4

median

Accelerated Pre-Calculus

Date: ____________________

9.01: Review of Measures of Center and Spread

1 2 4 9 100 1 1 1 2 2 5 6

1 2 4 6 12 15

3 MEASURES OF CENTRAL TENDENCY		
Mean	**Median**	**Mode**
Denoted as $\bar{x}$, "x-bar" the average $\bar{x} = \frac{\sum x}{n}$ Sum of all the values # of terms	The number in the middle when the data is arranged in ascending order. If there are 2 numbers in the middle, then find their average.	The number which occurs most frequently. There does not have to be a mode. There can be more than one mode. Bimodal – 2 modes Trimodal – 3 modes

Example 1: Given scores from the latest test: 90, 89, 78, 81, 68, 100, 84, 83, 83, 74, 88, 80, 73, 89, 32

a) Find the measures of central tendency. Don't forget to put the data in ascending order!!!

n = 15

32 68 73 74 78 80 81 $\boxed{83}$ 83 84 88 89 89 90 100

Q1 (74) Q2 89 (Q3)

$\frac{1192}{15} =$

Mean: 79.47 Median: 83 Mode: 83, 89

5 NUMBER SUMMARY				
Minimum (Lower Extreme)	**Lower (1st) Quartile** Q_1	**Median (2nd Quartile)** Q_2	**Upper (3rd) Quartile** Q_3	**Maximum (Upper Extreme)**
Smallest number	The median of the lower half. If there are 2 numbers find their average.	Divides the data into a lower and upper half.	The median of the upper half. If there are 2 numbers find their average.	Largest number

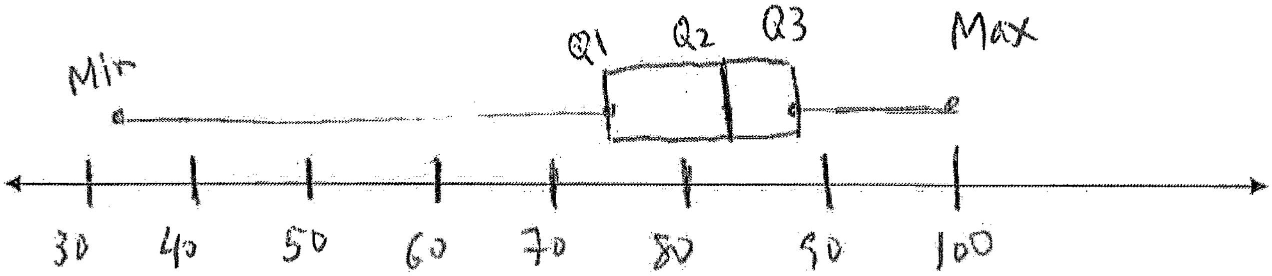
b) Find the 5-Number Summary of the test data above.

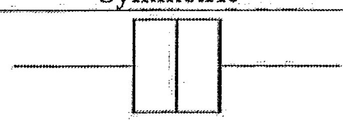
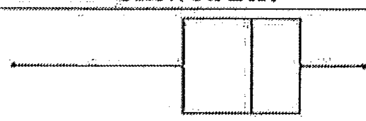
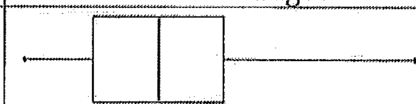
Min: 32 Q_1: 74 Median: 83 Q_3: 89 Max: 100

Box and Whisker Plot – A plot that displays the 5 number summary:

1. Draw a number line and scale it appropriately. Keep the minimum and maximum in mind.
2. Place points above the number line for each number in the 5 number summary.
3. Connect the minimum and Q_1 with a segment as well as Q_3 and the maximum.
4. Draw a box from Q_1 to Q_3.
5. Draw a vertical segment through the median.

c) Draw a box and whisker plot for the previous test data.



SHAPE OF A BOX AND WHISKER PLOT		
Symmetric	Skewed Left	Skewed Right
		

mean = median   mean < median   mean > median

MEASURES OF DISPERSION (SPREAD)		
Range	**Interquartile Range**	**Mean Absolute Deviation**
The difference in the maximum and the minimum. (Max – Min)	The difference in the upper quartile and lower quartile. ($Q_3 - Q_1$) $Q3 = 89$ $Q1 = 74$	$\text{MAD} = \dfrac{\Sigma \lvert x_i - \bar{x} \rvert}{n}$

mean = 79.47

d) Find the measures of spread for the given data set of test scores.

Range = 68   IQR = 15   MAD = ______

$\text{MAD} = \dfrac{144.65}{15}$

$100 - 32 =$

X	$\bar{X}$	$X-\bar{X}$
90	79.47	10.53
89	79.47	9.53
78	79.47	1.47
81	79.77	1.53
68	79.47	11.47

X	$\bar{X}$	$X-\bar{X}$
100	79.47	20.53
84		4.53
83		3.53
83		3.53
74		5.47
88		8.53

X	$\bar{X}$	$X-\bar{X}$
80	79.47	0.53
73		6.47
89		9.53
32		47.47

$\boxed{\text{MAD} = 9.64}$

Example 2:

a) List the number of pets from 8 of your classmates. 1 1 2 2 2 3 3 7

Q1 (1.5) Q2 (median) Q3 (3)

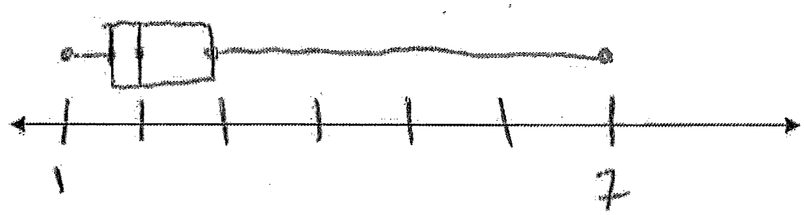
$\bar{x} = \frac{21}{8} = 2.625$

b) Calculate all measures of center, and the 5 number summary for the data.

$\bar{x}$ = 2.63 Median: 2 Mode: 2

Min: 1 Q_1: 1.5 Median: 2 Q_3: 3 Max: 7

c) Construct a box plot and describe the shape of the data.



Shape: skewed right

d) Calculate the measures of spread.

Range = $7-1=\boxed{6}$ IQR = $3-1.5=\boxed{1.5}$ MAD = 1.282

MAD

x	$\bar{x}$	$x-\bar{x}$
1	2.63	1.63
1	2.63	1.63
2	2.63	0.63
2	2.63	0.63

x	$\bar{x}$	$x-\bar{x}$
2	2.63	0.63
3		0.37
3		0.37
7		4.37

$MAD = \frac{10.26}{8} = 1.282$

BEST MEASURE OF CENTER AND SPREAD	
SYMMETRIC WITH NO OUTLIERS	SKEWED or WITH OUTLIERS
Mean and Mean Absolute Deviation (MAD)	Median and Interquartile Range (IQR)

9.01 Homework: Statistics Review

Date: ____________

1. Calculate all measures of center, spread, and the 5 number summary for the data provided. Construct a box plot and describe the shape of the data. Indicate if there are any outliers.

Homerun distances (in feet) to center field in 13 ballparks: n=13

387, 400, 400, 410, 410, 410, $\boxed{414}$, 415, 420, 420, 421, 457, 461

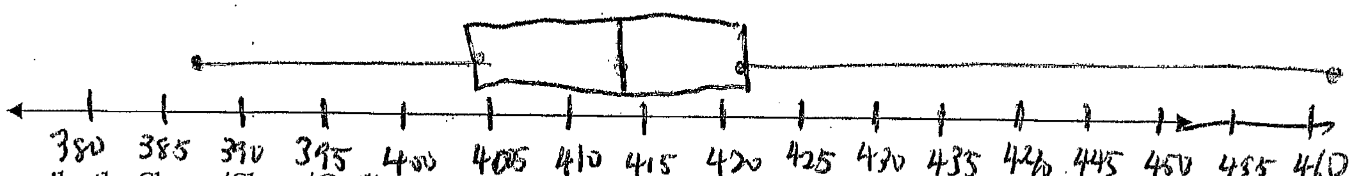
405 (Q1) Q2 (median) 420.5 (Q3)

$\bar{x}$ = 417.308 Median: 414 Mode: 410

Min: 387 Q_1: 405 Median: 414 Q_3: 420.5 Max: 461

461-387 Q3-Q1

Range = 74 IQR = 15.5 MAD = 14.225



Describe the Shape/Skew/Outliers:

Skewed Right

outliers: 457, 461

2. Suppose that the numbers of unnecessary procedures recommended by five doctors ~~in a 1-month~~ period are 2, 2, 8, 10, and 18. If we ask a 6th doctor and find out that they recommend 35 procedures.

(a) How will the Median and Mean be affected?

$\boxed{\text{first 5}}$ $\bar{x}=8$
Median=8

$\boxed{\text{6 doctors}}$
$\bar{x}=12.5$
Median=10

Both mean and median will both increase but mean increases by larger amount since data is skewed right.

(b) How will the IQR and Mean Absolute Deviation be affected?

IQR = 8
MAD = 4.8

IQR = 16
MAD = 9

Both IQR and MAD ~~both~~ increases by double the value.

3. Suppose the salaries (in dollars) of six employees are: 8000, 10000, 15000, | 16000, 20000 and 39000.

Q2

a. What are the Median and Mean salaries?

Median: $15,500

Mean: 18,000

b. Why are they such different numbers?

$39,000 is an outlier that more significantly impacts the mean

c. Which measure of center is the better pick to describe this data? Why?

Median b/c it's less affected by the outlier

4. Based solely on the given mean and median, decide on the shape of each distribution (skewed left, skewed right, or approximately symmetric):

a. Mean = 100 Median = 98 Shape: approximately symmetric

close together

b. Mean = 20 Median = 41 Shape: Skewed left

mean significantly less

c. Mean = 934 Median = 850 Shape: Skewed right

mean significantly more

9.02 Standard Deviation Notes

Date ____________

Statisticians use the Standard Deviation to discuss dispersion (spread) of data rather than the Mean Absolute Deviation (MAD).

The average of the squared differences of the mean is the Variance.
The standard deviation is the average distance from the mean. It tells us how tightly the data values are clustered around the mean.

MORE MEASURES OF DISPERSION (SPREAD)		
Variance	Standard Deviation for the whole population	Standard Deviation for a sample
$\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}$	$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$	$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$

Example 1: Compare the spread of the data for the two sets.

Falcons Football
Points Scored in 2008
Regular Season Games

n=8

Home Games	Away Games
34	9
38	9
22	27
34	14
20	24
45	22
13	25
31	24

a. Calculate the range of each set: Home: 32

$45 - 13 =$

Away: 18

$27 - 9$

b. The range of the home games is > the range of the away games.
Explain what this means: The difference b/t the largest and least points scored is greater in Home games than away games

c. Calculate the Mean of each set:

$\bar{x} = \frac{237}{8} = 29.625$ | $\frac{154}{8} = 19.25$

$\bar{x}$ (Home games) = 29.625

$\bar{x}$ (Away games) = 19.25

d. Calculate the IQR for each set:

Q1 (21) Q3 (36)

$\underbrace{13 \quad 20 \quad 22 \quad 31}$ | $\underbrace{34 \quad 34 \quad 38 \quad 45}$

$\overbrace{9 \quad 9 \quad 14 \quad 22}$ $\overbrace{24 \quad 24 \quad 25 \quad 27}$

Q3-Q1 → 36-21

Home: = 15

Away: 13

Q1=11.5
Q3=24.5

e. The IQR of the home games is > the IQR of the away games. Explain what this means:
The spread of the middle half of points scored is greater for Home games than for Away Games.

To calculate the Standard Deviation of a data set, find how far (the difference) each data value ($x_1, x_2, x_3, \ldots, x_n$) is from the mean ($\bar{x}$). These are the deviations from the mean. Square the differences and then find the average by adding them together and dividing by the number of data values for a population (n) or by (n – 1) for a sample. This number is the variance: σ^2. The Standard Deviation, σ, is the square root of the variance.

Ex 1 (cont'd) Falcons Points Scored in 2008 Regular Season Games

Home: $\bar{x}$ = 29.625	Away: $\bar{x}$ = 19.25
$(34 - 29.625)^2$ = 19.141	$(9 - 19.25)^2$ = 105.063
$(38 - 29.625)^2$ = 70.141	$(9 - 19.25)^2$ = 105.063
$(22 - \quad)^2$ = 58.141	$(27 - \quad)^2$ = 60.063
$(34 - \quad)^2$ = 19.141	$(14 - \quad)^2$ = 27.563
$(20 - \quad)^2$ = 92.641	$(24 - \quad)^2$ = 22.563
$(45 - \quad)^2$ = 236.391	$(22 - \quad)^2$ = 7.563
$(13 - \quad)^2$ = 276.391	$(25 - \quad)^2$ = 33.063
$(31 - \quad)^2$ = 1.891	$(24 - \quad)^2$ = 22.563
Sum of Dev²: 773.878	Sum of Dev²: 383.504
To calculate the variance, σ^2, divide the sum of deviation² by n (since this is the whole population and not sample)	
σ^2= 96.73475	σ^2= 47.938
To calculate the standard deviation, σ, take the square root of the variance.	
σ = 9.835	σ = 6.924

$\frac{773.878}{8}$

f. σ of home games:

9.835

g. σ of away games:

6.924

σ (home) > σ (away)

h. Explain the difference in σ:

The spread of the data around the mean is greater for Home games than for Away games.

