circle into standard form. Identify the center and radius.

$x^2+y^2-8x-4y-5=0$

$x^2-8x+\underline{16}+y^2-4y+\underline{4}=5+16+4$

$(x-4)(x-4)+(y-2)(y-2)=25$

$\boxed{(x-4)^2+(y-2)^2=25}$

$\boxed{C(4,2) \quad r=5}$

8. Find the equation of a circle whose center is at $\overset{h\ k}{(5, 9)}$ and contains the point $\overset{x\ y}{(7, 8)}$.

$(x-h)^2+(y-k)^2=r^2$

$(7-5)^2+(8-9)^2=r^2$

$2^2+1^2=r^2$

$r^2=5$

$\boxed{(x-5)^2+(y-9)^2=5}$

9. Find the equation of a circle whose diameter has endpoints at (-5,-8) and $\overset{x\ y}{(7, 2)}$.

Midpt: $\left(\frac{-5+7}{2}, \frac{-8+2}{2}\right)$

Center $\overset{h\ \ k}{(1, -3)}$

$(x-h)^2+(y-k)^2=r^2$

$(7-1)^2+(2+3)^2=r^2$

$6^2+5^2=r^2$

$36+25=r^2$

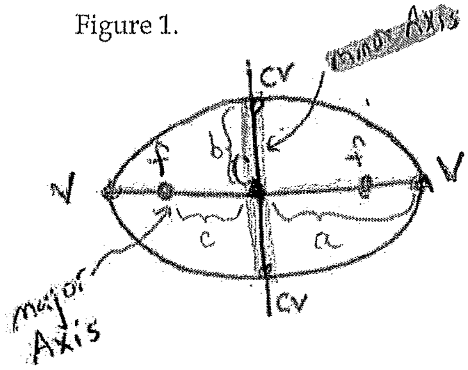
$r^2=61$

$\boxed{(x-1)^2+(y+3)^2=61}$

8.08 Ellipses – Day 1 Date: ______________

Ellipse: A conic section where the sum of the distance from 2 fixed points (foci) is a constant.

Figure 1.



Center:

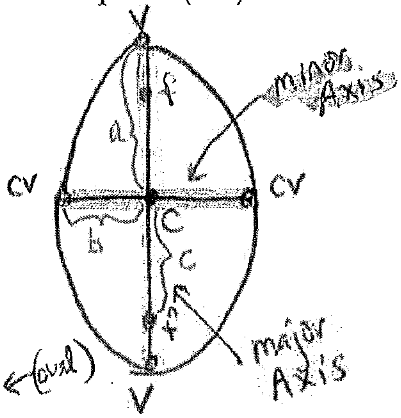
Vertices:

Co-vertices:

Foci: plural for focus fok-sie

Eccentricity:

(circle) → $0 < \frac{c}{a} < 1$ ← (oval)



Horizontal Major Axis	Vertical Major Axis
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ *where* $a > b$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ *where* $a > b$
Center: (h, k) Vertices: (h ± a, k) Co-vertices: (h, k ± b) Foci: (h ± c, k) Eccentricity = $\frac{c}{a}$ $a^2 - b^2 = c^2$	Center: (h, k) Vertices: (h, k ± a) Co-vertices: (h ± b, k) Foci: (h, k ± c) Eccentricity = $\frac{c}{a}$ $a^2 - b^2 = c^2$

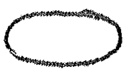
Examples: Graph the ellipse. State the center, vertices, co-vertices, foci, and eccentricity.

$a^2=25$
$a=5$
$b^2=9$
$b=3$

* horizontal major Axis

1. $\frac{(x-3)^2}{4}+\frac{(y-1)^2}{1}=1$ $a^2=4$ $a=2$ $b^2=1$ $b=1$

2. $\frac{x^2}{9}+\frac{(y+2)^2}{25}=1$



Center: (3, 1) Vertices: (1,1) (5,1)

Co-Vertices: (3,2) (3,0)

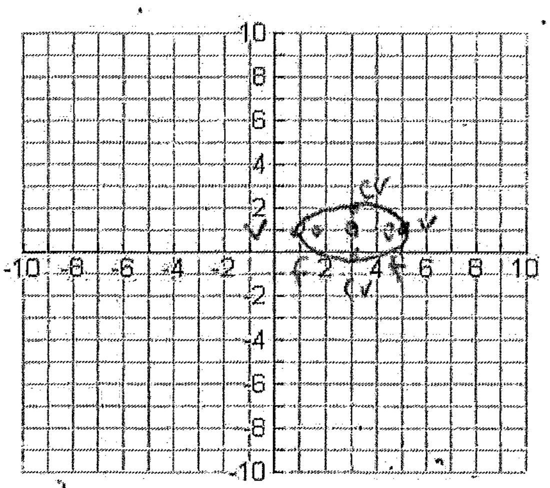
Foci: $(3\pm\sqrt{3}, 1)$ Eccentricity= $\frac{c}{a}\boxed{\frac{\sqrt{3}}{2}}$

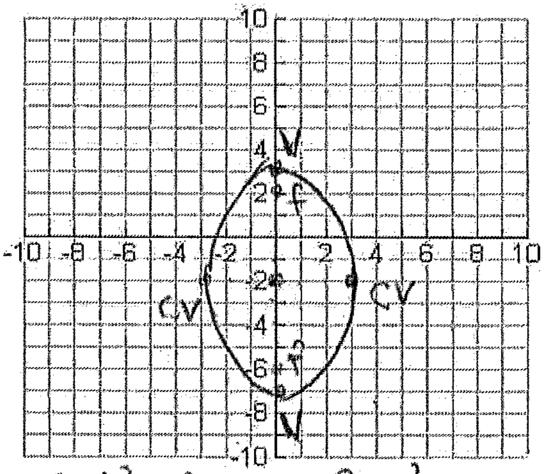
Center: (0,-2) Vertices: (0,3) (0,-7)

Co-Vertices: (-3,-2) (3,-2)

Foci: (0,-6) (0,2) Eccentricity= $\frac{c}{a}\rightarrow\boxed{\frac{4}{5}}$

$a=\underline{2}$
$b=\underline{1}$
$c=\underline{\sqrt{3}}$



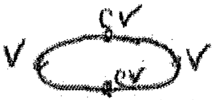


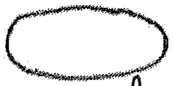
$a=\underline{5}$
$b=\underline{3}$
$c=\underline{4}$

$a^2-b^2=c^2$ $4-1=c^2 \rightarrow c=\sqrt{3}$

$a^2-b^2=c^2$ $25-9=c^2$ $c=4$

$16=c^2$





horizontal major Axis

8.08 Practice: Graph the ellipse. State the center, vertices, co-vertices, foci and eccentricity.

horizontal major Axis

1. $\frac{(x-3)^2}{81}+\frac{(y+5)^2}{25}=1$

$a^2 \nearrow$ 81 $\nwarrow b^2$ 25

$c^2=a^2-b^2$

$c^2=81-25$

$c=\sqrt{56}=2\sqrt{14}\approx7.5$

a = 9 b = 5 $c=2\sqrt{14}$

Center: (3, -5) Vertices: (-6, -5) (12, -5)

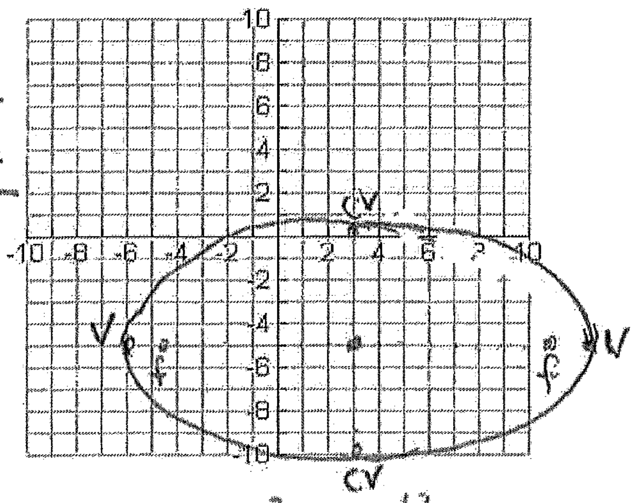
Co-Vertices: (3, -10)(3, 0)

Foci: $(3\pm2\sqrt{14}, -5)$ Eccentricity= $\frac{2\sqrt{14}}{9}$ $\boxed{\frac{c}{a}}$

a = 9

b = 5

$c=2\sqrt{14}$



2. $\frac{(x+2)^2}{64}+\frac{(y-6)^2}{1}=1$

$a^2 \nearrow$ 64 $\nwarrow b^2$ 1

$c^2=a^2-b^2$

$c^2=64-1=63$

$c=\sqrt{63}=3\sqrt{7}\approx7.9$

a = 8 b = 1

Center: (-2, 6) Vertices: (-10, 6), (6, 6)

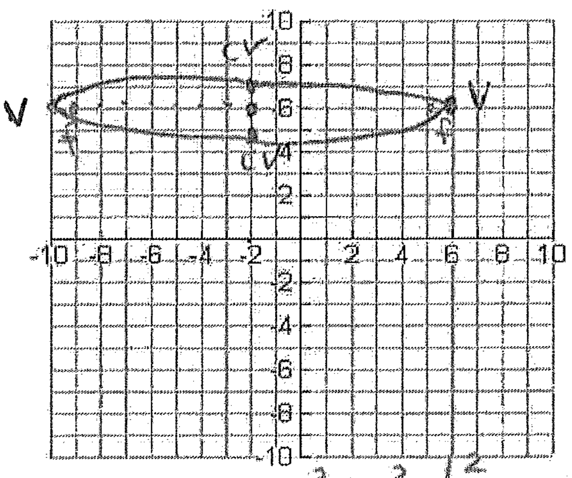
Co-Vertices: (-2, 5) (-2, 7)

Foci: $(-2\pm3\sqrt{7}, 6)$ Eccentricity= $\frac{c}{a}\rightarrow\boxed{\frac{3\sqrt{7}}{8}}$

a = 8

b = 1

$c=3\sqrt{7}$



vertical major Axis

3. $\frac{x^2}{9}+\frac{y^2}{64}=1$

$b^2 \nearrow$ 9 $\nwarrow a^2$ 64

$c^2=a^2-b^2$

$c^2=64-9=55$

$c=\sqrt{55}\approx7.4$

a = 8 b = 3

Center: (0, 0) Vertices: (0, -8) (0, 8)

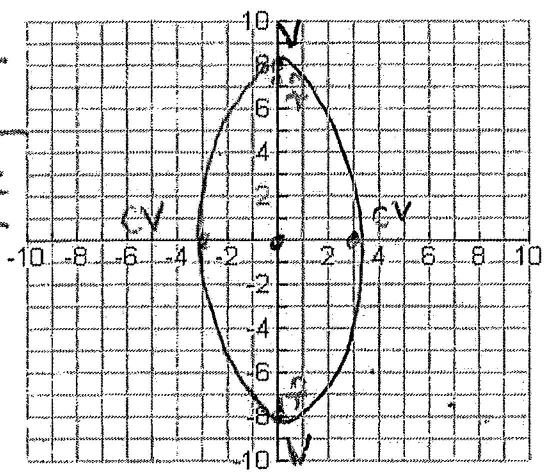
Co-Vertices: (3, 0) (-3, 0)