Example 2: Use test scores (from the previous lesson): 90, 89, 78, 81, 68, 100, 84, 83, 83, 74, 88, 80, 73, 89, 32 to calculate the following:

Show calculations for standard deviation here:

$\bar{x}$ = __________ σ = __________

Median = ________ IQR = __________

Which measure of center and spread should be used to describe the data? Justify your response.

9.02 Standard Deviation Homework

Date ______________

Mrs. Durand has a problem and needs your input. She has to give one math award this year to a deserving student, but she can't make a decision. Here are the test grades for her two best students:

Emily: 90, 90, 80, 100, 99, 81, 98, 82

Jacob: 90, 90, 91, 89, 91, 89, 90, 90

1. Which of the two students should get the math award? Discuss why he/she should be the recipient.

Maybe Jacob since he is the more consistent performer

2. Calculate the *mean deviation, variance,* and *standard deviation* of Emily's distribution. The formulas are below. Fill out the table to help calculate by hand.

Mean deviation: $\frac{\sum |x_i - \bar{x}|}{n}$ variance: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

$\frac{720}{8} =$

The mean of Emily's test scores: 90

$n=8$

x_i for Emily	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$	$(x_i - \bar{x})^2$
90	0	0	0
90	0	0	0
80	-10	10	100
100	10	10	100
99	9	9	81
81	-9	9	81
98	8	8	64
82	-8	8	64

a. Mean deviation for Emily: $\frac{54}{8} = 6.75$

b. Variance for Emily: $\frac{490}{8} = 61.25$

c. Standard deviation for Emily: $\sqrt{61.25} = \boxed{7.826}$
(square root of variance)

d. What do these measures of spread tell you?

Emily performs well on tests, but her scores vary from the mean quite a bit.

3. Calculate the *mean deviation, variance,* and *standard deviation* of Jacob's distribution. The formulas are below. Fill out the table to help calculate by hand.

Mean deviation: $\frac{\sum |x_i - \bar{x}|}{n}$ variance: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

The mean of Jacob's test scores: 90

x_i for Jacob	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$	$(x_i - \bar{x})^2$
90	0	0	0
90	0	0	0
91	1	1	1
89	-1	1	1
91	1	1	1
89	-1	1	1
90	0	0	0
90	0	0	0

a. Mean deviation for Jacob: $\frac{4}{8} = 0.5$

b. Variance for Jacob: $\frac{4}{8} = 0.5$

c. Standard deviation for Jacob: $\sqrt{0.5} = \boxed{0.707}$

d. What do these measures of spread tell you?

Jacob is the more consistent low A student.

4. Based on this information about the spread of test scores for Emily and Jacob, which of the two students should get the math award and discuss why he/she should be the one to receive it.

Even though they have the same mean (average), Jacob is more consistent in his performance. (High B/Low A)

9.03 Using Technology to Calculate Standard Deviation

Date: ________

Mean, standard deviation, and many other statistical measurements can be calculated using a scientific or graphing calculator. Use the steps that apply to your type of calculator:

TI-30XS MultiView or TI-36X Pro

1. Hit [data]
2. To clear a list, hit [data] again and select the list to clear.
3. Enter your data in a list (under L1).
4. Hit [2nd] [data] and then select "1-Var Stats".
5. Select the List your data is stored in (L1), and the frequency for each value recorded in that list (FRQ:One), and then enter "CALC".

TI Graphing

1. Hit [STAT] and select "1:Edit...".
2. To clear a list, arrow up to the list name above the list "L1", hit [CLEAR] and then [ENTER].
3. Enter your data under L1.
4. Hit [STAT] again, move over to "CALC" and then select "1-Var Stats".
5. Indicate the list you put your data in, like L1. If you need to specify a different list, press [2nd] and [1] or [2] or [3] etc, depending on the list name needed.
6. Select "Calculate" or hit [ENTER] (wording depends on OS).

You will receive a list of the following info:

- $\bar{x}$ = 35.818 mean
- $\sum x$ = 394 sum of all data
- $\sum x^2$ = 14348 sum of all squared data
- Sx = 4.854 standard deviation of a SAMPLE
- σx = 4.628 standard deviation of a POPULATION
- n = 11 how many pieces of data in the set
- minX = 27 minimum
- Q1 = 33 lower quartile
- Med = 37 median
- Q3 = 40 upper quartile
- maxX = 41 maximum

Example 1. Find the mean, median, range, interquartile range, and standard deviation for the following data sets.

a. 35, 27, 39, 41, 41, 38, 28, 33, 35, 37, 40

Mean	35.818
Median	37
Range	14
IQR	7
Standard Deviation	4.628

b. 84, 85, 105, 76, 73, 93, 81, 74, 84, 80, 72

Q1=74
Q3=85

Mean	82.455
Median	81
Range	33
IQR	11
Standard Deviation σx	9.297

105 − 72

Example 2. Compare the mean and standard deviation of the following data sets.

a. mpg of hybrids: 50, 37, 42, 40, 39, 38, 41

Mean: 41 Std. Dev.: 4

b. mpg of sedans: 28, 19, 24, 22, 18, 24, 26

Mean: 23 Std. Dev.: 3.338

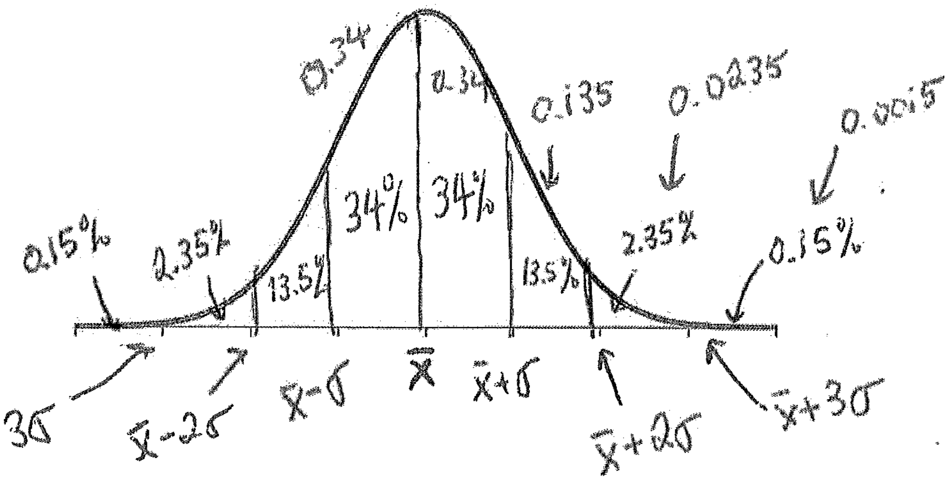
What are the implications of these statistics? higher avg. for hybrids, more spread amongst hybrid mpg data

9.04 Normal Distribution - The Empirical Rule

Date ____________

Normal Distribution is modeled by a normal (bell) curve and is symmetric about the mean.

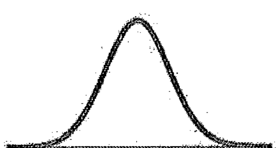
- It is formed using the mean and standard deviation.
- The total area under the curve is 100% because it represents all of the probability = 100 %
- Empirical Rule (68–95–99.7 Rule): the percent (probability) of the area under the curve for each standard deviation is shown on the graph below.



The normal distribution curve is used to find probability. It must be a normal distribution in order to use the above percentages (probabilities).

Example 1: For a normal curve, find the following probabilities of a randomly selected x-value from the distribution. If may be helpful to use a sketch.

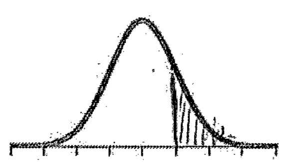
a. $P(x \le \bar{x} - 2\sigma)$



$2.35\% + 0.15\%$

$2.5\% \rightarrow \boxed{0.025}$

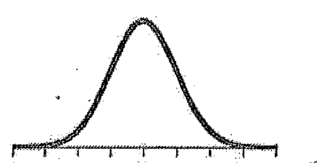
b. $P(x \ge \bar{x} + \sigma)$



$13.5 + 2.35 + 0.15 = 16\%$

$\boxed{0.16}$

c. $P(\bar{x} - 2\sigma \le x \le \bar{x} + \sigma)$



$34 + 34 + 13.5 = 81.5$

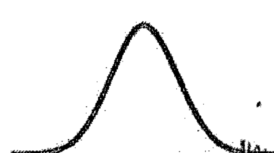
$\boxed{0.815}$

d. $P(x \ge \bar{x})$

50% $\boxed{0.5}$

$x \le$ means probability to the left

e. $P(x \ge \bar{x} + 3\sigma)$

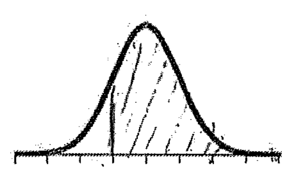


0.15%

$0.15\% \rightarrow \boxed{0.0015}$

$x \ge$ means probability to the right

f. $P(\bar{x} - \sigma \le x \le \bar{x} + 3\sigma)$



83.85%

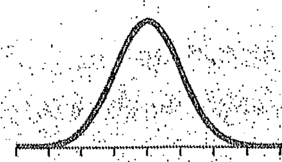
$34 + 34 + 13.5 + 2.35 = \boxed{0.8385}$

$\le x \le$ means between

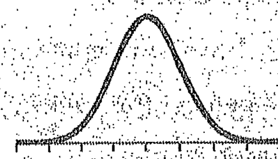
9.04 Homework – Normal Distribution & the Empirical Rule

1. Find the indicated probability for a randomly selected x-value from the distribution.

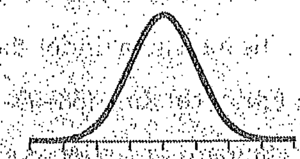
a. $P(x \le \bar{x} + \sigma)$



b. $P(x \ge \bar{x} - 2\sigma)$

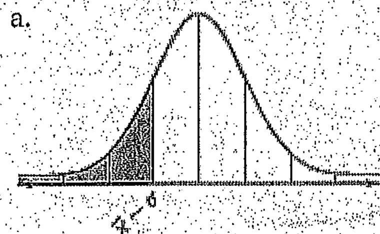


c. $P(\bar{x} - \sigma \le x \le \bar{x} + \sigma)$

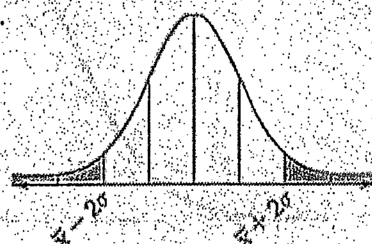


2. Give the percent of the area under the normal curve represented by the shaded region.

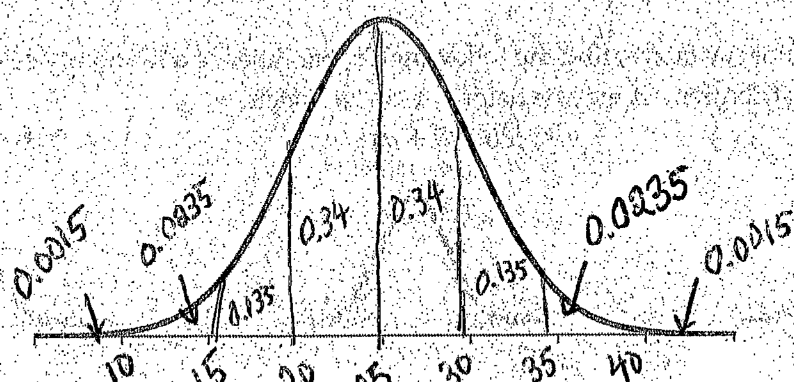
a.



b.



3. A normal distribution has a mean of 25 and a standard deviation of 5. Find the probability that a randomly selected x-value from the distribution is in the given interval. Label the x-axis and the probabilities under the curve below.



a. Between 20 and 30

$\boxed{0.68}$

b. Between 10 and 25

$0.0235 + 0.135 + 0.34 = \boxed{0.4985}$

c. Between 15 and 35

$0.135 + 0.34 + 0.34 + 0.135 = \boxed{0.95}$

d. At least 20 (20 or more)

$0.50 + 0.34 = \boxed{0.84}$

e. At least 35 (35 or more)

$0.0235 + 0.0015 = \boxed{0.025}$

f. At most 30 (30 or less)

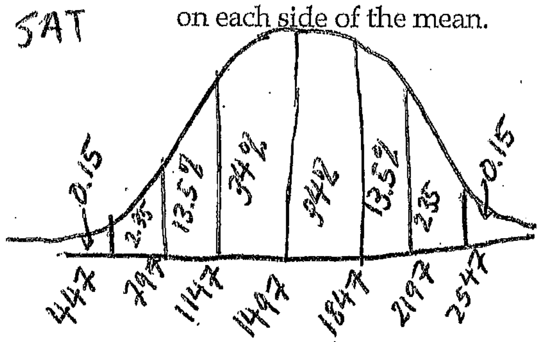
$0.50 + 0.34 = \boxed{0.84}$

9.05 Applications of the Empirical Rule Date: ____________

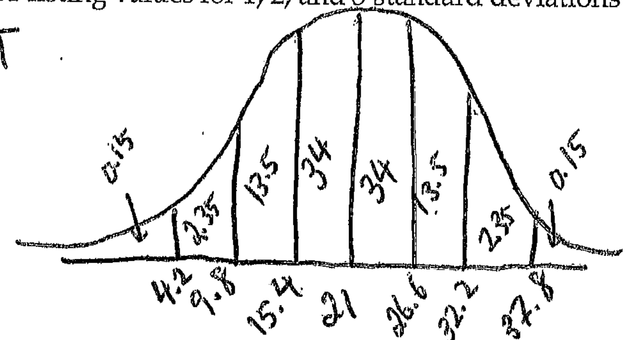
A) Comparing the SAT and ACT: college admissions offices need to compare scores of students who take the Scholastic Aptitude Test (SAT) with those who take the American College Test (ACT). Suppose that for recent college applicants who took the SAT, scores have a mean of 1497 (out of 2400) and a standard deviation of 350. Further, suppose that for recent college applicants who took the ACT, scores have a mean of 21 (out of 36) and a standard deviation of 5.6.

1. Sketch normal curves for both the SAT and ACT listing values for 1, 2, and 3 standard deviations on each side of the mean.

SAT



ACT



Apply the empirical rule to approximate the following: Empirical Rule (68-95-99.7 Rule)

2. About 95% of SAT takers score between what two values?

   within $(\pm 2\sigma) \rightarrow (797, 2197)$

3. About 95% of ACT takers score between what two values?

   within $(\pm 2\sigma)$ $(9.8, 32.2)$

4. What is the proportion of students who score between 1147 and 1847 on the SAT?

   68%

5. What is the proportion of students who score between 15.4 and 32.2 on the ACT?

   $68 + 13.5 = \boxed{81.5\%}$

6. If John scored at the 84th percentile on the ACT, what score did he achieve?

7. College Board reports that 1,672,395 students took the SAT in 2014. About how many students achieved a score of at least 2197?

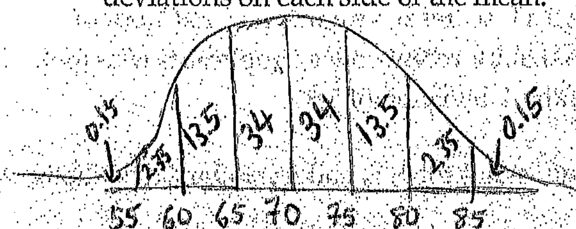
   1,672,395(0.025) = 41,810 students

8. ACT, Inc. reports that 1,845,787 students took the ACT in 2014. About how many students achieved a score of at most 21?

   1,845,787(0.5) = 922894 students

B) Last spring, 250 students took the Algebra 2 final exam. The scores were distributed normally with a mean of 70 and a standard deviation of 5.

9. Sketch the normal curve for the final exam scores, listing values for 1, 2, and 3 standard deviations on each side of the mean.



Apply the empirical rule to approximate the following:

10. What percentage of scores is between scores 65 and 75?

$34+34=68\%$

11. What percentage of scores is between scores 60 and 70?

$13.5+34=47.5\%$

12. What percentage of scores is between scores 60 and 85?

$13.5+34+34+13.5+2.35=97.35\%$

13. What percentage of scores is less than a score of 55?

0.15%

14. What percentage of scores is at least a score of 80?

$2.35+0.15=2.5\%$

(out of 250 total students)

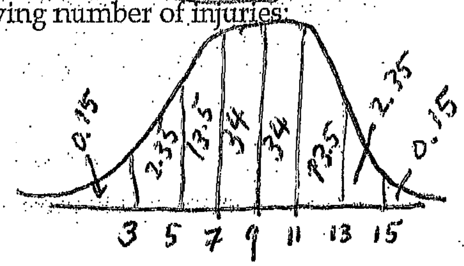
15. How many Algebra 2 students achieved a score between 70 and 80?

$0.34+0.135=0.475$ (47.5%) | $250(0.475)=$ $\boxed{119 \text{ students}}$

16. How many Algebra students achieved a score of at most 75? (75 and less)

$0.0015+0.0235+0.135+0.34+0.34=0.84$ (84%) | $250(0.84)=$ $\boxed{210 \text{ students}}$

C) Statistics kept for NFL football teams regarding the number of injuries suffered by NFL players during their careers showed the distribution is approximately normal with the mean number of injuries per player to be 9 with a standard deviation of 2. If there are 1696 NFL players in the current season, determine how many players will have the following number of injuries:



17. Less than 9 injuries in their career. (50%)

$1696(0.5)=848$ players

18. At least 7 injuries in their career. (7 or more)

$50+34=84\%$

$1696(0.84)=1425$ players

19. More than 5 but less than 11 injuries in their career.

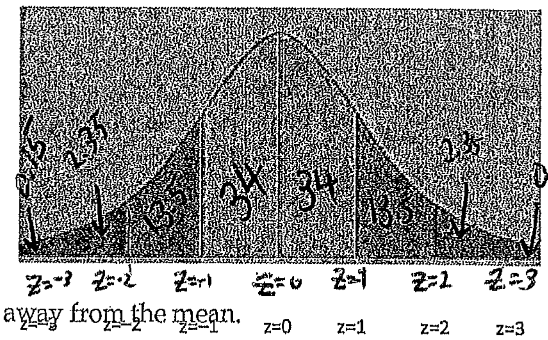
$13.5+34+34 \rightarrow 81.5\%$ | $1696(0.815)=1382$ players

9.06 Standard Normal Distributions Date ____________

What do you do when you are looking for the probability of an x-value in a normal distribution, but that value does not fall on one of the standard deviations?

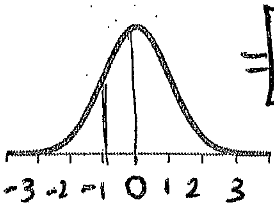
Standard Normal Distribution:

- Formed using a mean of 0 and a standard deviation of 1.
- Used when the x-value does not fall on a standard deviation.
- The Empirical Rule still applies to a standard normal distribution.
- To change an x-value from a normal distribution with mean $\bar{x}$ and standard deviation σ use the z-score formula: $z = \frac{x-\bar{x}}{\sigma}$
- The z-score is the number of standard deviations the x-value lies away from the mean.



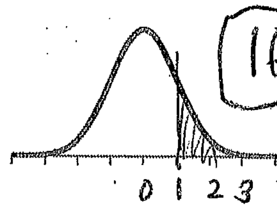
Example 1: For a standard normal distribution, find the following probabilities of a randomly selected x-value from the distribution. If may be helpful to draw a sketch.

a. P(z ≤ −1)



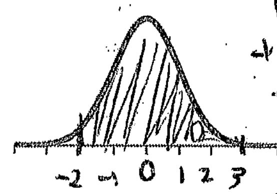
$= \boxed{16\%}$

b. P(z ≥ 1)



$\boxed{16\%}$

c. P(−2 ≤ z ≤ 3)



13.5
+34
+34
+13.5
+2.35

$\boxed{97.35\%}$

Converting to a z-score:

Example 2: Consider a normal distribution with a mean of 72 and a standard deviation of 3. Convert the following into z-scores. $\bar{x}=72$ $\sigma=3$

a. x = 65

$z = \frac{x-\bar{x}}{\sigma} \rightarrow \frac{65-72}{3} = \boxed{-2.33}$

b. x = 81

$z = \frac{81-72}{3} = \boxed{3.00}$

c. x = 76

$z = \frac{76-72}{3} = \boxed{1.33}$

WHAT'S THE POINT???

Z-scores enable us to find any probability with a table of values. The probability given on the z-score table is the probability that is less than the z-score. BE CAREFUL! We are not always looking for less than!!!!

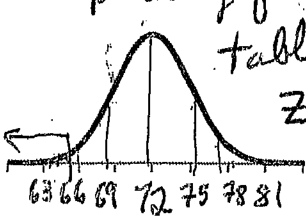
$\bar{x}=72$ $\sigma=3$

Example 3: Consider a normal distribution with a mean of 72 and a standard deviation of 3. Sketch a graph and find the following:

a. P(x ≤ 65)

≤ take the probability from the table

z=-2.33

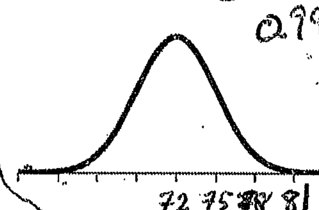


$P(x \leq 65) = \boxed{0.0099}$

b. P(x ≥ 81)

≥ 1 − probability

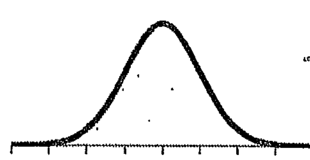
z=3
0.9987



$P(x \geq 81) = 1 - 0.9987 = \boxed{0.0013}$

c. P(76 ≤ x ≤ 81)

*subtract the 2 probabilities from table.



0.9987
−0.9082

$\boxed{0.0905}$

9.06 Standard Normal Distribution Homework $\bar{x}=70$ $\sigma=10$ $z=\frac{x-\bar{x}}{\sigma}$

1) A normal distribution has a mean of 70 and a standard deviation of 10. Calculate the z-score and use the z-score table to find the indicated probability.

a. $P(x \le 65)$

$z=\frac{65-70}{10}=-0.5$

$P(x\le 65)=\boxed{0.3085}$

b. $P(x \ge 47)$

$z=\frac{47-70}{10}=-2.3$

$1-P(x\le 47)$

$1-0.0107=\boxed{0.9893}$

c. $P(54 \le x \le 83)$

$z=\frac{54-70}{10}=-1.6$ | $z=\frac{83-70}{10}=1.3$

0.0548 0.9032

$0.9032-0.0548=\boxed{0.8484}$

d. $P(x \le 91)$

$z=\frac{91-70}{10}=2.1$

$P(x\le 91)=\boxed{0.9821}$

e. $P(x \ge 39)$

$z=\frac{39-70}{10}=-3.1$

$1-0.0010=\boxed{0.999}$

f. $P(79 \le x \le 101)$

$z=\frac{79-70}{10}$ $z=\frac{101-70}{10}$

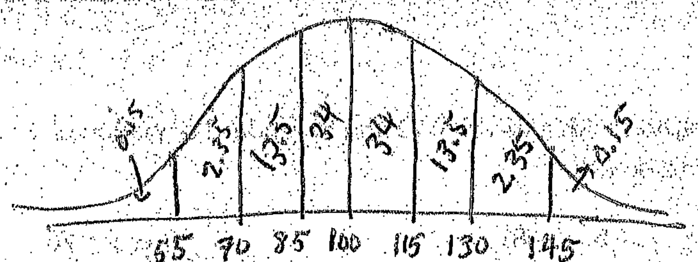
$z=0.9$ $z=3.1$

0.8159 0.9990

$0.9990-0.8159=\boxed{0.1831}$

2) The scores on an Intelligence Quotient (IQ) test are normally distributed with a mean of 100 and a standard deviation of 15.

a. Draw and label the normal distribution.



b. What is the probability that a person will score at least a 121? $P(x \ge 121)$

$z=\frac{121-100}{15}=1.4$

$P(x\ge 121)=1-0.9192=\boxed{0.0808}$

c. What is the probability that a person will score no more than 79? $P(x\le 79)$

$z=\frac{79-100}{15}=-1.4$

$P(x\le 79)=\boxed{0.0808}$

d. What is the probability that a person will score between 92 and 111?

$z_{92}=\frac{92-100}{15}=-0.53\rightarrow 0.2981$

$z_{111}=\frac{111-100}{15}\rightarrow 0.73\rightarrow 0.7673$

$0.7673-0.298$

$=\boxed{0.4692}$

e. What minimum score would someone need to score higher than 80% of those taking an IQ test?

$z=\frac{x-100}{15}$

$\frac{0.84}{1}=\frac{x-100}{15}$

$x-100=15(0.84)$

$x-100=12.6$

$\boxed{x=112.6}$

$* \; z = \frac{x - \bar{x}}{\sigma}$

9.07 Standard Normal Distribution Applications Date ______

All data in the following exercises is normally distributed. $\bar{x} = 125 \quad \sigma = 15$

1. Jack took a standardized achievement test with a mean of 125 and a standard deviation of 15. Jack's score was 148. $x = 148$
   a. What is the percentile rank for Jack's score on this test? Percentile rank means that Jack scored as well or better than that percentage of students taking the test.

   $z = \frac{148 - 125}{15} = 1.53 \qquad 0.937 \rightarrow \boxed{93.7\%}$

   b. If Jill scored at the 67th percentile, what was her score on the test?

   $67\% \rightarrow z = 0.44 \;\Big|\; z = \frac{x - \bar{x}}{\sigma} \;\Big|\; \frac{0.44}{1} = \frac{x - 125}{15} \;\Big|\; x - 125 = 15(0.44)$

   $x - 125 = 6.6$

   $\boxed{x = 131.6}$

2. The average number of absences for 1st graders is 15 with a standard deviation of 6.
   a. What is the probability of a 1st grader having fewer than 6 absences?

   $\bar{x} = 15,\ \sigma = 6 \;\Big|\; x = 6 \;\Big|\; z = \frac{x - \bar{x}}{\sigma},\ z = \frac{6 - 15}{6} \;\Big|\; z = -1.5 \;\Big|\; \boxed{P(x \leq 6) = 0.0668}$

   b. What is the probability of a 1st grader having more than 20 absences?