Foci: $(0, 0\pm\sqrt{55})$ Eccentricity= $\frac{c}{a}$ $\boxed{\frac{\sqrt{55}}{8}}$

a = 8

b = 3

$c=\sqrt{55}$



4. $\frac{(x+2)^2}{16}+\frac{(y+3)^2}{36}=1$

$b^2 \nearrow$ 16 $\nwarrow a^2$ 36

$c^2=a^2-b^2$

$c^2=36-16=20$

$c=\sqrt{20}\approx4.5$

$c=2\sqrt{5}\nearrow$

a = 6 b = 4

Center: (-2, -3) Vertices: (-2, -9), (-2, 3)

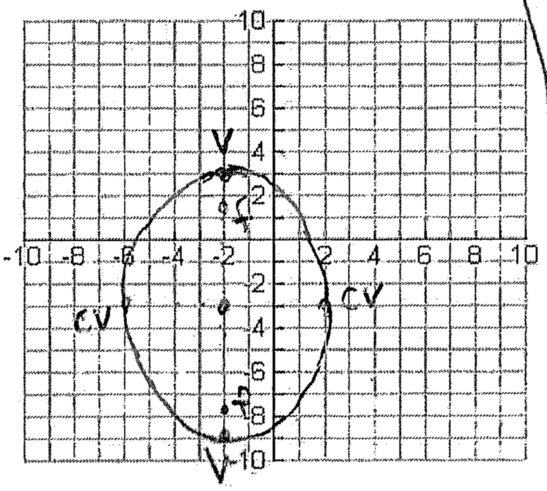
Co-Vertices: (-6, -3) (2, -3)

Foci: $(-2, -3\pm2\sqrt{5})$ Eccentricity= $\frac{c}{a}\rightarrow\frac{2\sqrt{5}}{6}$ or $\frac{\sqrt{5}}{3}$

a = 6

b = 4

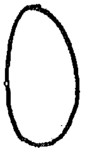
$c=2\sqrt{5}$



$C(-6,4)$

$a=5$

$b=3$ $c=4$

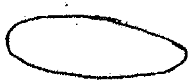


18 $C=(1,0)$

$a=9$

$b=8$

$c=\sqrt{17}$



Find focus

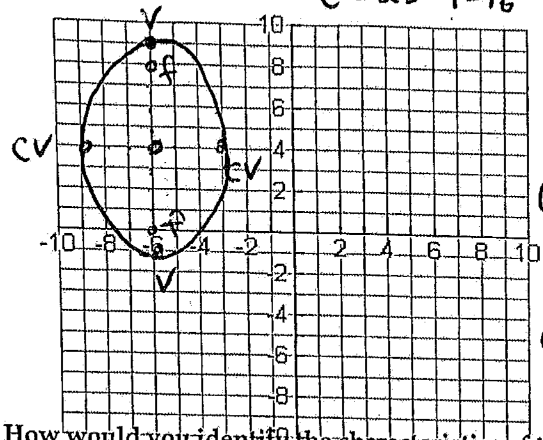
8.09 Ellipses – Day 2 Date: ____________

Review: For each ellipse, identify the coordinates of the center, the vertices, and co-vertices. Then graph.

1. $\frac{(x+6)^2}{9}+\frac{(y-4)^2}{25}=1$ $c^2=a^2-b^2$

$c^2=25-9=16$

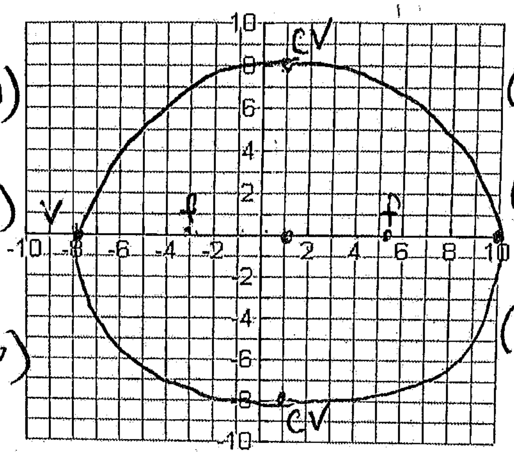
2. $\frac{(x-1)^2}{81}+\frac{y^2}{64}=1$ $c^2=81-64=17$



V: $(-6,9)(-6,-1)$

CV: $(-9,4)(-3,4)$

f: $(-6,8),(-6,0)$



V: $(10,0)(-8,0)$

CV: $(1,8)(1,-8)$

F: $(1\pm\sqrt{17},0)$

How would you identify the characteristics of the ellipse if the equation is not in standard form?

Write the standard form of the equation of each ellipse and then graph the equation. List the coordinates of the center, foci, and the major and minor axis vertices. State the eccentricity of the ellipse.

3. $3x^2+y^2+18x-2y+4=0$ $a=\sqrt{24}$ $b=\sqrt{8}$

$\frac{(x+3)^2}{8}+\frac{(y-1)^2}{24}=1$ | $c^2=24-8=16$ $c=4$

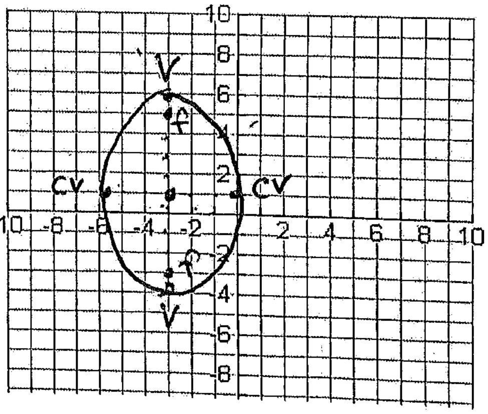
a = $\sqrt{24}$
b = $\sqrt{8}$
c = 4

Standard Form: ______________________

Center: $(-3,1)$ Vertices: $(-3,1\pm\sqrt{24})$

Co-Vertices: $(-3\pm\sqrt{8},1)$

Foci: $(-3,5)$ $(-3,-3)$ Eccentricity = $\frac{c}{a}\rightarrow\boxed{\frac{4}{\sqrt{24}}}$



8.09 Practice

1. $4x^2+56x+9y^2+108y=-484$

$\frac{(x+7)^2}{9}+\frac{(y+6)^2}{4}=1$ | $c^2=9-4=5$ $c=\sqrt{5}$

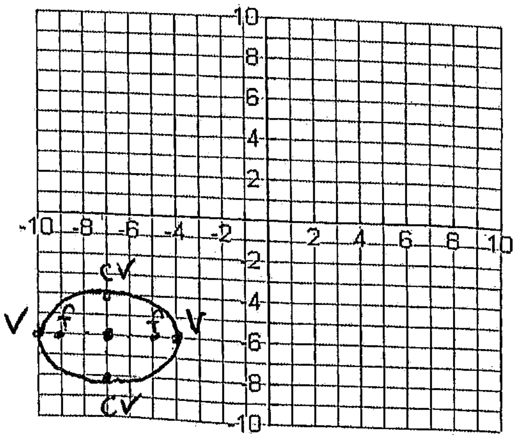
a = 3
b = 2
c = $\sqrt{5}$

Standard Form: ______________________

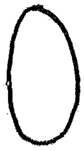
Center: $(-7,-6)$ Vertices: $(-4,-6),(-10,-6)$

Co-Vertices: $(-7,-4)(-7,-8)$

foci: $(-7\pm\sqrt{5},-6)$ eccentricity : $\frac{c}{a}=\boxed{\frac{\sqrt{5}}{3}}$



C(-6,4)



a=5

b=3 c=4

C=(1,0)

18 a=9

b=8

c=√17

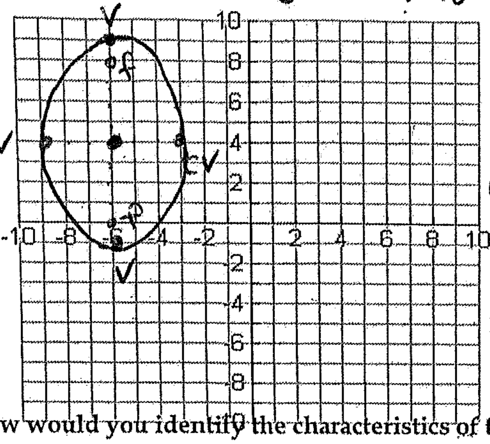
8.09 Ellipses – Day 2

Date: ____________ Find focus

Review: For each ellipse, identify the coordinates of the center, the vertices, and co-vertices. Then graph.

1. $\frac{(x+6)^2}{9}+\frac{(y-4)^2}{25}=1$ $c^2=a^2-b^2$

$c^2=25-9=16$

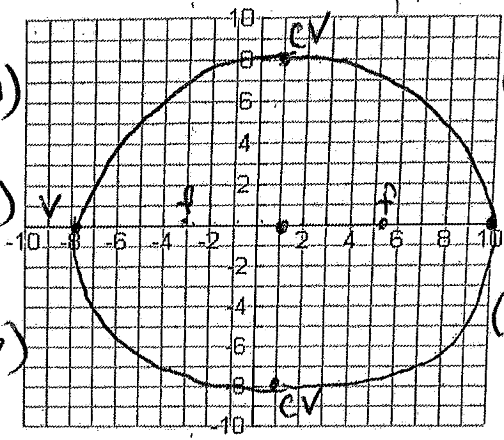


V: (-6,9)(-6,-1)

CV: (-9,4)(-3,4)

f: (-6,8),(-6,0)

2. $\frac{(x-1)^2}{81}+\frac{y^2}{64}=1$ $c^2=81-64=17$



V: (10,0)(-8,0)

CV: (1,8)(1,-8)

F: $(1\pm\sqrt{17},0)$

How would you identify the characteristics of the ellipse if the equation is not in standard form?

Write the standard form of the equation of each ellipse and then graph the equation. List the coordinates of the center, foci, and the major and minor axis vertices. State the eccentricity of the ellipse.