   $\bar{x} = 15,\ \sigma = 6,\ x = 20 \;\Big|\; z = \frac{20 - 15}{6},\ z = 0.83 \;\Big|\; P(x \geq 20) = 1 - 0.7967 \;\Big|\; \boxed{P(x \geq 20) = 0.2033}$

   c. If a student is absent more often than 30.85% of other 1st graders, how many days did she miss?

   $z = -0.5 \;\Big|\; z = \frac{x - \bar{x}}{\sigma} \;\Big|\; \frac{-0.5}{1} = \frac{x - 15}{6} \;\Big|\; x - 15 = 6(-0.5)$

   $\boxed{x = 12 \text{ days}}$

3. On the Scholastic Aptitude Test (SAT), scores have a mean of 500 and a standard deviation of 100.
   a. Mo scored 600 on the math section. What percentile did he achieve?

   $\bar{x} = 500,\ \sigma = 100 \;\Big|\; x = 600 \;\Big|\; z = \frac{x - \bar{x}}{\sigma} \;\Big|\; z = \frac{600 - 500}{100} = 1 \;\Big|\; 0.8413 \rightarrow \boxed{84.13\%}$ percentile

   b. Larry scored 750 on the math section. What percentile did he achieve?

   $\bar{x} = 500,\ \sigma = 100,\ x = 750 \;\Big|\; z = \frac{750 - 500}{100} = 2.5 \;\Big|\; 0.9938 \rightarrow \boxed{99.38\%}$ percentile

   c. Curley scored better than 97.72% of students on the math section of the SAT. What was his score?

   $0.9772 \rightarrow z = 2.0 \;\Big|\; z = \frac{x - \bar{x}}{\sigma},\ 2 = \frac{x - 500}{100} \;\Big|\; 200 = x - 500$

   $700 = x$

   $\boxed{x = 700}$

$\bar{x}=52 \quad \sigma=5 \quad x=45$

4. A patient recently diagnosed with Alzheimer's disease took a cognitive abilities test. The mean score on this test is 52 with a standard deviation of 5.
   a. If the patient scored a 45 on the test, what is his percentile rank?

$z=\frac{x-\bar{x}}{\sigma} \rightarrow \frac{45-52}{5}=-1.4$ | $P(z\le -1.4)=0.0808 \rightarrow$ $\boxed{8.08\% \text{ percentile}}$

   b. If a patient scored higher than 87.7% of other Alzheimer's patients, what was her score on the test?

0.877, $z=1.16$ | $z=\frac{x-\bar{x}}{\sigma}$ | $1.16=\frac{x-52}{5}$ | $x-52=5(1.16)$, $x=5(1.16)+52$ | $\boxed{x=57.8}$

5. Mrs. Durand's Algebra 2 unit test scores have a mean of 82 with a standard deviation of 5.5.
   a. What is the probability that a student will score at least a 92? $1-P(x\le 92)$

$\bar{x}=82$, $\sigma=5.5$, $x=92$ | $z=\frac{92-82}{5.5}$, $z=1.82 \rightarrow 0.9656$ | $P(x\ge 92)=1-P(x\le 92)$ $=1-0.9656$ | $P(x\ge 92)=\boxed{0.0344}$

   b. What is the probability a student will get a B?

$P(79.5\le x\le 89.4)$ $=0.9115-0.3264$ $=\boxed{0.5851}$ | $z_{79.5}=\frac{79.5-82}{5.5}=-0.45$, 0.3264 | $z_{89.4}=\frac{89.4-82}{5.5}=1.35$, 0.9115

   c. What is the probability a student will fail?

$P(x\le 69.4)$ | $z=\frac{69.4-82}{5.5}$, $z=-2.29$ | $P(x\le 69.4)=P(z\le -2.29)=\boxed{0.0110}$

6. The diameter of maple trees in a Canadian forest have a mean of 10 inches and standard deviation of 3.2 inches. What is the percentage of trees with a diameter between 8 and 15 inches?

$\bar{x}=10$, $\sigma=3.2$ | $P(8\le x\le 15)$, $0.9406-0.2643$, $\boxed{0.6763}$ | $z_8=\frac{8-10}{3.2}=-0.63$, 0.2643 | $z_{15}=\frac{15-10}{3.2}=1.56$, 0.9406

7. The weights of 1800 fish in a lake have a mean of 3 kilograms and standard deviation of .5 kilograms.
   a. What percentage of fish weigh at least 3.75 kilograms? (3.75 kg or more) $\bar{x}=3 \quad \sigma=0.5$

$P(x\ge 3.75)=1-P(x\le 3.75)$ | $1-0.9332$

$z=\frac{3.75-3}{0.5}=1.5 \rightarrow 0.9332$ | $P(x\ge 3.75)=0.0668 \rightarrow \boxed{6.68\%}$

   b. Approximately how many fish in the lake weigh at least 3.75 kilograms?

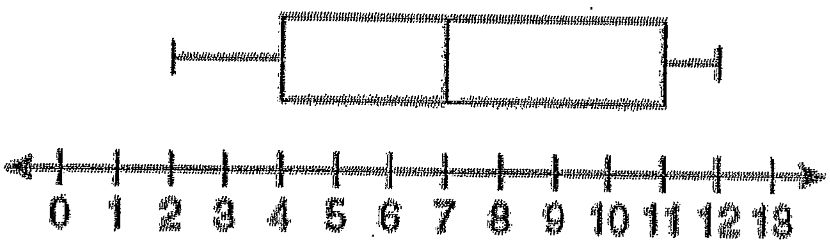
$0.0668\times 1800=\boxed{120 \text{ fish}}$

Accelerated Pre-Calculus

Name: ______________________

9.07b Quiz Review: Centers, Spread, & Normal Distribution

Use the following box and whisker plot to identify the information requested in #1 – 5.



1. What percent of values fall between 4 and 11? ________ 2. What percent of values are below 7? ________ 3. What percent of values are above 11? ________ 4. Describe the shape of the distribution of test scores. ________________	5. Identify the test scores for each value: Q1: ________ Q2: ________ Q3: ________ Minimum: ________ Maximum: ________ IQR: ________ Range: ________

Notes:

In a normally distributed distribution, the mean is the best measure of central tendency.

With skewed distributions (left or right),however, median can be the better measure of central tendency

The interquartile range (IQR) is the range of values within which reside the middle 50% of the scores.

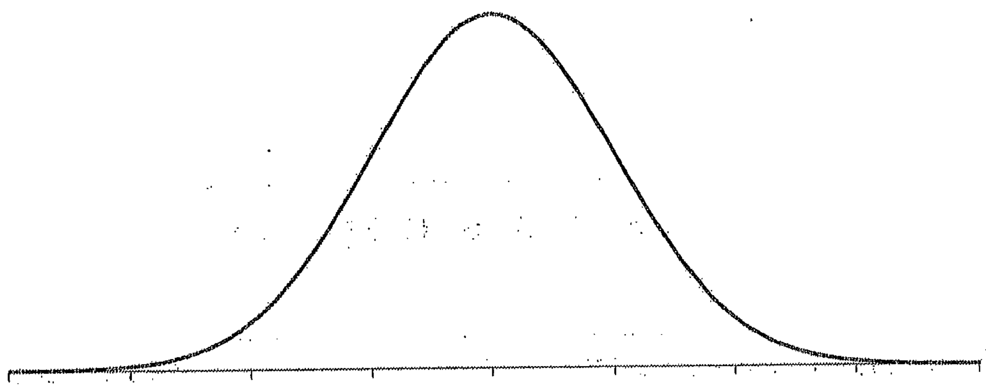
The first quartile (Q1) indicates that 25% of the scores have a value lower than Q1

The third quartile (Q3) indicates that 75% of the scores have a value lower than Q3

The median is the value that is in the "middle" of the distribution (Q2), with 50% of the scores having a value less than the median

6) A machine is used to put bolts into boxes. It does so such that the actual number of bolts in a box is normally distributed with a mean of 106 and a standard deviation of 2.

a) Label the x-axis and the Empirical rule probabilities under the Normal distribution graph below.



Use the scenario and graph above to answer the following questions. Show all work and write a probability statement for each. Answers may be given in percent or decimal form.

a) What percentage of boxes contain no more than 108 bolts? ____________________

b) What percentage of boxes contain at least 104 bolts? ____________________

c) What is the probability of boxes containing between 102 and 112 bolts? ____________________

d) What number represents the 78h percentile? ____________________

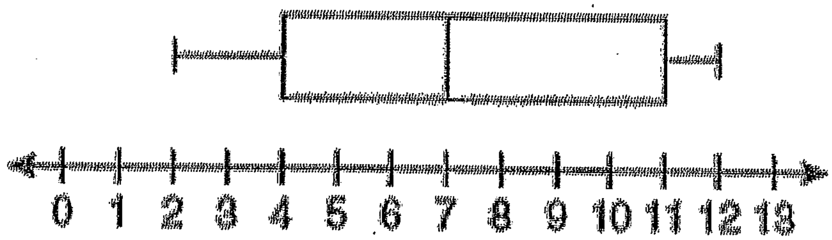
7) A test was given to 120 students, and the scores approximated a normal distribution. If the mean score was 72 with a standard deviation of 7, approximately what percent of the scores were 65 or higher?

Accelerated Pre-Calculus

Name: Key

9.07b Quiz Review: Centers, Spread, & Normal Distribution

Use the following box and whisker plot to identify the information requested in #1 – 5.



1. What percent of values fall between 4 and 11? between Q1 and Q3 is the middle 50% 50° 2. What percent of values are below 7? 50° 3. What percent of values are above 11? 25° 4. Describe the shape of the distribution of test scores. skewed left *longer tail on the left	5. Identify the test scores for each value: Q1: 4 (median) Q2: 7 Q3: 11 Minimum: 2 Maximum: 12 Q3 – Q1 (11 – 4 = 7) IQR: 7 12 – 2 = 10 → Range: 10

Notes:

In a normally distributed distribution, the mean is the best measure of central tendency.

With skewed distributions (left or right),however, median can be the better measure of central tendency

The interquartile range (IQR) is the range of values within which reside the middle 50% of the scores.

The first quartile (Q1) indicates that 25% of the scores have a value lower than Q1

The third quartile (Q3) indicates that 75% of the scores have a value lower than Q3

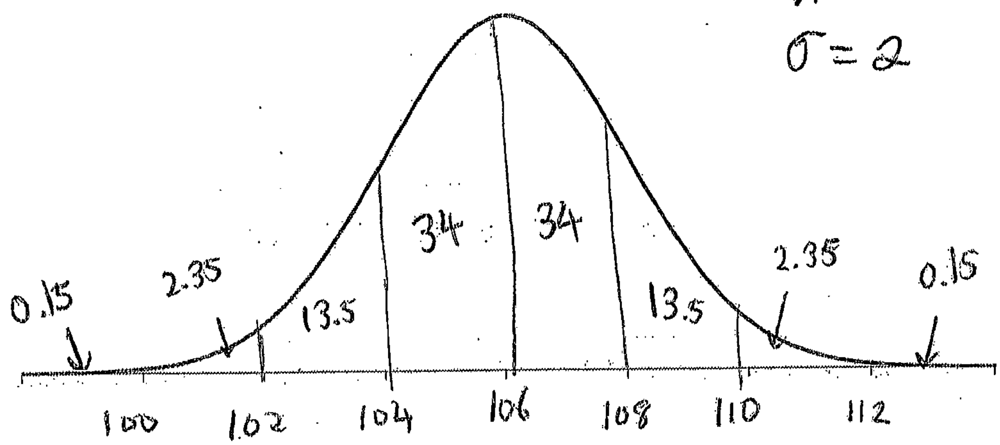
The median is the value that is in the "middle" of the distribution (Q2), with 50% of the scores having a value less than the median

6) A machine is used to put bolts into boxes. It does so such that the actual number of bolts in a box is normally distributed with a mean of 106 and a standard deviation of 2.

a) Label the x-axis and the Empirical rule probabilities under the Normal distribution graph below.

$\bar{x} = 106$

$\sigma = 2$



Use the scenario and graph above to answer the following questions. Show all work and write a probability statement for each. Answers may be given in percent or decimal form.

a) What percentage of boxes contain no more than 108 bolts? 84%

$P(x \leq 108) =$

b) What percentage of boxes contain at least 104 bolts? 84%

$P(x \geq 104)$

c) What is the probability of boxes containing between 102 and 112 bolts? 97.35%

$13.5 + 34 + 34 + 13.5 + 2.35$

d) What number represents the 78th percentile? ____________

$0.78 \rightarrow z = 0.77$

$z = \frac{x - \bar{x}}{\sigma} \rightarrow 0.77 = \frac{x - 106}{2}$

$1.54 = x - 106$

$\boxed{x = 107.54}$

7) A test was given to 120 students, and the scores approximated a normal distribution. If the mean score was 72 with a standard deviation of 7, approximately what percent of the scores were 65 or higher?

$\bar{x} = 72$

$\sigma = 7$

$x = 65$

$z = \frac{65 - 72}{7}$

$z = -1$

0.1587

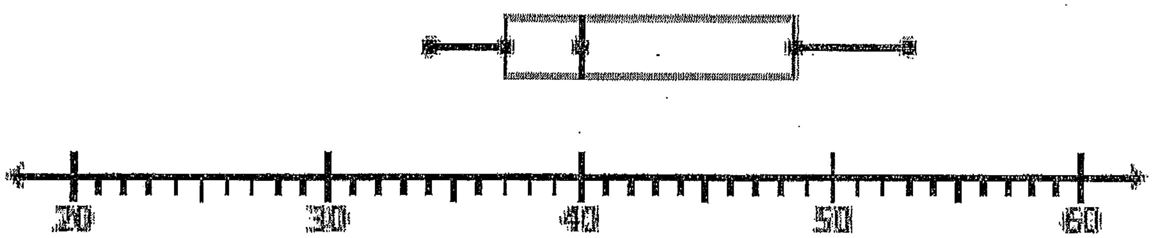
$P(x \geq 65) = 1 - 0.1587 = \boxed{0.8413}$

$\boxed{84.13\%}$

Accelerated Pre-Calculus Name: ____________________

9.07b Review WS 3: Centers, Spread, & Normal Distribution

Use the following box and whisker plot to identify the information requested in #1 – 5.



1. What percent of values fall between 40 and 49? ________

2. What percent of values are below 49? ________

3. What percent of values are above 37? ________

4. Describe the shape of the distribution of test scores. ____________________

5. Identify the values from the data:

Q1: ________

Q2: ________

Q3: ________

Minimum: ________

Maximum: ________

IQR: ________

Range: ________

Notes:

In a normally distributed distribution, the mean is the best measure of central tendency.

With skewed distributions (left or right),however, median can be the better measure of central tendency

The interquartile range (IQR) is the range of values within which reside the middle 50% of the scores.

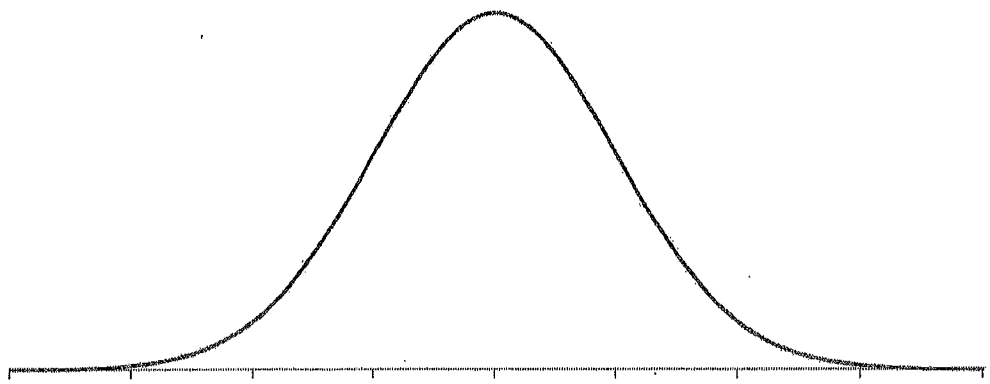
The first quartile (Q1) indicates that 25% of the scores have a value lower than Q1

The third quartile (Q3) indicates that 75% of the scores have a value lower than Q3

The median is the value that is in the "middle" of the distribution (Q2), with 50% of the scores having a value less than the median

6) A set of scores with a normal distribution has a mean of 50 and a standard deviation of 7.

a) Label the x-axis and the Empirical rule probabilities under the Normal distribution graph below.



Use the scenario and graph above to answer the following questions. Show all work and write a probability statement for each. Answers may be given in percent or decimal form.

a) What percentage of scores is no more than 35? ______________________

b) What percentage of scores is at least 68? ______________________

c) What percentage of scores is between 29 and 64? ______________________

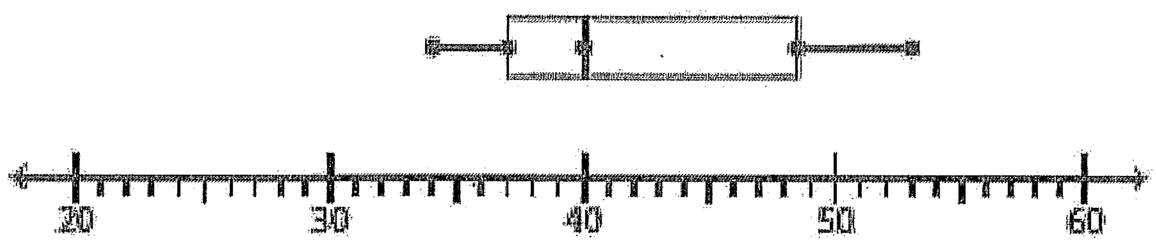
d) What number represents the 45^{h} percentile? ______________________

7) The monthly income of 5,000 workers at the Microsoft plant are distributed normally. Suppose the mean monthly income is $1,250 and the standard deviation is $250. What percentage of the workers earn less than $1750 per month? (Answer .977)

Accelerated Pre-Calculus Name: Key

9.07b Review WS 3: Centers, Spread, & Normal Distribution

Use the following box and whisker plot to identify the information requested in #1 – 5.



1. What percent of values fall between 40 and 49? 25%

2. What percent of values are below 49? 75°

3. What percent of values are above 37? 75°

4. Describe the shape of the distribution of test scores. skew right

5. Identify the values from the data:

Q1: 37

Q2: 40

Q3: 49

Minimum: 34

Maximum: 53

Q3 – Q1
49 – 37 = 12 IQR: 12

53 – 34 Range: 19

Notes:

In a normally distributed distribution, the mean is the best measure of central tendency.

With skewed distributions (left or right),however, median can be the better measure of central tendency

The interquartile range (IQR) is the range of values within which reside the middle 50% of the scores.

The first quartile (Q1) indicates that 25% of the scores have a value lower than Q1

The third quartile (Q3) indicates that 75% of the scores have a value lower than Q3

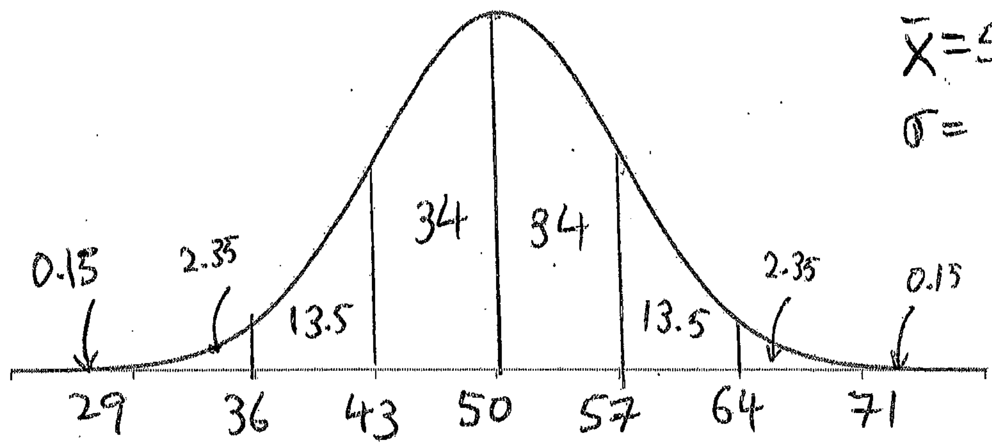
The median is the value that is in the "middle" of the distribution (Q2), with 50% of the scores having a value less than the median

6) A set of scores with a normal distribution has a mean of 50 and a standard deviation of 7.

a) Label the x-axis and the Empirical rule probabilities under the Normal distribution graph below.

$\bar{x} = 50$

$\sigma = 7$



Use the scenario and graph above to answer the following questions. Show all work and write a probability statement for each. Answers may be given in percent or decimal form.

a) What percentage of scores is no more than 35? 0.0162 or 1.62%

$z = \frac{x-\bar{x}}{\sigma} \rightarrow \frac{35-50}{7} = -2.143 \rightarrow 0.0162$

b) What percentage of scores is at least 68? 0.0051 or 0.51%

$P(x \geq 68) = 1 - P(x \leq 68)$

$= 1 - 0.9949$

$= 0.0051$

$z = \frac{68-50}{7} = 2.57 \rightarrow 0.9949$

c) What percentage of scores is between 29 and 64? 97.35

$2.35 + 13.5 + 34 + 34 + 13.5 \nearrow$

d) What number represents the 45h percentile?

$\boxed{x = 49.09}$

$0.45 \rightarrow z = -0.13$ | $z = \frac{x-\bar{x}}{\sigma}$ | $-0.13 = \frac{x-50}{7}$

$x - 50 = 7(-0.13)$ | $x - 50 = -0.91$

$x = 49.09$

7) The monthly income of 5,000 workers at the Microsoft plant are distributed normally. Suppose the mean monthly income is $1,250 and the standard deviation is $250. What percentage of the workers earn less than $1750 per month? (Answer .977)

$\bar{x} = 1250$

$\sigma = 250$

$x = 1750$

$z = \frac{1750-1250}{250}$

$z = 2$

$\boxed{0.9772 \text{ or } 97.72\%}$

Key

9.09 Confidence Intervals Date: ________

Opener: We plan to meet Saturday morning for a fun day at Six Flags. If I tell you that I will be there at 10:30, what time do you expect me to arrive? 10:30 Would any other times also be reasonable? If so, what are they? 10:25, 10:35, .. 10:15 10:30 ± ____ minutes

Would you be more confident that I will arrive "on time" if you make my window of arrival times wider or narrower? Why?

Because there are more options for the reasonable arrival times.

Population vs. Sample:

Population: includes all elements of a set of data
example: all U.S. adults

Sample: includes a portion of a set of data
example: 3500 adults called randomly

Parameter: a number relating to the population
example: N = 252,063,800 US adults
p = 30.11% US adults have college degree

Statistic: a number relating to the sample
example: n = 3500
sample percentage $\hat{p}$ = 28% of adults called have college degree.

Identify the population, sample, and statistic for each of the following scenarios:

A survey of 1300 American households found that 32% of those households have basements.

Population: All American households Sample: 1300 American households surveyed Statistic: $\hat{p} = 0.32$ have basements.

The average bill from every 6th person getting food at Chipotle in a 3-hour period was $19.61.

Population: all Chipotle customers Sample: Every 6th customer in the 3 hr period Statistic: $\bar{x} = 19.61$ avg. meal cost.

Confidence Intervals are intervals of plausible values for estimating a parameter, with a given percent confidence. We use a sample mean to estimate the population mean. We use a sample proportion to estimate a population proportion.

Consider this: The Milton Parks and Recreation Department wants to build a new park in Crabapple. To allocate funds to build the park, they need to determine if residents in the area want one. They mail a survey to residents within 1 mile of the proposed location and find that 78% of residents who responded are in favor of building the new park. They'll find confidence intervals to project what all residents in the area may think of the new park.

Confidence Interval for Proportion:	Confidence Interval for Mean:
$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$
$\hat{p}$ = sample proportion in decimal	$\bar{x}$ = sample mean
z = z-score for probability from $\boxed{\frac{1-c\%}{2}}$	σ = population standard deviation
n = sample size	z = z-score for probability of $\frac{1-c\%}{2}$
Margin of error = $z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ doubling makes interval width	n = sample size
p = true population proportion	Margin of error = $z\frac{\sigma}{\sqrt{n}}$
	μ = true population mean

$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Examples: $n=1150$ $\hat{p}=0.846$

1. A survey of 1150 people found that 84.6% of respondents believed a toilet paper roll should roll over (not under). Construct the following confidence intervals for the proportion of people whose toilet paper rolls over and state the margin of error for each.

$\frac{1-c\%}{2}$

90%

$\frac{1-0.90}{2}=0.05 \rightarrow z=-1.65$

$CI=0.846 \pm 1.65\sqrt{\frac{0.846(1-0.846)}{1150}}$

$CI=0.846 \pm 0.018$ ← margin of error (M.E)

$CI=(0.828, 0.864)$

95%

$\frac{1-0.95}{2} \rightarrow 0.025$

$z=-1.96$

$CI=0.846 \pm 1.96\sqrt{\frac{0.846(1-0.846)}{1150}}$

$=0.846 \pm 0.021$ ← (M.E)

$CI=(0.825, 0.867)$

2. In a sample of 2500 people, 770 people separate their Skittle's by color before eating them. Construct an 85% confidence interval for the proportion of people who "taste the rainbow" with colors separated.

$n=2500$

$\hat{p}=\frac{770}{2500}=0.308$

$\frac{1-0.85}{2}=0.075$

$z=-1.44$

$CI=0.308 \pm 1.44\sqrt{\frac{0.308(1-0.308)}{2500}}$

$CI=0.308 \pm 0.01329$

$\boxed{(0.295, 0.321)}$

3. A recent survey of 133 Milton students found their average daily screen time is 5.402 hours. If the population standard deviation is 1.565 hours, construct the following confidence intervals for the average daily screen time for all Milton students and state the margin of error for each.

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

$n=133$

$\bar{x}=5.402$

$\sigma=1.565$

80%

$\frac{1-0.8}{2}=0.1$ $z=-1.28$

$CI=5.402 \pm 1.28\left(\frac{1.565}{\sqrt{133}}\right)$

$CI=5.402 \pm 0.174$

$\boxed{(5.228, 5.576)}$

99%

$\frac{1-0.99}{2}=0.005$

$z=-2.58$

$CI=5.402 \pm 2.58\left(\frac{1.565}{\sqrt{133}}\right)$

$CI=5.402 \pm 0.350$

$\boxed{(5.052, 5.752)}$

4. A recent survey found that Milton students get an average of 6.303 hours of sleep each night. Given the sample size of 540 students and population standard deviation of 0.926 hours, construct an 88% confidence interval for the average amount of sleep by Milton students.