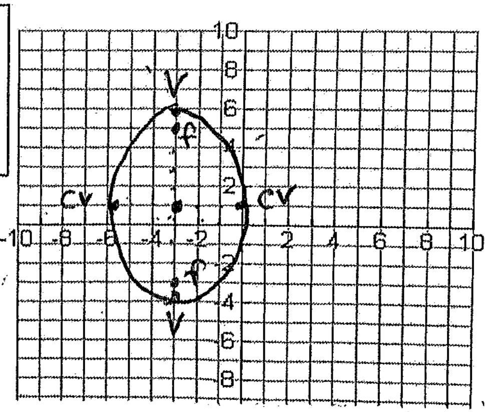
3. $3x^2+y^2+18x-2y+4=0$ $a=\sqrt{24}$ $b=\sqrt{8}$

$\frac{(x+3)^2}{8}+\frac{(y-1)^2}{24}=1$ | $c^2=24-8=16$ $c=4$

a = $\sqrt{24}$

b = $\sqrt{8}$

c = 4



Standard Form: ____________

Center: (-3,1) Vertices: $(-3,1\pm\sqrt{24})$

Co-Vertices: $(-3\pm\sqrt{8},1)$

Foci: (-3,5) (-3,-3) Eccentricity = $\frac{c}{a}\rightarrow\boxed{\frac{4}{\sqrt{24}}}$

8.09 Practice

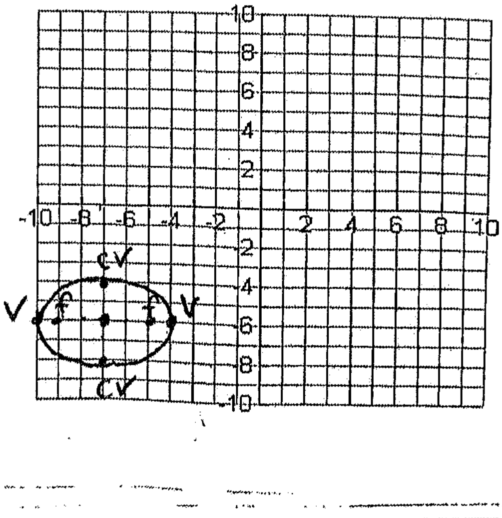
1. $4x^2+56x+9y^2+108y=-484$

$\frac{(x+7)^2}{9}+\frac{(y+6)^2}{4}=1$ | $c^2=9-4=5$ $c=\sqrt{5}$

a = 3

b = 2

c = $\sqrt{5}$



Standard Form: ____________

Center: (-7,-6) Vertices: (-4,-6),(-10,-6)

Co-Vertices: (-7,-4) (-7,-8)

foci: $(-7\pm\sqrt{5},-6)$ eccentricity : $\frac{c}{a}=\boxed{\frac{\sqrt{5}}{3}}$

Pg. 18

3) $3x^2+y^2+18x-2y+4=0$

$$3x^2+18x \quad +y^2-2y \quad =-4$$

$$3(x^2+6x+\underset{\uparrow}{\underline{9}})+y^2-2y+\underline{1}=-4+\underline{27}+\underline{1}$$

$$\left(\frac{b}{2}\right)^2=\left(\frac{6}{2}\right)^2= \qquad \left(\frac{2}{2}\right)^2=1$$

$$3(x+3)(x+3)+(y-1)(y-1)=24$$

$$\frac{3(x+3)^2}{24}+\frac{(y-1)^2}{24}=\frac{24}{24}$$

$$\boxed{\frac{(x+3)^2}{8}+\frac{(y-1)^2}{24}=1}$$

1) $4x^2+56x+9y^2+108y=-484$ 8:09 $\boxed{pg.18}$

$4x^2+56x \quad +9y^2+108y \quad =-484$

$4(x^2+14x+\underline{49})+9(y^2+12y+\underline{36})=-484+\underline{196}+\underline{324}$

$\left(\frac{b}{2}\right)^2 \to \left(\frac{14}{2}\right)^2=49$ $\left(\frac{12}{2}\right)^2=6^2=36$

$\frac{4(x+7)^2}{36}+\frac{9(y+6)^2}{36}=\frac{36}{36}$

$\boxed{\frac{(x+7)^2}{9}+\frac{(y+6)^2}{4}=1}$

0

Foci: ______________________ Eccentricity = __________

2. $49x^2 + 16y^2 + 160y - 384 = 0$

$$\frac{x^2}{16} + \frac{(y+5)^2}{49} = 1$$

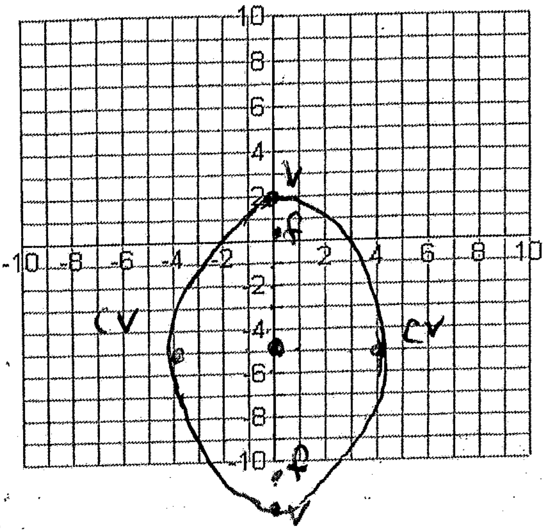
$c^2 = 49 - 16$

$c^2 = 33$

$a =$ 7

$b =$ 4

$c =$ $\sqrt{33}$



Standard Form: ______________________

Center: (0, -5) Vertices: (0, -12)(0, 2)

Co-Vertices: (4, -5)(-4, -5)

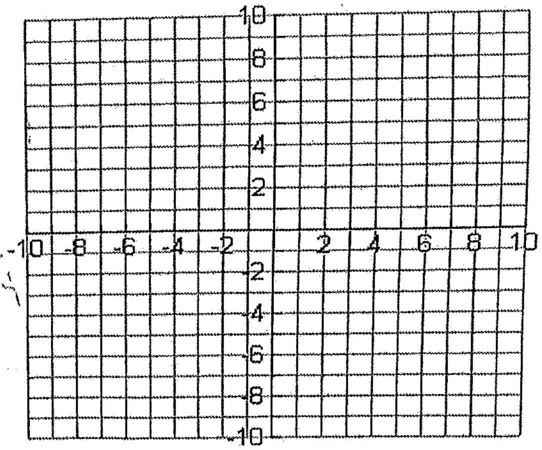
Foci: $(0, -5 \pm \sqrt{33})$ Eccentricity = $\frac{c}{a} \to \frac{\sqrt{33}}{7}$

3. $2x^2 + 18y^2 + 8x + 108y + 99 = 1$

$a =$ ________

b = ________

c = ________



Standard Form: ______________________

Center: __________ Vertices: ______________________

Co-Vertices: ______________________

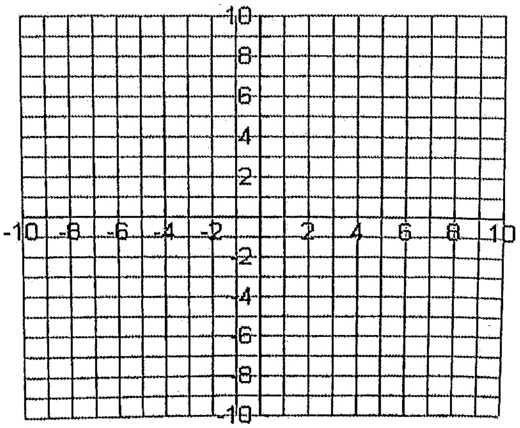
Foci: ______________________ Eccentricity = __________

4. $18y^2 + 12x^2 - 144y - 48x = -120$

$a =$ ________

b = ________

c = ________



Standard Form: ______________________

Center: __________ Vertices: ______________________

Co-Vertices: ______________________

Foci: ______________________ Eccentricity = __________

$\boxed{pg.19}$

2) $49x^2+16y^2+160y-384=0 \quad \left(\frac{10}{2}\right)^2=25$

$$49x^2+16(y^2+10y+\underline{25}) = 384+\underline{400}$$

$$\frac{49x^2}{784}+\frac{16(y+5)^2}{784}=\frac{784}{784} \quad \Bigg| \quad \frac{x^2}{16}+\frac{(y+5)^2}{49}=1$$

8.10 Ellipses – Day 3

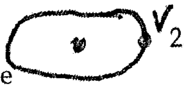
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Date: ____________

Write the equation of the ellipse in standard form that meets each set of conditions. Calculate *a*, b, and c. Graph, then list the coordinates of the center, foci, vertices, and co-vertices.

1. The center is at (–3, –1), the length of the horizontal semi-major axis is 7 units, and the length of the semi-minor axis is 5 units.

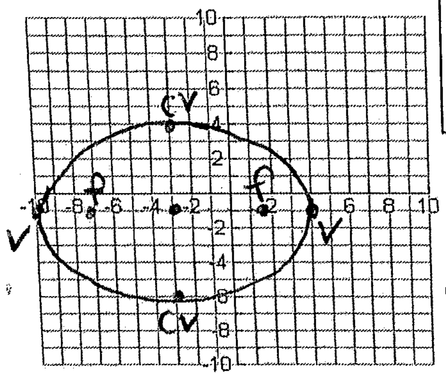
h k



2. The foci are at (~~–2, 0~~) and (~~2, 0~~) and *a* = 7.

(0, 2) (–4, 2)

Center (–2, 2)



a = 7

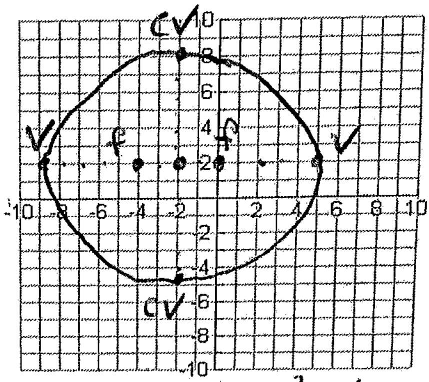
b = 5

c = √24

$c^2 = 49-25$

$c^2 = 24$

$c = \sqrt{24}$



a = 7

b = √45

c = 2

$c^2 = a^2 - b^2$

$4 = 7^2 - b^2$

$4 = 49 - b^2$

$b^2 = 45 \quad b = \sqrt{45}$

Standard Form: $\frac{(x+3)^2}{49}+\frac{(y+1)^2}{25}=1$

Vertices: (–10, –1)(4, –1) Co–Vertices: (–3, 4)(–3, –6)

Foci: $(-3\pm\sqrt{24}, -1)$ Eccentricity: $\frac{\sqrt{24}}{7}$

Standard Form: $\frac{(x+2)^2}{49}+\frac{(y-2)^2}{45}=1$

Vertices: (–9, 2)(5, 2) Co–Vertices: $(-2, 2\pm\sqrt{45})$

Center: (–2, 2) Eccentricity: $\frac{2}{7}$

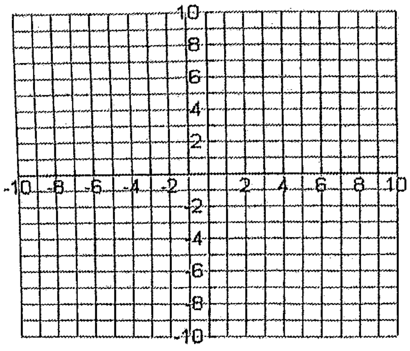
8.09 Practice

1. The length of the semi-major axis is twice the length of the horizontal semi-minor axis, the center is at the origin, and b = 3.

a = ________

b = ________

c = ________



Standard Form: ______________________________

Vertices: ____________ Co–Vertices: ____________

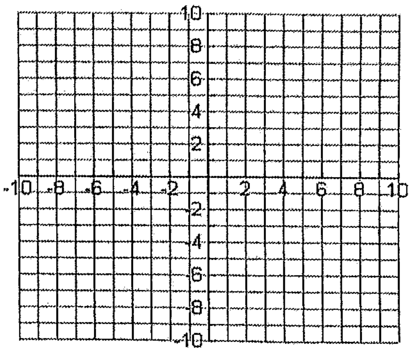
Foci: ____________ Eccentricity: ____________

2. The semi-major axis has a length of 6 units and the foci are at (–1, 1) and (–1, –5).

a = ________

b = ________

c = ________



Standard Form: ______________________________

Vertices: ____________ Co–Vertices: ____________

Center: ____________ Eccentricity: ____________

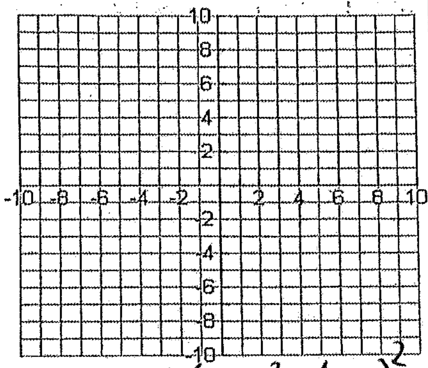
8.10 Ellipses – Day 3

Soh-sie Date: ____________

Write the equation of the ellipse in standard form that meets each set of conditions. Calculate *a*, b, and c. Graph, then list the coordinates of the center, foci, vertices, and co-vertices.

1. The center is at ($\overset{h}{-3}$, $\overset{k}{-1}$), the length of the horizontal semi-major axis is 7 units, and the length of the semi-minor axis is 5 units.

a = 7
b = 5
c = $2\sqrt{6}$



Standard Form: $\frac{(x+3)^2}{49}+\frac{(y+1)^2}{25}=1$

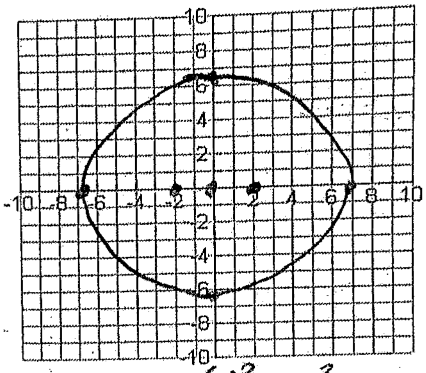
Vertices: ____________ Co-Vertices: ____________

Foci: $-3\pm2\sqrt{6}, -1$ Eccentricity: $\frac{2\sqrt{6}}{7}$

2. The foci are at (–2, 0) and (2, 0) and a = 7.

$C=2$

a = 7
b = $\sqrt{45}$
c = 2



$c^2=a^2-b^2$

$4=49-b^2$

$b^2=45$

$b=\sqrt{45}\rightarrow3\sqrt{5}$

Standard Form: $\frac{(x)^2}{49}+\frac{y^2}{45}=1$

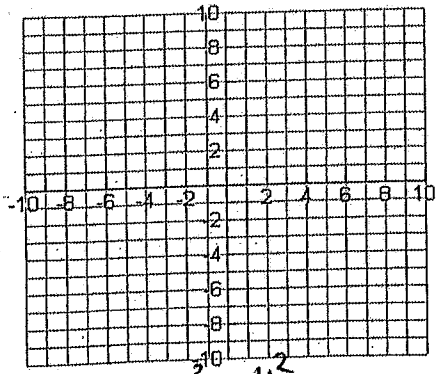
Vertices: (-7, 0) (7, 0) Co-Vertices: $(0, 0\pm\sqrt{45})$

Center: (0, 0) Eccentricity: $\frac{2}{7}$

8.09 Practice

1. The length of the semi-major axis is twice the length of the horizontal semi-minor axis, the center is at the origin, and b = 3.

a = 6 $\updownarrow$
b = 3 $\leftrightarrow$
c = $3\sqrt{3}$



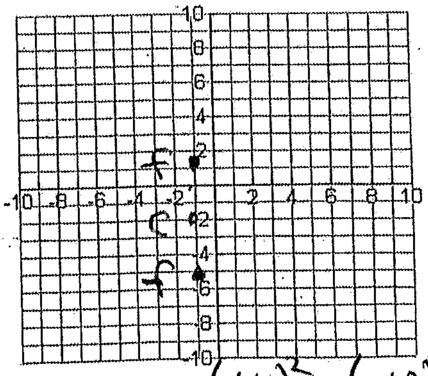
Standard Form: $\frac{x^2}{9}+\frac{y^2}{36}=1$

Vertices: ____________ Co-Vertices: ____________

Foci: $(0, 0\pm3\sqrt{3})$ Eccentricity: $\frac{\sqrt{3}}{2}$

2. The semi-major axis has a length of 6 units and the foci are at (–1, 1) and (–1, –5).

a = 6
b = $3\sqrt{3}$
c = 3



Standard Form: $\frac{(x+1)^2}{27}+\frac{(y+2)^2}{36}=1$

Vertices: (-1, 4) (-1, -8) Co-Vertices: $-1\pm3\sqrt{3}, -2$

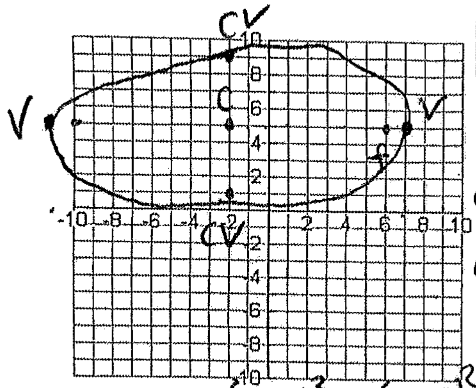
Center: ____________ Eccentricity: ____________

3. The endpoints of the major axis are (-11, 5) and (7, 5). The endpoints of the minor axis are at (-2, 9) and (-2, 1).

$a =$ 9 ↔

$b =$ 4 ↕

$c =$ $\sqrt{65}$ ↔



$c^2=a^2-b^2$

$c^2=81-16$

$c^2=65$

Standard Form: $\frac{(x+2)^2}{81}+\frac{(y-5)^2}{16}=1$

Center: $(-2,5)$ Foci: $(-2\pm\sqrt{65},5)$

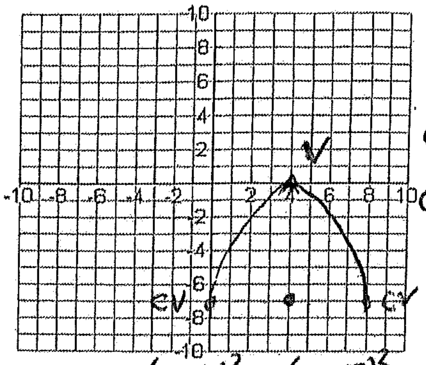
Eccentricity: $\frac{\sqrt{65}}{9}$

4. The ellipse is tangent to the x-axis and the y-axis and the center is (4, -7).

$a =$ 7 ↕

$b =$ 4 ↔

$c =$ $\sqrt{33}$



$c^2=a^2-b^2$

$c^2=49-16$

$c^2=33$

Standard Form: $\frac{(x-4)^2}{16}+\frac{(y+7)^2}{49}=1$

Vertices: $(4,0)\ (4,-14)$ Co-Vertices: $(0,-7)\ (8,-7)$

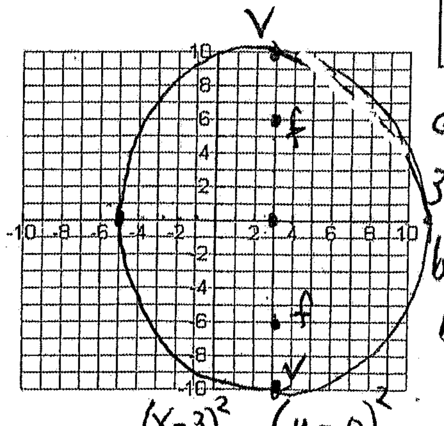
Foci: $(4,-7\pm\sqrt{33})$ Eccentricity: $\frac{\sqrt{33}}{7}$

5. The vertical major axis is 20 units, the center is at (3, 0), and the eccentricity equals $\frac{3}{5}$. $\frac{c}{a}$

$a =$ 10 ↕

$b =$ 8 ↔

$c =$ 6 ↕



$c^2=a^2-b^2$

$36=100-b^2$

$b^2=64$

$b=8$

Standard Form: $\frac{(x-3)^2}{64}+\frac{(y-0)^2}{100}=1$

Vertices: $(3,10)\ (3,-10)$ Co-Vertices: $(-5,0)\ (11,0)$

Foci: $(3,6),(3,-6)$

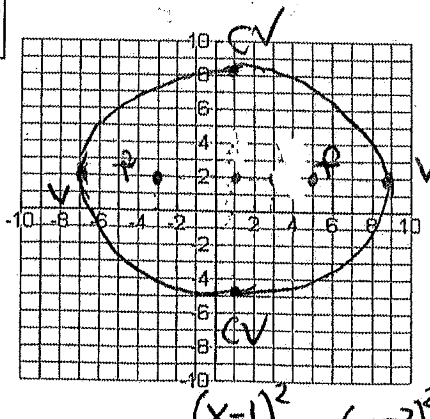
6. The foci are at (-3, 2) and (5, 2) and the eccentricity equals 0.5

$\frac{c}{a}\rightarrow\frac{1}{2}\rightarrow\frac{4}{\boxed{8}}$

$a =$ 8

$b =$ $4\sqrt{3}$

$c =$ 4



$c^2=a^2-b^2$

$16=64-b^2$

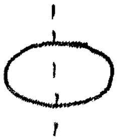
$48=b^2$

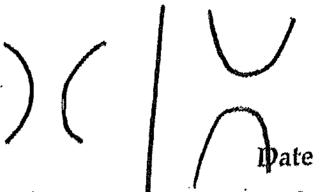
$b=\sqrt{48}=4\sqrt{3}$

Standard Form: $\frac{(x-1)^2}{64}+\frac{(y-2)^2}{48}=1$

Vertices: $(9,2)\ (-7,2)$ Co-Vertices: $(2,8)\ (2,-5)$

Center: $(1,2)$



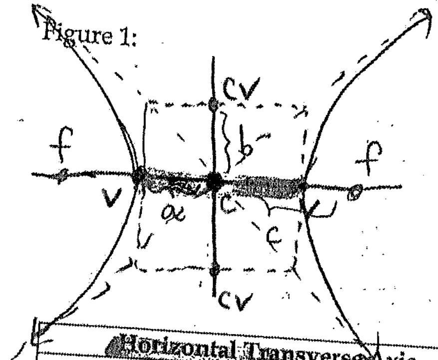


8.11 Hyperbolas – Day 1

Date: ______________

Hyperbola: a conic section where the difference of the distance from 2 fixed points (foci) is a constant.