$\bar{x}=6.303$

$n=540$

$\sigma=0.926$

$\frac{1-0.88}{2}=0.06$

$z=-1.56$

$CI=6.303 \pm 1.56\left(\frac{0.926}{\sqrt{540}}\right)$

$=6.303 \pm 0.0621$

$\boxed{(6.241, 6.365)}$

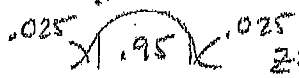
9.09 Homework:

Key

1. High school students who take the SAT Mathematics exam a second time generally score higher than on their first try. The change in score has a normal distribution with standard deviation $\sigma = 50$. A random sample of 1000 students gain an average of 22 points on their second try. $\bar{x}$

a) Construct a 95% confidence interval for the mean score gain μ in the population.

$$95\%\,CI = \bar{x} \pm z \frac{\sigma}{\sqrt{n}} = 22 \pm (1.96)\frac{50}{\sqrt{1000}} = \boxed{(18.901, 25.099)}$$



z = look for .025 in chart (or .975)

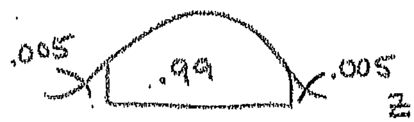
b) Construct a 90% confidence interval for μ.

$$90\%\,CI = 22 \pm (1.65)\frac{50}{\sqrt{1000}} = \boxed{(19.391, 24.609)}$$

z = look for .05 in chart (or .95)

c) Construct a 99% confidence interval for μ.

$$99\%\,CI = 22 \pm (2.58)\frac{50}{\sqrt{1000}} = \boxed{(17.921, 26.079)}$$



z = look for .005 in chart (or .995)

d) What is the margin of error for each of the confidence intervals calculated above?

in part a) $\pm 1.96\left(\frac{50}{\sqrt{1000}}\right)$ or $22 - 18.901 = \boxed{\pm 3.099}$

in part b) $\pm 1.65 \cdot \frac{50}{\sqrt{1000}}$ or $22 - 19.391 = \boxed{\pm 2.609}$

in part c) $\pm 2.58 \cdot \frac{50}{\sqrt{1000}}$ or $22 - 17.921 = \boxed{\pm 4.079}$

2. The National Survey of Student Engagement found that 87% of students report that their peers at least "sometimes" copy information from the Internet in their reports without citing the source. Assume that the sample size is 400. Construct an 88% confidence interval and find the margin of error.

$\hat{p} = .87$ ← proportion from a sample

$n = 400$

$$88\%\,CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .87 \pm 1.55\sqrt{\frac{.87(.13)}{400}} = .87 \pm .026$$

z = look for .06 in chart (or .94)

$\boxed{(.844, .896)}$

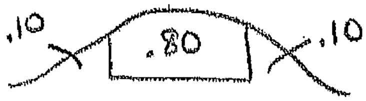
$$\begin{array}{r} 1.00 \\ -\ .88 \\ \hline .12 \div 2 \end{array}$$

$\boxed{\text{Margin of Error: } \pm .026}$

3. A recent survey of 1366 adults found that 1127 of those respondents like hot sauce on their eggs. Construct an 80% confidence interval.

n_2

$\hat{p} = \frac{1127}{1366}$

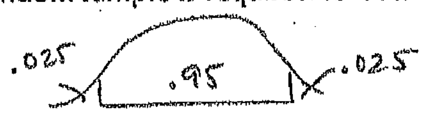


z = look for .10 on chart (or .90)

$$80\%\,CI: \frac{1127}{1366} \pm 1.28\sqrt{\frac{\frac{1127}{1366}\left(\frac{239}{1366}\right)}{1366}} = .825 \pm .013 = \boxed{(.812, .838)}$$

4. A national opinion poll found that 44% of all American adults agree that parents should be given vouchers good for education at any public or private school of their choice. The result was based on a small sample. How large of a random sample is required to obtain a margin of error of 0.03 in a 95% confidence interval?

$$ME = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$



$z = 1.96$

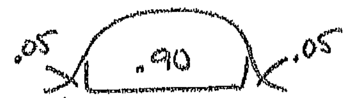
$$\frac{.03}{1.96} = \frac{1.96}{1.96}\sqrt{\frac{.44(.56)}{n}} \Rightarrow \left(\frac{.03}{1.96}\right)^2 = \left(\sqrt{\frac{.44(.56)}{n}}\right)^2 \Rightarrow \left(\frac{.03}{1.96}\right)^2 = \frac{.44(.56)}{n}$$

$$n = \frac{.44(.56)}{\left(\frac{.03}{1.96}\right)^2} \text{ or } \left(\frac{1.96}{.03}\right)^2(.44)(.56) = 1051.745$$

$\boxed{\text{sample 1052 adults}}$

5. A radio talk show invites listeners to enter a dispute about a proposed pay increase for city council members. "What yearly pay do you think council members should get? Call us with your number." In all, 958 people call. The mean pay they suggest is \$8740 per year, and the standard deviation of the responses is \$1125. Calculate a 90% confidence interval for the mean pay μ that all citizens would propose for council members.

$n = 958 \qquad \bar{x} = \$8740 \qquad \sigma = \$1125$



z @ .05 probability is 1.65

$$90\%\,CI = \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 8740 \pm 1.65\left(\frac{1125}{\sqrt{958}}\right)$$

$$= 8740 \pm 59.973$$

$$= \boxed{(\$8680.03, \$8799.97)}$$

9.10 More Confidence Intervals Practice $\hat{p} = \frac{130}{1100} \approx 0.118$ $CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

1. A town takes a poll of its residents to find out how many people would be willing to pay a new tax to repair the town's sidewalks. Out of 1100 people polled, only 130 said that they would be willing to pay.
   (a) Find a 90% confidence interval for the proportion of the whole town that would be willing to pay the extra tax.

$\frac{1-c\%}{2} \rightarrow \frac{1-0.9}{2} = 0.05$ $z = -1.65$

$CI = 0.118 \pm 1.65\sqrt{\frac{0.118(1-0.118)}{1100}}$

$CI = 0.118 \pm 0.01604$

$\boxed{(0.1020, 0.1340)}$

   (b) If 15,000 people live in this town, then we are 90% confident that between 1529 and 2011 will be willing to pay this tax. (Fill in with numbers of people.)

$0.1020(15000) = 1529.26$ $0.1340(15000) = 2010.74$

2. A company that produces white bread is concerned about the distribution of the amount of sodium in its bread. The company takes a simple random sample of 100 slices of bread and computes the sample mean to be 103 milligrams of sodium per slice.
   (a) Construct a 99% confidence interval for the unknown mean sodium level assuming that the population standard deviation is 10 milligrams.

$n = 100$, $\bar{x} = 103$, $\sigma = 10$

$CI = \bar{x} + z\left(\frac{\sigma}{\sqrt{n}}\right)$

$CI = 103 \pm 2.58\left(\frac{10}{\sqrt{100}}\right)$

$CI = 103 \pm 2.58$

$\boxed{(100.42\text{ mg}, 105.58\text{ mg})}$

$\frac{1-c\%}{2}$ $\frac{1-0.99}{2}$ 0.005 $z = -2.58$

   (b) Interpret the 99% confidence interval found in (a).

We are 99% confident that the mean sodium level per slice is between 100.42mg and 105.58mg.

3. You work for a consumer advocate agency and want to find the mean repair cost of a washing machine. In the past, the standard deviation of the cost of repairs for washing machines has been $17.50. As part of your study, you randomly select 40 repair costs and find the mean to be $100.00.
   (a) Calculate a 85% confidence interval for the population mean.

$\sigma = 17.50$, $n = 40$, $\bar{x} = 100.00$

$CI = \bar{x} \pm z\left(\frac{\sigma}{\sqrt{n}}\right)$

$CI = 100 \pm 1.44\left(\frac{17.50}{\sqrt{40}}\right)$

$CI = 100 \pm 3.9844$

$\boxed{(\$96.02, \$103.98)}$

$\frac{1-0.85}{2}$ 0.075 $z = -1.44$

   (b) Interpret the interval found in (a).

We are 85% confident that the avg. repair cost for washing machine is between $96.02 and $103.98

4. You want to estimate the mean fuel efficiency for all Ford Focus cars with 99% confidence and a margin of error of no more than 1 mile per gallon. Preliminary data suggests that $\sigma = 2.4$ miles per gallon is a reasonable standard deviation for all cars of this make and model. How large a sample do you need?

M.E. $\leq$ 1 mpg, $\sigma = 2.4$, $n = \_\_?$

$z_{99} = -2.58$

$M.E. = z\left(\frac{\sigma}{\sqrt{n}}\right)$

$1 = \frac{-2.58(2.4)}{\sqrt{n}}$

$(\sqrt{n})^2 = (-6.192)^2$

$n = 38.341$

$38.341 \leq n$

$n \geq 38.341$

sample of at least 39 cars of this model.

5. The actual time it takes to cook a 10-pound turkey is a Normal random variable with a mean of 2.8 hours and a standard deviation of 0.24 hours. Suppose that a random sample of 35 10-pound turkeys is taken.

   (a) What is the probability that a randomly selected 10-pound turkey will take less than 3.1 hours to cook?

$P(x \le 3.1)$ | $z = \frac{x - \bar{x}}{\sigma}$ | $\bar{x} = 2.8$, $x = 3.1$, $\sigma = 0.24$ | $z = \frac{3.1 - 2.8}{0.24}$, $z = 1.25$ | $\boxed{P(x \le 3.1) = 0.8944}$

   (b) What is the probability that the average cooking time of a 10-pound turkey will take between 2.7 and 2.95 hours to cook?

$z = \frac{2.7 - 2.8}{0.24}$, $z = -0.42 \rightarrow 0.3372$ | $z = \frac{2.95 - 2.8}{0.24}$, $z = 0.63 \rightarrow 0.7357$ | $P(2.7 \le x \le 2.95) = 0.7357 - 0.3372 = \boxed{0.3985}$

   (c) Given that an average of 2.9 hours was found for a sample of 35 turkeys, calculate an 80% confidence interval for the average cooking time of a 10-pound turkey.

$\bar{x} = 2.9$, $\sigma = 0.24$, $n = 35$ | $\frac{1 - c\%}{2} \rightarrow \frac{1 - 0.80}{2} \rightarrow 0.1 \rightarrow z = -1.28$ | $CI = \bar{x} \pm z\left(\frac{\sigma}{\sqrt{n}}\right)$, $CI = 2.9 \pm 1.28\left(\frac{0.24}{\sqrt{35}}\right)$ | $CI = 2.9 \pm 0.0519$, $\boxed{(2.848 \text{ hrs}, 2.952 \text{ hrs})}$

6. Weight Watchers takes a poll of 250 members and finds that 95 of them include exercise with their diet program, while the rest do not. Find a 99% confidence interval for the proportion of all members that do exercise.

$\hat{p} = \frac{95}{250} = 0.38$, $n = 250$ | $\frac{1 - 0.99}{2} \rightarrow 0.005$, $z = -2.58$ | $CI = \hat{p} \pm z\left(\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$, $CI = 0.38 \pm 2.58\sqrt{\frac{0.38(1-0.38)}{250}}$ | $CI = 0.38 \pm 0.0792$, $\boxed{(0.301, 0.459)}$

7. A magazine polls 395 readers and finds that 95 of them bought the magazine in the store, while the rest had a subscription. Find an 87% confidence interval for the proportion of all readers who have a subscription.

$\hat{p} = \frac{300}{395} = 0.759$, $n = 395$ | $\frac{1 - c\%}{2} \rightarrow \frac{1 - 0.87}{2}$, 0.065, $z = -1.51$ | $CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, $CI = 0.759 \pm 1.51\sqrt{\frac{0.759(1-0.759)}{395}}$ | $CI = 0.759 \pm 0.325$, $\boxed{(0.727, 0.791)}$

8. Fill in the blanks with one of the following for how the margin of error is impacted: *increases, decreases,* or *stays the same* where $ME = z\left(\frac{\sigma}{\sqrt{n}}\right)$.

As the sample size (n) increases, the margin of error (ME) decreases (dividing by larger #'s)

As the confidence level (C%) increases, the margin of error (ME) increases (wider window, $z\uparrow$)

As the standard deviation (σ) increases, the margin of error (ME) increases (more variability in the data)

9.12 Review

Date: Key

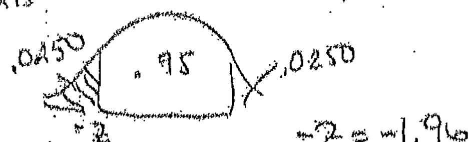
1) A sample of 16 students finds that the average age is 22 years. All student ages have a standard deviation of 6 years. Construct a 95% confidence interval for the average age of students.

$\bar{x} = 22$ yrs

$n = 16$ students

$\sigma = 6$ yrs

$$CI = \bar{x} \pm z \frac{\sigma}{\sqrt{n}} = 22 \pm 1.96 \frac{6}{\sqrt{16}} = \boxed{(19.06 \text{ yrs}, 24.94 \text{ yrs})}$$



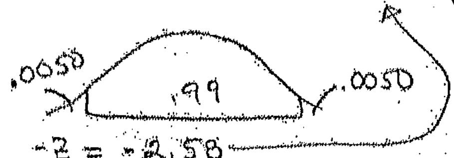
2) Construct a 99% confidence interval for the population mean lifetime of fluorescent lightbulbs. Assume the population has a Normal distribution with a standard deviation of 31 hours. A sample of 16 fluorescent light bulbs have a mean life of 645 hours.