Figure 1:

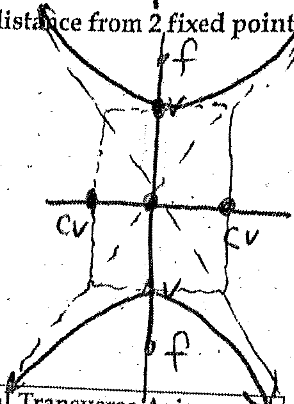


Center:

Vertices:

Foci:

Asymptotes:



Horizontal Transverse Axis	Vertical Transverse Axis
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Center: (h, k) Vertices: $(h \pm a, k)$ Foci: $(h \pm c, k)$ Eccentricity $= \frac{c}{a}$ $a^2 + b^2 = c^2$ Asymptotes: $y - k = \pm\frac{b}{a}(x - h)$	Center: (h, k) Vertices: $(h, k \pm a)$ Foci: $(h, k \pm c)$ Eccentricity $= \frac{c}{a}$ $a^2 + b^2 = c^2$ Asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

Transverse axis:
Axis that connect the vertices:

Conjugate axis:
Axis connecting the covertices

Examples: Graph the Hyperbola. State the center, vertices, foci, eccentricity, and asymptotes.

1. $\frac{(x+1)^2}{16} - \frac{(y-1)^2}{9} = 1$

$c^2 = 16+9$
$c^2 = 25$
$c = 5$

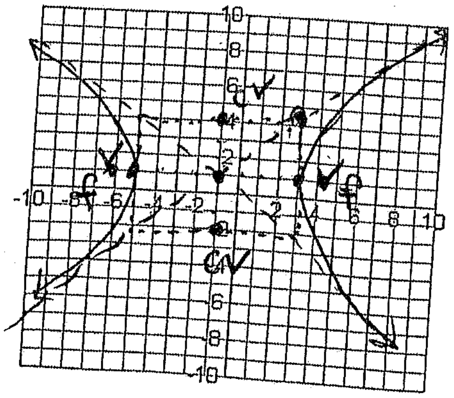
$a=4$
$b=3$

Center: (-1,1) Eccentricity: $\frac{5}{4}$

Vertices: (-5,1) (3,1)

Foci: (-6,1),(4,1)

Asymptotes: $y-1 = \pm\frac{3}{4}(x+1)$



2. $\frac{(y-3)^2}{4} - \frac{(x+5)^2}{25} = 1$

$a=2$
$b=5$

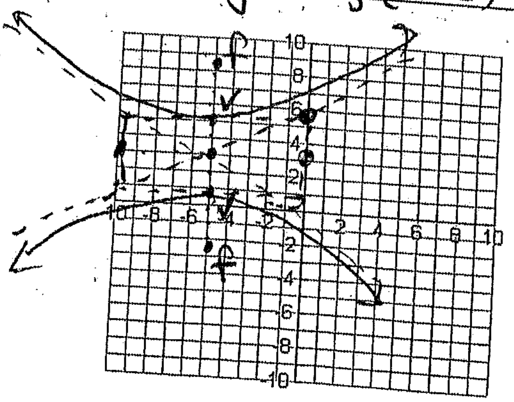
Center: (-5,3) Eccentricity: ______

Vertices: (-5,5)(-5,1)

Foci: $(-5, 3\pm\sqrt{29})$

Asymptotes: $y-3 = \pm\frac{2}{5}(x+5)$

$c^2 = a^2+b^2$
$c^2 = 4+25$
$c^2 = 29$
$c = \sqrt{29}$



8.12 Hyperbolas – Day 2

Date: ____________

Write the standard form of the equation of each hyperbola and list the coordinates of the center, vertices, foci and the equation of the asymptotes. Then graph the hyperbola.

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3. $x^2 - 4y^2 + 6x - 8y - 11 = 0$

$x^2+6x+\_ -4y^2-8y+\_ = 11+\_+\_$

$$\frac{(x+3)^2}{16} - \frac{(y+1)^2}{4} = 1$$

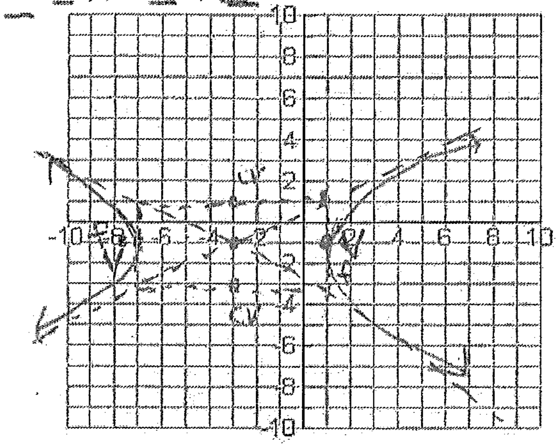
$c^2=a^2+b^2$

$c^2=16+4=20$

$c=\sqrt{20}$

$a=4 \leftrightarrow$

$b=2 \updownarrow$



Standard Form: ____________

Center: $(-3,-1)$

Vertices: $(1,-1)\ (-7,-1)$ Foci: $(-3\pm\sqrt{20}, -1)$

Asymptotes: $(y+1) = \pm\frac{1}{2}(x+3)$

Eccentricity = $\frac{\sqrt{20}}{4}$ or $\frac{\sqrt{5}}{2}$

$\smile$ $\frown$

4. $-4x^2 + 9y^2 - 90y - 24x + 153 = 0$

$$\frac{(y-5)^2}{4} - \frac{(x+3)^2}{9} = 1$$

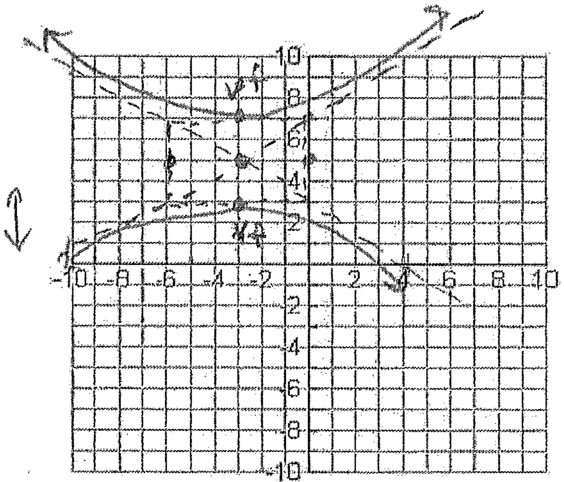
$c^2=a^2+b^2$

$c^2=4+9=13$

$c^2=13,\ c=\sqrt{13} \updownarrow$

$a=2 \updownarrow$

$b=3 \leftrightarrow$



Standard Form: ____________

Center: $(-3,5)$

Vertices: $(-3,7)\ (-3,3)$ Foci: $(-3\ 5\pm\sqrt{13})$

Asymptotes: $y-5=\pm\frac{2}{3}(x+3)$

Eccentricity = $\sqrt{13}/2$

)(

5. $36x^2 - 49y^2 - 72x - 294y - 2169 = 0$

$$\frac{(x-1)^2}{49} - \frac{(y+3)^2}{36} = 1$$

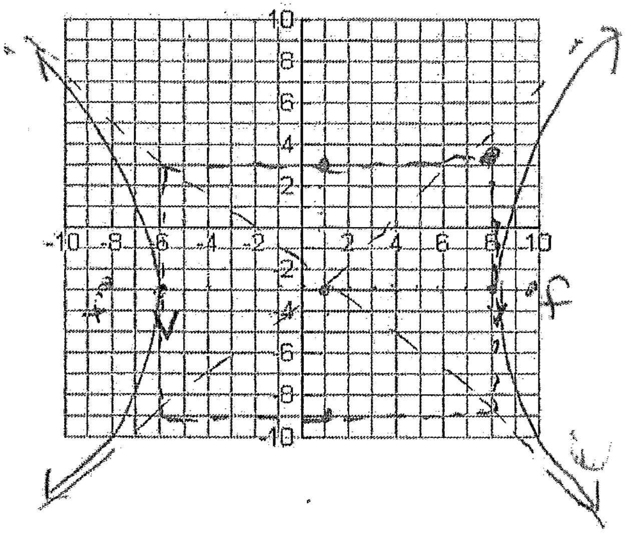
$c^2=a^2+b^2$

$c^2=49+36$

$c=\sqrt{85} \leftrightarrow$

$a=7 \leftrightarrow$

$b=6 \updownarrow$



Standard Form: ____________

Center: $(1,-3)$

Vertices: $(8,-3)\ (-6,-3)$ Foci: $(1\pm\sqrt{85}, -3)$

Asymptotes: $y+3=\frac{\pm 6}{7}(x-1)$

eccentricity: $\frac{\sqrt{85}}{7}$

Hyperbolas Day 2

3) $x^2-4y^2+6x-8y-11=0$

$x^2+6x+\underline{9}\ -4y^2-8y+\underline{\ \ }\ =11+\underline{9}+\underline{\ \ }$

$\left(\frac{b}{2}\right)^2=\left(\frac{6}{2}\right)^2=9$ $\quad x^2+6x+\underline{9}-4(y^2+2y+\underline{1})=11+\underline{9}+\underline{-4}$

$\left(\frac{2}{2}\right)^2=1$

$$\frac{(x+3)^2}{16}-\frac{4(y+1)^2}{16}=\frac{16}{16} \rightarrow \boxed{\frac{(x+3)^2}{16}-\frac{(y+1)^2}{4}=1}$$

---

4) $-4x^2+9y^2-90y-24x+153=0$

$9y^2-90y\quad -4x^2-24x\quad =-153$

$9(y^2-10y+\underline{25})-4(x^2+6x+\underline{9})=-153+\underline{225}+\underline{-36}$

$\left(\frac{10}{2}\right)^2=25$ $\quad \left(\frac{6}{2}\right)^2=9$

$$\frac{9(y-5)^2}{36}-\frac{4(x+3)^2}{36}=\frac{36}{36}$$

$$\boxed{\frac{(y-5)^2}{4}-\frac{(x+3)^2}{9}=1}$$

5) $36x^2 - 49y^2 - 72x - 294y - 2169 = 0$

$$36x^2 - 72x \quad - 49y^2 - 294y = 2169 + \_\_ + \_\_$$

$$36(x^2 - 2x + \underline{1}) - 49(y^2 + 6y + \underline{9}) = 2169 + \underline{36} + \underline{-441}$$

$\left(\frac{2}{2}\right)^2 = 1$

$(x-1)(x-1)$

$\left(\frac{6}{2}\right)^2 = 9$

$$\frac{36(x-1)^2}{1764} - \frac{49(y+3)^2}{1764} = \frac{1764}{1764}$$

$$\boxed{\frac{(x-1)^2}{49} - \frac{(y+3)^2}{36} = 1}$$

$a = 3$

$b =$

$c = 5$

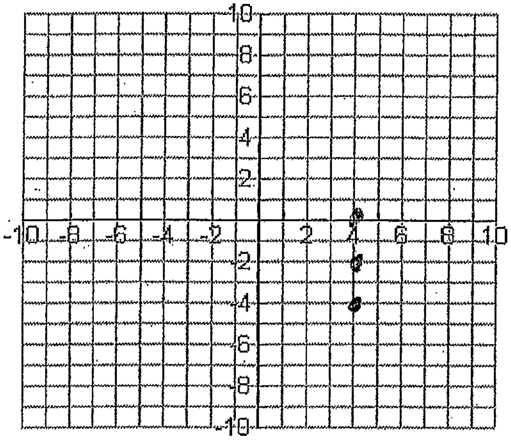
Eccentricity = ____________

Write the equation of the hyperbola in standard form that meets each set of conditions. Use the grid if it helps.

4. The center is at (4, -2), a = 2, b = 3, and the transverse axis is vertical.

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Equation: $\frac{(y+2)^2}{4} - \frac{(x-4)^2}{9} = 1$



center $(-2, 2)$

5. The length of the transverse axis is 6 units and the foci are at (3, 2) and (-7, 2).

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

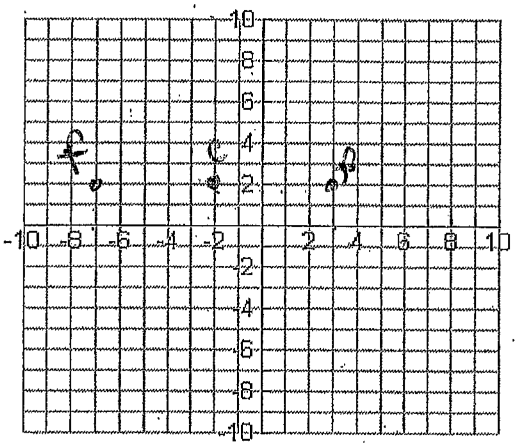
$c^2 = a^2 + b^2$

$25 = 9 + b^2$

$16 = b^2$

$b = 4$

Equation: $\frac{(x+2)^2}{9} - \frac{(y-2)^2}{16} = 1$



$b = 4$

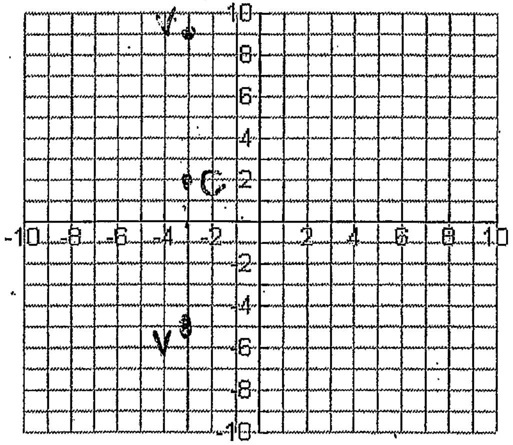
6. The length of the conjugate axis is 8 units and the vertices are (-3, 9) and (-3, -5)

$a = 7$

center $(-3, 2)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Equation: $\frac{(y-2)^2}{49} - \frac{(x+3)^2}{16} = 1$



7. The center is at (0, 1), one focus is at (10, 1) and the eccentricity is $\frac{5}{3}$

$c = 10$ ↔

$a = 6$ ↔

$e = \frac{c}{a} \rightarrow \frac{5}{3} \rightarrow \frac{10}{\boxed{6}}$

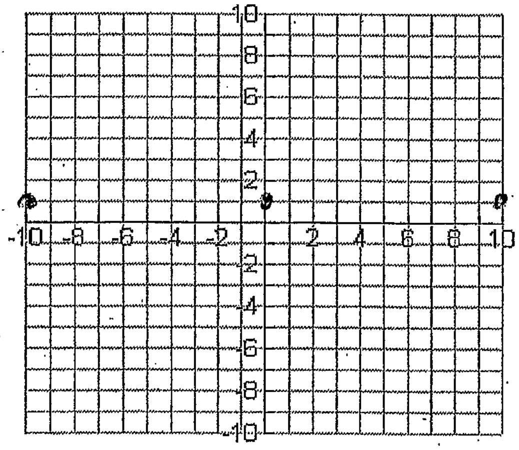
$c^2 = a^2 + b^2$

$100 = 36 + b^2$

$b^2 = 64$

$b = 8$ ↕

Equation: $\frac{(x-0)^2}{36} - \frac{(y-1)^2}{64} = 1$



Identify conics: circle, ellipse, parabola, hyperbola

a) $3x^2-6x+3y^2-9y=16$	a) circle
b) $3x^2-6x+5y=12$	b) parabola
c) $3x^2-6x+4y^2-9y=16$	c) ellipse
d) $4y^2-6y-4x^2+9x=12$	d) hyperbola

8.13 Ellipses and Hyperbolas Review

Date Key

1. Write the equation of the ellipse in standard form. Graph the ellipse and identify requested parts.

$9x^2 + 16y^2 + 72x - 160y + 400 = 0$

$9x^2 + 72x + 16y^2 - 160y = -400$

$9(x^2+8x+\underline{16}) + 16(y^2-10y+\underline{25}) = -400 + \underline{144} + \underline{400}$

$\left(\frac{8}{2}\right)^2 = 16 \qquad \left(\frac{10}{2}\right)^2 = 25$

$\frac{9(x+4)^2}{144} + \frac{16(y-5)^2}{144} = \frac{144}{144}$

$\frac{(x+4)^2}{16} + \frac{(y-5)^2}{9} = 1$

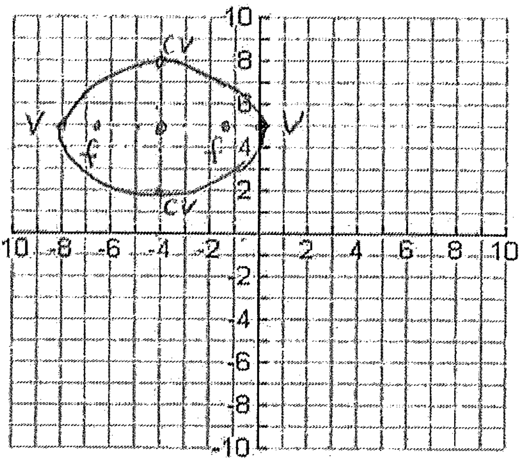
$a = 4 \leftrightarrow$

$b = 3 \updownarrow$

$c^2 = a^2 - b^2$

$c^2 = 16 - 9 = 7$

$c = \sqrt{7} \leftrightarrow$



Center: $(-4, 5)$

Foci: $(-4 \pm \sqrt{7}, 5)$

Eccentricity = $\frac{\sqrt{7}}{4}$

vertices: $(-8, 5), (0, 5)$

co-vertices: $(-4, 2), (-4, 8)$

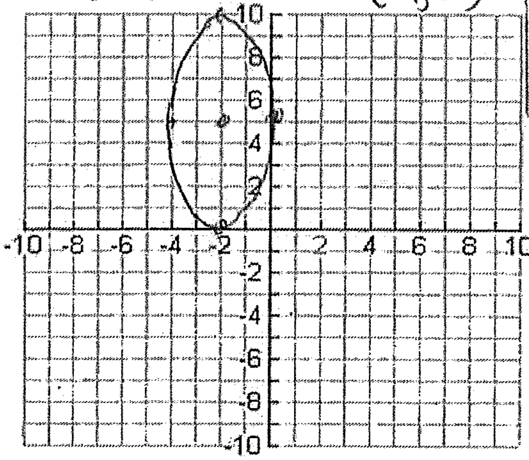
2. Write the equation of the ellipse in standard form which is tangent to the x-axis and the y-axis and the center is (-2, 5) then graph.

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Equation:

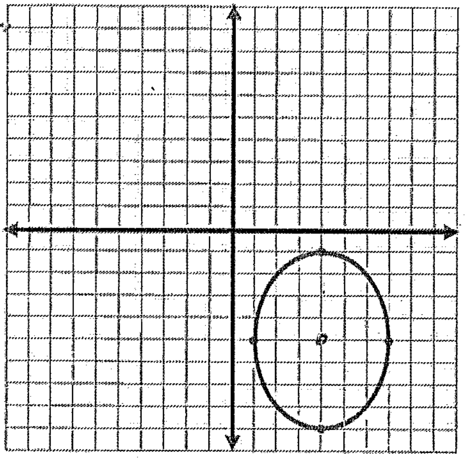
$a = 5 \quad b = 2 \quad$ center $(-2, 5)$

$$\boxed{\frac{(x+2)^2}{4} + \frac{(y-5)^2}{25} = 1}$$



3. Given the graph of the following ellipse, write the standard form of the ellipse equation.

Equation: $\frac{(x-4)^2}{9} + \frac{(y+5)^2}{16} = 1$



$a = 4 \updownarrow$

$b = 3 \leftrightarrow$

center: $(4, -5)$

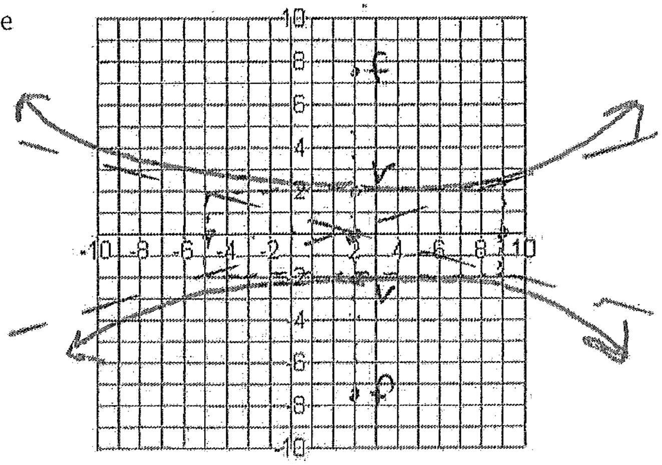
4. Identify the characteristics of the hyperbola. Then, graph the hyperbola and label all parts.

$$\frac{y^2}{4} - \frac{(x-2)^2}{49} = 1$$

$c^2 = a^2 + b^2$

$c^2 = 4 + 49 = 53$

↕ a = 2 b = 7 ↔ c = $\sqrt{53}$ ↕ $c = \sqrt{53}$



Center: (2, 0)

Vertices: (2, 2) (2, -2)

Foci: $(2, 0 \pm \sqrt{53})$

Asymptotes: $y - 0 = \pm\frac{2}{7}(x-2)$

Eccentricity: $\frac{c}{a} \to \frac{\sqrt{53}}{2}$

5. Write the equation of the hyperbola 16x² – 9y² + 32x – 54y – 209 = 0 in standard form. Identify the center, vertices, foci, asymptotes, and eccentricity. Graph the hyperbola and label all parts.