$\sigma = 31$ hrs

$n = 16$ bulbs

$\bar{x} = 645$ hrs

$$CI = 645 \pm \underline{2.58} \frac{31}{\sqrt{16}} = \boxed{(625.005 \text{ hrs}, 664.995 \text{ hrs})}$$



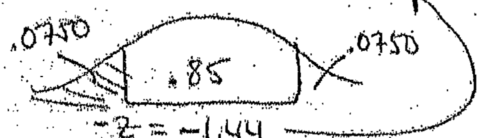
3) A sample of 100 bean cans showed an average weight of 13 ounces. If all been cans have a standard deviation of 0.8 ounces, construct an 85% confidence interval for the mean weight of the population.

$n = 100$ bean cans

$\bar{x} = 13$ oz

$\sigma = 0.8$ oz

$$CI = 13 \pm \underline{1.44} \frac{0.8}{\sqrt{100}} = \boxed{(12.885 \text{ oz}, 13.115 \text{ oz})}$$



4) A researcher wants to know the percentage of Columbus residents who would favor a two cent increase in the gasoline tax to fund road repairs. A random sample of 900 residents finds 278 favor the increase.

n x

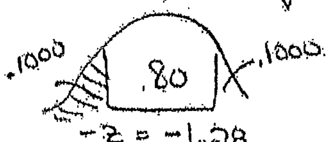
a) Specify the parameter and statistic for this problem.

Parameter: Proportion of all residents in favor of paying 2¢ increase, p

Statistic: $\hat{p}$ $\frac{278}{900} = .309$ proportion of sample willing to pay 2¢ increase.

b) Find an 80% confidence interval for the parameter.

$$CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .309 \pm 1.28\sqrt{\frac{.309(1-.309)}{900}}$$



$$= \boxed{(0.289, 0.329)}$$

5) A random sample of female college students has a mean height of 64.5 inches, which is greater than the 63-inch mean height of all adult American women. Determine if each bold-faced number is a parameter or a statistic.

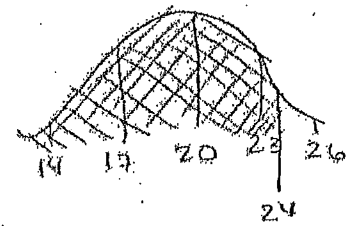
$\bar{x} = 64.5''$ is a statistic because it came from a sample of female college students

$\mu = 63''$ is a parameter because it describes all American women.

6) In a certain Normal distribution of scores, the mean is 20 and the standard deviation is 3. $N(20,3)$

a. Find the z-score corresponding to a score of 24.

$z = \frac{24-20}{3} = \frac{4}{3} = 1.33$



b. Find the percentile for a score of 24.

$P(x \le 24) = P(z \le 1.33) = .9082$

$\boxed{90.82\text{ \%ile}}$

7) The Jackson triplets, Jenny, John, and James are in different math classes at City High. On their final exams, Jenny scored 82 on a test with a mean of 76 and a standard deviation of 7.5; John scored 77 on a test with a mean of 72 and a standard deviation of 10.5; and James scored 78 on a test with a mean of 66 and a standard deviation of 10.5. Who had the best z-score and what does this say about that triplet in relation to their test score in relation to their peers?

Jenny $= \frac{82-76}{7.5} = \frac{6}{7.5} = 0.8$ stdev above class avg.

John $= \frac{77-72}{10.5} = \frac{5}{10.5} = 0.476$ stdev above class avg

James $= \frac{78-66}{10.5} = \frac{12}{10.5} = 1.143$ std dev above class avg

James has the highest z-score meaning he did better than his class peers.

8) Some test scores were Normally distributed with a mean of 55 and a standard deviation of 5. Approximately what percentage of the scores lie between 45 and 65?

$N(55,5)$

According to the empirical rule, about $\boxed{95\%}$.

.9544 so 95.44% according to z chart.

9) The heights of a certain group of adult parrots were found to be Normally distributed. The mean height is 36 cm with a standard deviation of 8 cm. In a group of 1000 of these birds, how many would be more than 28 cm tall?

$N(36,8)$ $\quad P(x > 28) = 50\% + 34\% = 84\%$

$n = 1000$

84% of 1000 = $\boxed{840 \text{ birds}}$

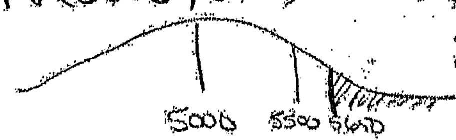
10) The life expectancy (in hours) of a fluorescent tube is normally distributed with a mean of 5000 and a standard deviation of 500. Find the probability that a tube lasts for at least 5650 hours.

$N(5000, 500)$ $\quad P(x \ge 5650) = P(z \ge 1.30) = 1 - P(z \le 1.30)$

$= 1 - .9032$

$= \boxed{.0968}$

$z = \frac{5650-5000}{500}$

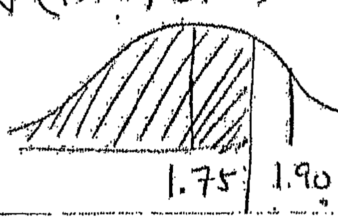


11) A potato chip company sells a small bag of chips that has a mean volume of 1.75 ounces with a standard deviation of 0.15 ounces. What is the probability that a bag contains at most 1.84 ounces?

Normal

$N(1.75, 0.15)$ $\quad P(x \le 1.84\text{ oz}) = P(z \le 0.60) = \boxed{0.7257}$

$z = \frac{1.84-1.75}{0.15} = 0.60$

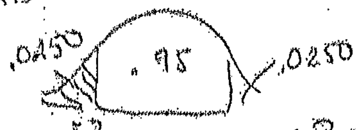


9.12 Review Date: Key

1) A sample of 16 students finds that the average age is 22 years. All student ages have a standard deviation of 6 years. Construct a 95% confidence interval for the average age of students.

$\bar{X} = 22$ yrs
$n = 16$ students
$\sigma = 6$ yrs

$CI = \bar{X} \pm z \frac{\sigma}{\sqrt{n}} = 22 \pm 1.96 \frac{6}{\sqrt{16}} = \boxed{(19.06 \text{ yrs}, 24.94 \text{ yrs})}$

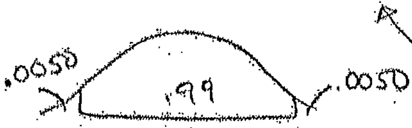


$-z = -1.96$

2) Construct a 99% confidence interval for the population mean lifetime of fluorescent lightbulbs. Assume the population has a Normal distribution with a standard deviation of 31 hours. A sample of 16 fluorescent light bulbs have a mean life of 645 hours.

$\sigma = 31$ hrs
$n = 16$ bulbs
$\bar{x} = 645$ hrs

$CI = 645 \pm \underline{2.58} \frac{31}{\sqrt{16}} = \boxed{(625.005 \text{ hrs}, 664.995 \text{ hrs})}$

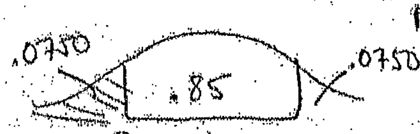


$-z = -2.58$

3) A sample of 100 bean cans showed an average weight of 13 ounces. If all been cans have a standard deviation of 0.8 ounces, construct an 85% confidence interval for the mean weight of the population.

$n = 100$ bean cans
$\bar{X} = 13$ oz
$\sigma = 0.8$ oz

$CI = 13 \pm \underline{1.44} \frac{0.8}{\sqrt{100}} = \boxed{(12.885 \text{ oz}, 13.115 \text{ oz})}$



$-z = -1.44$

4) A researcher wants to know the percentage of Columbus residents who would favor a two cent increase in the gasoline tax to fund road repairs. A random sample of 900 residents finds 278 favor the increase.
n x

a) Specify the parameter and statistic for this problem.

Parameter: Proportion of all residents in favor of paying 2¢ increase, p

Statistic: $\hat{p} = \frac{278}{900} = .309$ proportion of sample willing to pay 2¢ increase.

b) Find an 80% confidence interval for the parameter.

$CI = \hat{p} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .309 \pm 1.28 \sqrt{\frac{.309(1-.309)}{900}}$

$= \boxed{(0.289, 0.329)}$

$-z = -1.28$

5) A random sample of female college students has a mean height of 64.5 inches, which is greater than the 63-inch mean height of all adult American women. Determine if each bold-faced number is a parameter or a statistic.

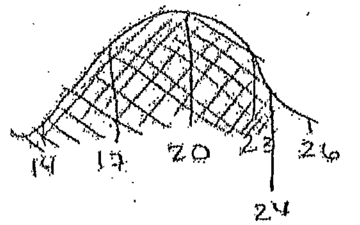
$\bar{X} = 64.5"$ is a statistic because it came from a sample of female college students

$\mu = 63"$ is a parameter because it describes all American women.

6) In a certain Normal distribution of scores, the mean is 20 and the standard deviation is 3. $N(20,3)$

a. Find the z-score corresponding to a score of 24.

$z = \frac{24-20}{3} = \frac{4}{3} = 1.33$



b. Find the percentile for a score of 24.

$P(x \le 24) = P(z \le 1.33) = .9082$

$\boxed{90.82\text{ \%ile}}$

7) The Jackson triplets, Jenny, John, and James are in different math classes at City High. On their final exams, Jenny scored 82 on a test with a mean of 76 and a standard deviation of 7.5; John scored 77 on a test with a mean of 72 and a standard deviation of 10.5; and James scored 78 on a test with a mean of 66 and a standard deviation of 10.5. Who had the best z-score and what does this say about that triplet in relation to their test score in relation to their peers?

Jenny $= \frac{82-76}{7.5} = \frac{6}{7.5} = 0.8$ stdev above class avg.

John $= \frac{77-72}{10.5} = \frac{5}{10.5} = 0.476$ stdev above class avg

James $= \frac{78-66}{10.5} = \frac{12}{10.5} = 1.143$ std dev above class avg

James has the highest z-score meaning he did better than his class peers.

8) Some test scores were Normally distributed with a mean of 55 and a standard deviation of 5. Approximately what percentage of the scores lie between 45 and 65?

$N(55,5)$

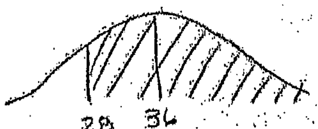
According to the empirical rule, about $\boxed{95\%}$.

.9544 so 95.44% according to z chart.

9) The heights of a certain group of adult parrots were found to be Normally distributed. The mean height is 36 cm with a standard deviation of 8 cm. In a group of 1000 of these birds, how many would be more than 28 cm tall?

$N(36,8)$ $\quad P(x > 28) = 50\% + 34\% = 84\%$

$n = 1000$



84% of 1000 = $\boxed{840 \text{ birds}}$

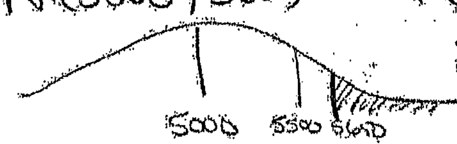
10) The life expectancy (in hours) of a fluorescent tube is normally distributed with a mean of 5000 and a standard deviation of 500. Find the probability that a tube lasts for at least 5650 hours.

$N(5000, 500)$ $\quad P(x \ge 5650) = P(z \ge 1.30) = 1 - P(z \le 1.30)$

$z = \frac{5650-5000}{500}$

$= 1 - .9032$

$= \boxed{.0968}$

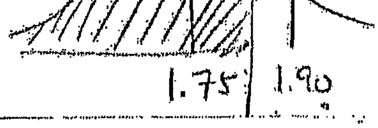


11) A potato chip company sells a small bag of chips that has a mean volume of 1.75 ounces with a standard deviation of 0.15 ounces. What is the probability that a bag contains at most 1.84 ounces?

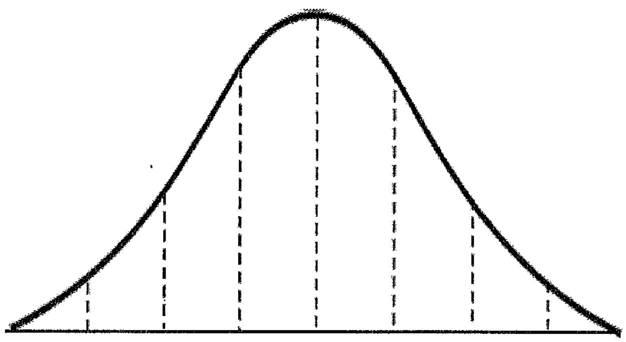
Normal

$N(1.75, 0.15)$ $\quad P(x \le 1.84\text{ oz}) = P(z \le 0.60) = \boxed{0.7257}$

$z = \frac{1.84-1.75}{0.15} = 0.60$



Unit 9.12b Test Review WS #2 - Statistics

<table>
<tr><td colspan="2">Directions: Draw and label normal distribution curves, then answer the questions.</td></tr>
<tr><td rowspan="4">1. The weights of the 50 football players are normally distributed with a mean of 178 pounds and a standard deviation of 8 pounds.
</td><td>a) What percent of the players weigh between 178 lbs and 194 lbs?</td></tr>
<tr><td>b) What is the probability that a player weighs at most 170 lbs?</td></tr>
<tr><td>c) What is the probability that a player weighs less than 162 lbs or greater than 194 lbs?</td></tr>
<tr><td>d) How many players weight between 170 lbs and 186 lbs?</td></tr>
</table>

2. Identify the population and the sample:

a) A survey of 1353 American households found that 18% of the households own a computer.

b) A recent survey of 2625 elementary school children found that 28% of the children could be classified obese.

c) The average weight of every sixth person entering the mall within 3 hour period was 146 lb.