$16x^2 + 32x - 9y^2 - 54y = 209$

$16(x^2 + 2x + \underline{1}) - 9(y^2 + 6y + \underline{9}) = 209 + \underline{16} + \underline{-81}$

$\frac{16(x+1)^2}{144} - \frac{9(y+3)^2}{144} = \frac{144}{144}$

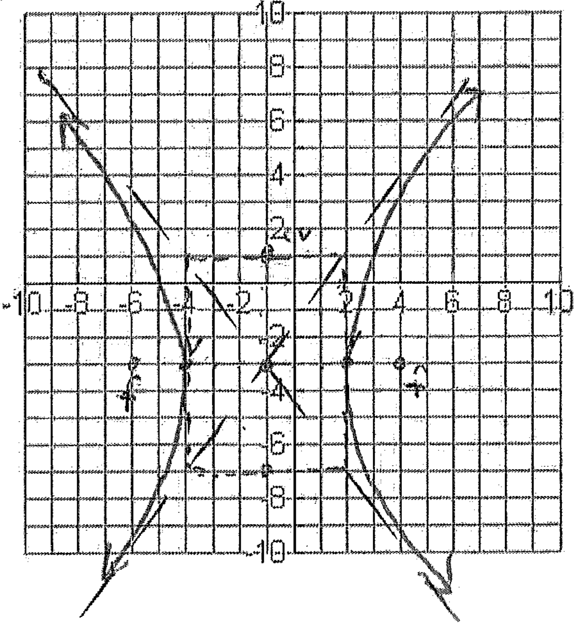
$\frac{(x+1)^2}{9} - \frac{(y+3)^2}{16} = 1$

Standard Form:

↔ a = 3 b = 4 ↕ c = 5 ↔ $c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c = 5$



Center: (-1, -3)

Vertices: (2, -3), (-4, -3)

Foci: (4, -3) (-6, -3)

Asymptotes: $y + 3 = \pm\frac{4}{3}(x+1)$

Eccentricity: $\frac{5}{3}$

connects vertex to vertex

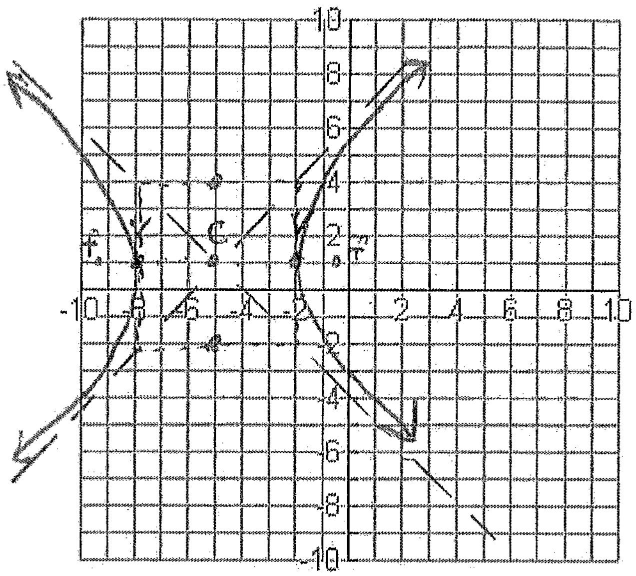
6. Write the standard form of the equation for the hyperbola whose transverse axis has endpoints P(−2, 1) and Q(−8, 1) and whose conjugate axis is a length of 6. Identify its characteristics. Then, graph the hyperbola and label all parts.

$b=3$

$c^2=a^2+b^2$

$c^2=9+9=18$

$\leftrightarrow$ a = 3 b = $3 \updownarrow$ c = $\sqrt{18}$ or $3\sqrt{2}$ $\leftrightarrow$



Standard Form: $\frac{(x+5)^2}{9} - \frac{(y-1)^2}{9} = 1$

Center: $(-5, 1)$

Vertices: $(-2, 1)$ $(-8, 1)$

Foci: $(-5 \pm 3\sqrt{2}, 1)$

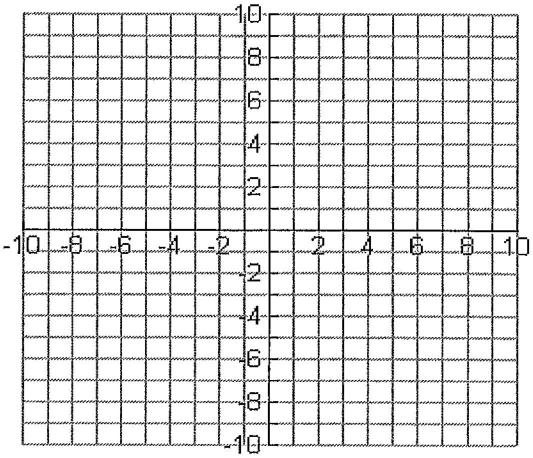
Asymptotes: $y-1 = \pm 1(x+5)$

Eccentricity: $\frac{c}{a} \rightarrow \frac{3\sqrt{2}}{3} = \boxed{\sqrt{2}}$

8.13b Ellipses and Hyperbolas Review WS #2 **Date** ____________

1. Write the equation of the ellipse in standard form. Graph the ellipse and identify requested parts.

$3x^2 + y^2 - 48x - 4y + 184 = 0$



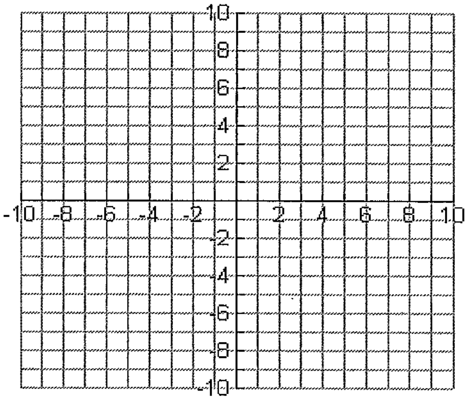
Standard Form: ______________________________

Center: ________ Vertices: ______________________________

Foci: ____________________________ Co-Vertices: ______________________________

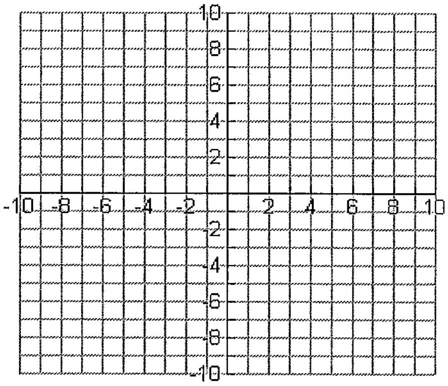
Eccentricity = ___________________

2. Write the equation of an ellipse with center (-2, -1), a horizontal major axis of length 10 and a minor axis of length 5.



Equation: ________________________

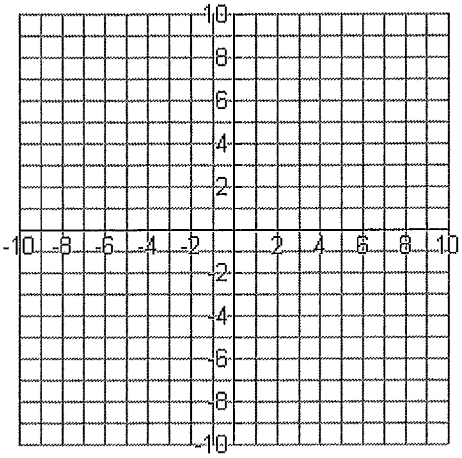
3. Write an equation of the ellipse with a vertex at (0,7) and a co-vertex at (-3, 0)



Equation: ________________________

4. Identify the characteristics of the hyperbola. Then, graph the hyperbola and label all parts.

$$\frac{(y+1)^2}{25} - \frac{(x-3)^2}{36} = 1$$



a = ________ b = ________ c = ________

Center: ________________

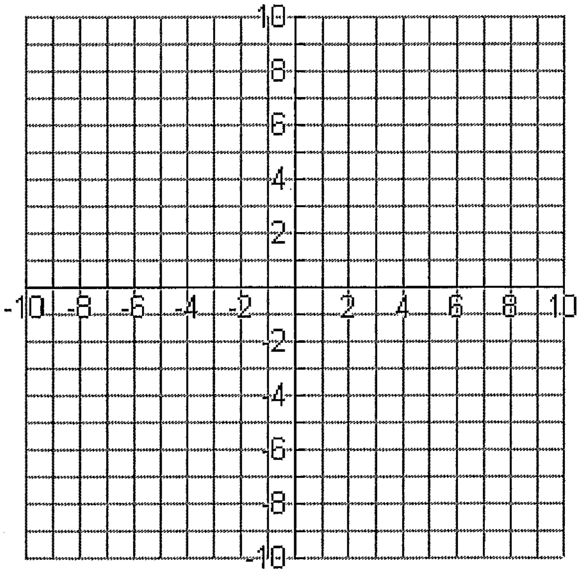
Vertices: ________________________________

Foci: ________________________________

Asymptotes: ________________________________

Eccentricity: ______________

5. Write the equation of the hyperbola in standard form. Identify the center, vertices, foci, asymptotes, and eccentricity. Graph the hyperbola and label all parts. $3x^2 - 4y^2 - 30x - 8y + 59 = 0$



Standard Form: ________________________________

a = ________ b = ________ c = ________

Center: ___________

Vertices: ________________________________

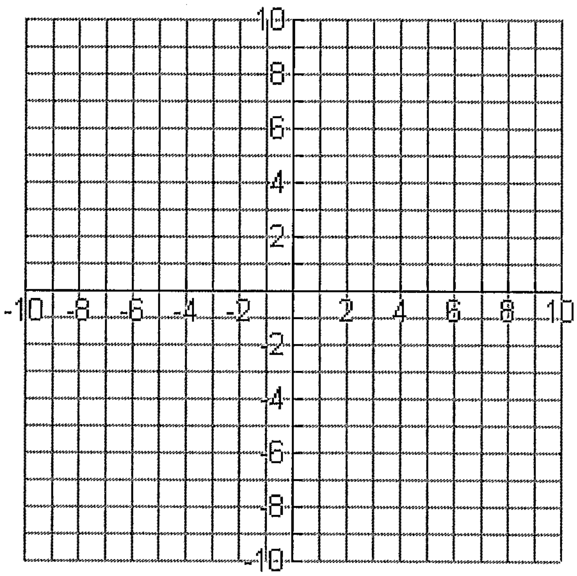
Foci: ________________________________

Asymptotes: ________________________________

Eccentricity: ______________

6. Write an equation of a hyperbola with center at (2, -3), a focus at (8, -3) and one vertex at (6,-3).

a = _______ b = ________ c = ________



Standard Form: ____________________________

Center: _____________

Vertices: ________________________________

Foci: ____________________________________

Asymptotes: ______________________________

Eccentricity: ______________

7. Match the general form equation to the type of conic section it represents.

A) $127-3x^2+3y^2-24x+10y=0$

B) $4-x^2-16x-14y=0$

C) $253-2x^2=2y^2+15x$

D) $1-5y^2-3x^2+12x=16y$

Parabola: _____

Circle: _______

Ellipse: _______

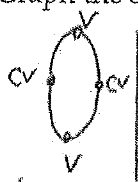
Hyperbola: ____

8.13b Ellipses and Hyperbolas Review WS #2

Date ____________ Key

1. Write the equation of the ellipse in standard form. Graph the ellipse and identify requested parts.

$3x^2+y^2-48x-4y+184=0$



$3x^2-48x+y^2-4y=-184$

$3(x^2-16x+\underline{64})+y^2-4y+\underline{4}=-184+\underline{192}+\underline{4}$

$\left(\frac{16}{2}\right)^2=64$ $\left(\frac{4}{2}\right)^2=4$

$\frac{3(x-8)^2}{12}+\frac{(y-2)^2}{12}=\frac{12}{12}$

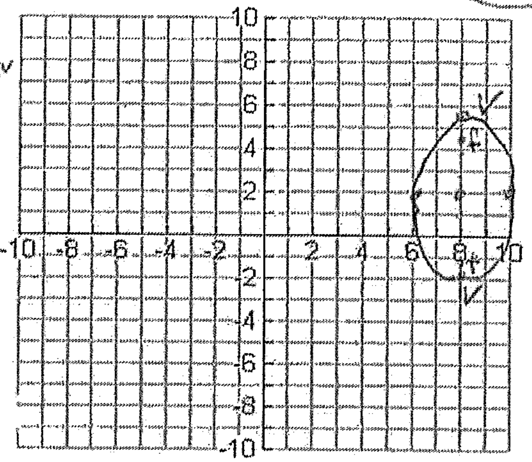
$a=\sqrt{12}=2\sqrt{3}$ ↕

$b=2$ ↔

$c^2=a^2-b^2$

$c^2=12-4=8$

$c=2\sqrt{2}$ ↕



Standard Form: $\frac{(x-8)^2}{4}+\frac{(y-2)^2}{12}=1$

Center: (8, 2) Vertices: $(8, 2\pm2\sqrt{3})$

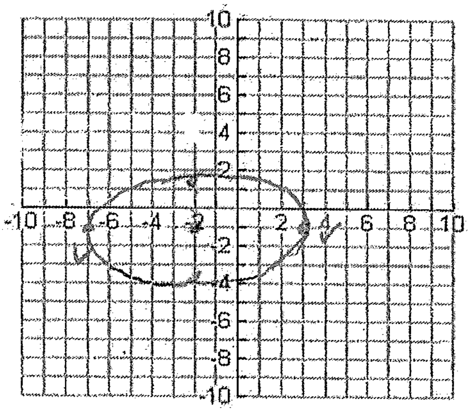
Foci: $(8, 2\pm2\sqrt{2})$ Co-Vertices: (6, 2), (10, 2)

Eccentricity = $\frac{c}{a}\rightarrow\frac{2\sqrt{2}}{2\sqrt{3}}=\frac{\sqrt{2}}{\sqrt{3}}$

2. Write the equation of an ellipse with center (-2, -1), a horizontal major axis of length 10 and a minor axis of length 5.

a=5 b=2.5

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $\frac{(x+2)^2}{5^2}+\frac{(y+1)^2}{2.5^2}=1$



Equation: $\frac{(x+2)^2}{25}+\frac{(y+1)^2}{6.25}=1$

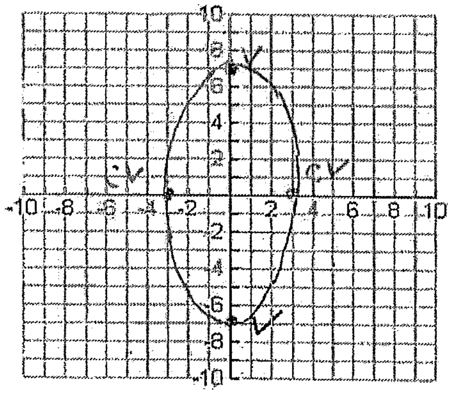
3. Write an equation of the ellipse with a vertex at (0,7) and a co-vertex at (-3, 0)

$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

a=7

b=3

center(0,0)



Equation: $\frac{(x-0)^2}{9}+\frac{(y-0)^2}{49}=1$

4. Identify the characteristics of the hyperbola. Then, graph the hyperbola and label all parts.

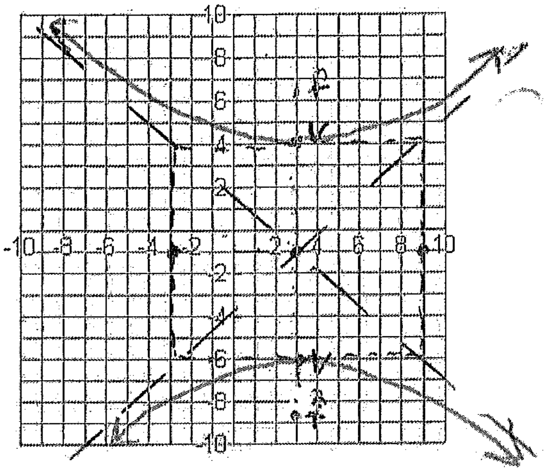
$$\frac{(y+1)^2}{25} - \frac{(x-3)^2}{36} = 1$$

$c^2 = a^2 + b^2$

$c^2 = 25 + 36 = 61$

$c = \sqrt{61}$

a = $\updownarrow 5$ b = $\overset{\longleftrightarrow}{6}$ c = $\updownarrow \sqrt{61}$



Center: (3, -1)

Vertices: (3, 4), (3, -6)

Foci: $(3, -1 \pm \sqrt{61})$

Asymptotes: $y+1 = \pm\frac{5}{6}(x-3)$

Eccentricity: $\frac{\sqrt{61}}{5}$

5. Write the equation of the hyperbola in standard form. Identify the center, vertices, foci, asymptotes, and eccentricity. Graph the hyperbola and label all parts. $3x^2 - 4y^2 - 30x - 8y + 59 = 0$

$3x^2 - 30x - 4y^2 - 8y = -59$

$3(x^2 - 10x + \underline{25}) - 4(y^2 + 2y + \underline{1}) = -59 + \underline{75} + \underline{-4}$

$\left(\frac{10}{2}\right)^2 = 25$ $\left(\frac{2}{2}\right)^2 = 1$

$\frac{3(x-5)^2}{12} - \frac{4(y+1)^2}{12} = \frac{12}{12}$

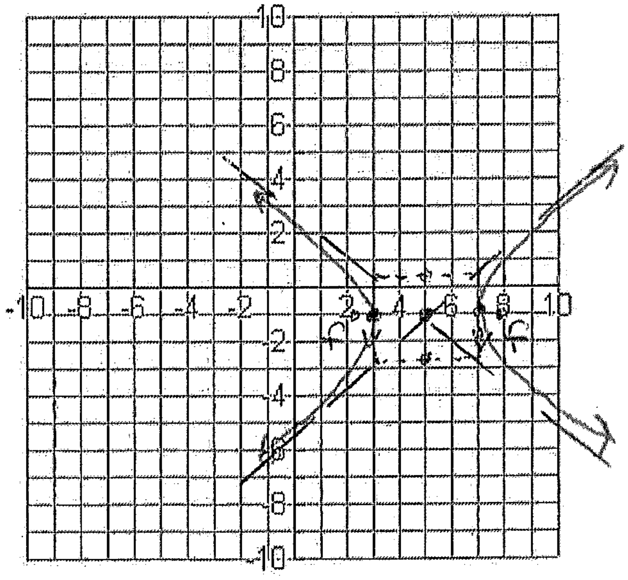
Standard Form: $\frac{(x-5)^2}{4} - \frac{(y+1)^2}{3} = 1$

a = $\underset{\longleftrightarrow}{2}$ b = $\sqrt{3} \updownarrow$ c = $\underset{\longleftrightarrow}{\sqrt{7}}$

$c^2 = a^2 + b^2$

$c^2 = 4 + 3$

$c = \sqrt{7}$



Center: (5, -1)

Vertices: (3, -1), (7, -1)

Foci: $(5 \pm \sqrt{7}, -1)$

Asymptotes: $y+1 = \pm\frac{\sqrt{3}}{2}(x-5)$

Eccentricity: $\frac{\sqrt{7}}{2}$

6. Write an equation of a hyperbola with center at (2, -3), a focus at (8, -3) and one vertex at (6,-3).

$c^2=a^2+b^2$

$6^2=4^2+b^2$

a = 4 ↔ b = $2\sqrt{5}$ ↕ c = 6 ↔

$b^2=20$

$b=\sqrt{20}$

$=2\sqrt{5}$

Standard Form: $\frac{(x-2)^2}{16}-\frac{(y+3)^2}{20}=1$

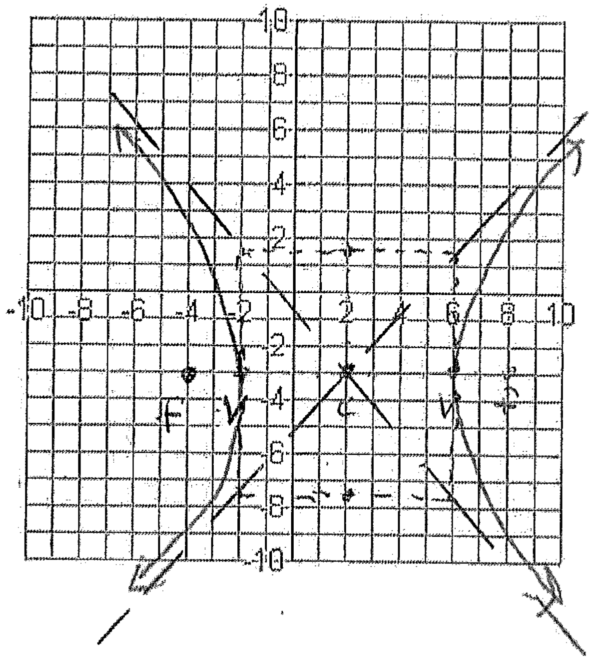
Center: (2,-3)

Vertices: (-2,-3), (6,-3)

Foci: (-4,-3), (8,-3)

Asymptotes: $y+3=\pm\frac{2\sqrt{5}}{4}(x-2)$

Eccentricity: $\frac{6}{4}$ or $\frac{3}{2}$



7. Match the general form equation to the type of conic section it represents.

A) $127-3x^2+3y^2-24x+10y=0$ opposite signs → hyperbola

B) $4-x^2-16x-14y=0$ only 1 variable is squared → parabola

C) $253-2x^2=2y^2+15x$ $253-2x^2-2y^2=15x$ same coefficient, same signs Circle

D) $1-5y^2-3x^2+12x=16y$ same signs, different coefficients → ellipse

Parabola: B

Circle: C

Ellipse: D

Hyperbola: A