3. Determine whether the numerical value is a parameter or a statistics (and explain):

a) A recent survey by the alumni of a major university indicated that the average salary of 10,000 of its 300,000 graduates was 125,000.

b) The average salary of all assembly-line employees at a certain car manufacturer is $33,000.

c) The average late fee for 360 credit card holders was found to be $56.75.

4. For the studies described, identify the population, sample, population parameters, and sample statistics:

a) In a USA Today Internet poll, readers responded voluntarily to the question "Do you consume at least one caffeinated beverage every day?"

b) Astronomers typically determine the distance to galaxy (a galaxy is a huge collection of billions of stars) by measuring the distances to just a few stars within it and taking the mean (average) of these distance measurements.

5) **The length of a certain fish species is normally distributed with a mean of 15 cm. If a fish in this species is 18.8 cm with a *z*-score of 1.9, what is the standard deviation?**

Use for questions 39-42: The shoe sizes of the 36 students in Samantha's PE class are normally distributed with a mean of 8.5 and a standard deviation of 1.5.

39. What percent of the students have a shoe size between 7 and 11?	**40.** What is the probability that a student will have a maximum shoe size of 9.5?
41. Approximately how many students wear at least a size 6?	**42.** Approximately how many students wear a shoe size between 8 and 10?

Answers:

2. a) population: all American households

sample: collection of 1353 American households surveyed

b) population: all elementary school children

sample: collection of 2625 elementary school children surveyed

c) population: all people entering the mall within the assigned 3 hour period

sample: every 6th person entering the mall within the 3 hour period

3. a) statistic – part of 300,000 graduates are surveyed

b) parameter – all assembly-line employees were included in the study

c) statistic – 360 credit cards were examined (not all)

4. a) population: all readers of USA Today;

sample: volunteers that responded to the survey;

population parameter: percent who have at least one caffeinated drink among all readers of USA Today;

sample statistic: percent who have at least one caffeinated drink among those who responded to the survey

b) population: all starts in the galaxy;

sample: the few stars selected for measurements;

population parameter: mean (average) of distances between all stars and Earth;

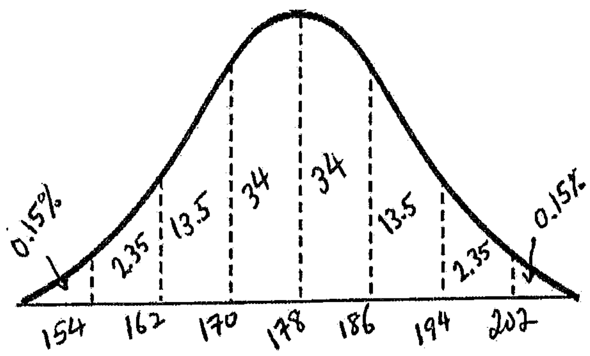
sample statistics: mean of distances between the stars in the sample and Earth

Key

Unit 9.12b Test Review WS #2 - Statistics

Directions: Draw and label normal distribution curves, then answer the questions.

1. The weights of the 50 football players are normally distributed with a mean of 178 pounds and a standard deviation of 8 pounds.



a) What percent of the players weigh between 178 lbs and 194 lbs?

47.5%

b) What is the probability that a player weighs at most 170 lbs?

$P(x \leq 170) = 16\%$

c) What is the probability that a player weighs less than 162 lbs or greater than 194 lbs?

$2.5\% + 2.5\% = 5\%$

d) How many players weight between 170 lbs and 186 lbs?

68% of 50 players | $50(0.68) =$ $\boxed{\text{34 players}}$

2. Identify the population and the sample:

a) A survey of 1353 American households found that 18% of the households own a computer.

population: all American households
sample: collection of 1353 households surveyed

b) A recent survey of 2625 elementary school children found that 28% of the children could be classified obese.

population: all elementary school children
sample: collection of 2625 children surveyed

c) The average weight of every sixth person entering the mall within 3 hour period was 146 lb.

population: all people entering mall in 3 hr. period
sample: every 6th person in 3 hr. period.

3. Determine whether the numerical value is a parameter or a statistics (and explain):

a) A recent survey by the alumni of a major university indicated that the average salary of 10,000 of its 300,000 graduates was 125,000.

statistic - part of 300,000 graduates

b) The average salary of all assembly-line employees at a certain car manufacturer is $33,000.

parameter - all assembly-line employees were included in the study.

c) The average late fee for 360 credit card holders was found to be $56.75.

statistic - 360 (but not all) credit cards were examined

4. For the studies described, identify the population, sample, population parameters, and sample statistics:

a) In a USA Today Internet poll, readers responded voluntarily to the question "Do you consume at least one caffeinated beverage every day?" population: all Readers of USA today

Sample: volunteers responding to survey

population parameter: percent of readers who have at least one caffeinated drink among all readers

sample statistic: percent of caffeine drinkers among those who responded.

b) Astronomers typically determine the distance to galaxy (a galaxy is a huge collection of billions of stars) by measuring the distances to just a few stars within it and taking the mean (average) of these distance measurements.

population: all stars in galaxy

Sample: the selected stars chosen for measurement

population parameter: mean (avg) of distances b/t all stars and Earth

sample statistic: mean of distances found in the sample of stars.

5) **The length of a certain fish species is normally distributed with a mean of 15 cm. If a fish in this species is 18.8 cm with a z-score of 1.9, what is the standard deviation?**

$$z = \frac{x - \bar{x}}{\sigma}$$

$\bar{x} = 15$

$x = 18.8$

$z = 1.9$

$\sigma = \_\_?$

$$\frac{1.9}{1} = \frac{18.8-15}{\sigma}$$

$1.9\sigma = 3.8$

$$\sigma = \frac{3.8}{1.9}$$

$\boxed{\sigma = 2}$

Use for questions 39-42: The shoe sizes of the 36 students in Samantha's PE class are normally distributed with a mean of 8.5 and a standard deviation of 1.5.

39. What percent of the students have a shoe size between 7 and 11?

$z = \frac{7-8.5}{1.5} = -1 \rightarrow 0.1587$

$z = \frac{11-8.5}{1.5} = 1.667 \rightarrow 0.9525$

$P(7 \leq x \leq 11)$
$= 0.9525 - 0.1587$
$= \boxed{0.7938}$

40. What is the probability that a student will have a maximum shoe size of 9.5?

$z = \frac{9.5-8.5}{1.5} = 0.67 \rightarrow 0.7486$

$P(x \leq 9.5) = \boxed{0.7486}$

41. Approximately how many students wear at least a size 6?

$$z = \frac{6-8.5}{1.5} = -1.67 \rightarrow 0.0475$$

$P(x \geq 6) = 1 - 0.0475$
$= \boxed{0.9525}$

42. Approximately how many students wear a shoe size between 8 and 10?

$$z_8 = \frac{8-8.5}{1.5} = -0.33 \rightarrow 0.3707$$

$$z_{10} = \frac{10-8.5}{1.5} = 1 \rightarrow 0.8413$$

$0.8413 - 0.3707 = 0.4706$

$0.4706(36) = 16.94 \boxed{\approx 17 \text{ students}}$

Unit 9 Test Review WS #3 - Statistics

6.10 **Apartment rental rates.** You want to rent an unfurnished one-bedroom apartment for next semester. The mean monthly rent for a random sample of 10 apartments advertised in the local newspaper is \$540. Assume that the standard deviation is \$80. Find a 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent in this community.

Confidence Intervals:

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$$

Compare the margin of error for intervals with 90, 95, and 99% confidence:

Suppose you desire a 90% confidence interval with a *width* of no more than \$50. What sample size is needed?

2)

A U.S. Coast Guard survey of 300 small boats in the Cape Cod area found 120 in violation of one or more safety regulations. Give a 99.8% confidence estimate for p, the proportion of all unsafe small boats.

3) The birth process of a newly discovered mammal is being studied, and the lengths of 18 observed pregnancies have been recorded. The mean gestation period was 97.3 days with s = 2.2 days. Find a 95% confidence interval for the mean time of pregnancy for this mammal.

4) The *New York Times* and CBS News conducted a nationwide poll of 1048 randomly selected 13- to 17-year-olds. We can consider the sample to be a SRS.

) Of these 1048 teenagers, 692 had a television in their room. Give a 95% confidence interval for the proportion of all people in this age group who had a TV in their room at the time of the poll

5)

Find n: A researcher wants to determine the 99% confidence interval for the mean number if hours per week that adults spend doing community service. How large of a sample should the researcher select so that the estimate will be within 1 hour of the population mean? Assume that the standard deviation for hours spent per week by adults doing community service is 3.
Answer: $n = 35$

Unit 9 Test Review WS #3 - Statistics

Key

6.10 Apartment rental rates. You want to rent an unfurnished one-bedroom apartment for next semester. The mean monthly rent for a random sample of 10 apartments advertised in the local newspaper is $540. Assume that the standard deviation is $80. Find a 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent in this community.

$\bar{x} = 540$, $n = 10$, $\sigma = 80$

$\frac{1-c\%}{2}$, $\frac{1-0.95}{2}$, $0.025 \rightarrow z = -1.96$

$CI = 540 \pm 1.96\left(\frac{80}{\sqrt{10}}\right)$

$CI = 540 \pm 49.585$

$(490.41, 589.58)$

Confidence Intervals:

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$$

Compare the margin of error for intervals with 90, 95, and 99% confidence:

90%: $\frac{1-0.9}{2} \rightarrow 0.05$, $z = -1.64$, $M.E. = -1.64\left(\frac{80}{\sqrt{10}}\right)$, $M.E. = 41.489$

95%: $M.E. = 49.585$

99%: $\frac{1-0.99}{2} = 0.005$, $z = -2.57$, $M.E. = 2.57\left(\frac{80}{\sqrt{10}}\right) = 65.0164$

*Margin of Error increases with higher confidence percentage.

Suppose you desire a 90% confidence interval with a *width* of no more than $50. What sample size is needed?

$M.E. = 50$, $90\% \rightarrow 0.05 \rightarrow$ $z = -1.64$

$M.E. = z\left(\frac{\sigma}{\sqrt{n}}\right)$

$50 = 1.64\left(\frac{80}{\sqrt{n}}\right)$

$50\sqrt{n} = 131.2$

$\sqrt{n} = 2.624$

$n = 6.88$

$\boxed{n \geq 7}$

2)

A U.S. Coast Guard survey of 300 small boats in the Cape Cod area found 120 in violation of one or more safety regulations. Give a 99.8% confidence estimate for p, the proportion of all unsafe small boats.

$n = 300$

$\hat{p} = \frac{120}{300} = 0.4$

$\frac{1-0.998}{2} = 0.001 \rightarrow z = 3.09$

$CI = 0.4 \pm 3.09\sqrt{\frac{0.4(1-0.4)}{300}}$

$CI = 0.4 \pm 0.0874$

$\boxed{(0.3126, 0.4874)}$

3) The birth process of a newly discovered mammal is being studied, and the lengths of 18 observed pregnancies have been recorded. The mean gestation period was 97.3 days with s = 2.2 days. Find a 95% confidence interval for the mean time of pregnancy for this mammal.

$\bar{x} = 97.3$

$\sigma = 2.2$

$n = 18$

$\frac{1-c\%}{2} \rightarrow \frac{1-0.95}{2} \rightarrow 0.025$

$z = -1.96$

$CI = 97.3 \pm 1.96\left(\frac{2.2}{\sqrt{18}}\right)$

$CI = 97.3 \pm 1.0163$

$\boxed{(96.284, 98.316)}$

4) The *New York Times* and CBS News conducted a nationwide poll of 1048 randomly selected 13- to 17-year-olds. We can consider the sample to be a SRS. Of these 1048 teenagers, 692 had a television in their room. Give a 95% confidence interval for the proportion of all people in this age group who had a TV in their room at the time of the poll

$n = 1048$

$\hat{p} = \frac{692}{1048} = 0.6603$

$\frac{1-c\%}{2} \rightarrow \frac{1-0.95}{2} \rightarrow z = 1.96$

$\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$CI = 0.6603 \pm 1.96\sqrt{\frac{0.6603(1-0.6603)}{1048}}$

$CI = 0.6603 \pm 0.0287$

$\boxed{(0.6316, 0.689)}$

5)

Find n: A researcher wants to determine the 99% confidence interval for the mean number if hours per week that adults spend doing community service. How large of a sample should the researcher select so that the estimate will be within 1 hour of the population mean? Assume that the standard deviation for hours spent per week by adults doing community service is 3.

$\frac{1-c\%}{2} \rightarrow \frac{1-0.99}{2} \rightarrow 0.005 \rightarrow z = -2.57$

$n = \_\_?$

$M.E. = 1$

$\sigma = 3$

$M.E. = z\left(\frac{\sigma}{\sqrt{n}}\right)$

$1 = -2.57\left(\frac{3}{\sqrt{n}}\right)$

$\frac{1}{1} = \frac{-2.57(3)}{\sqrt{n}}$

$\sqrt{n} = -7.71$

$n = 59.4441$

$\boxed{n = 60}$