Algebraic AIME Word Problems

1. 2009 AIME II

Before starting to paint, Bill had 130 ounces of blue paint, 164 ounces of red paint, and 188 ounces of white paint. Bill painted four equally sized stripes on a wall, making a blue stripe, a red stripe, a white stripe, and a pink stripe. Pink is a mixture of red and white, not necessarily in equal amounts. When Bill finished, he had equal amounts of blue, red, and white paint left. Find the total number of ounces of paint Bill had left.

After pink stripe is drawn, all 3 paint cans will have same amount of paint.

164 Red
−130 Blue
34 Red

188 white
−130 Blue
58 white

34 Red + 58 white = 92 pink

pink $\boxed{92 \text{ oz}}$
Red $\boxed{92 \text{ oz}}$
white $\boxed{92 \text{ oz}}$
Blue $\boxed{92 \text{ oz}}$

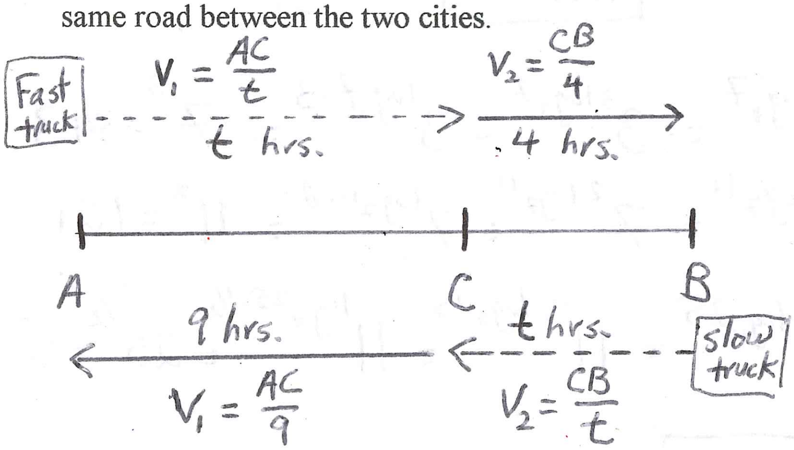
$130(3) - 92(3) = \boxed{114 \text{ ounces}}$

2. At exactly sunrise, a delivery truck starts driving from city A to B. At the same time, another delivery truck is going in the opposite direction from city B to A.

The two trucks pass each other at 12 noon. The truck driving to city B reaches its destination at 4pm. The truck going to city A takes much longer and reaches at 9pm.

What time was sunrise? Assume the two trucks moved at a constant rate, and they traveled along exactly the same road between the two cities.

$\boxed{D = v \cdot t \rightarrow v = \frac{d}{t}}$



$\frac{V_1}{V_2} = 1 \rightarrow \frac{\frac{AC}{t}}{\frac{CB}{4}} = \frac{AC}{t} \cdot \frac{4}{CB} = \boxed{\frac{4AC}{tBC} = 1}$

$\frac{V_1}{V_2} = 1 \quad \frac{\frac{AC}{9}}{\frac{CB}{t}} = \frac{AC}{9} \cdot \frac{t}{CB} = \boxed{\frac{ACt}{9BC} = 1}$

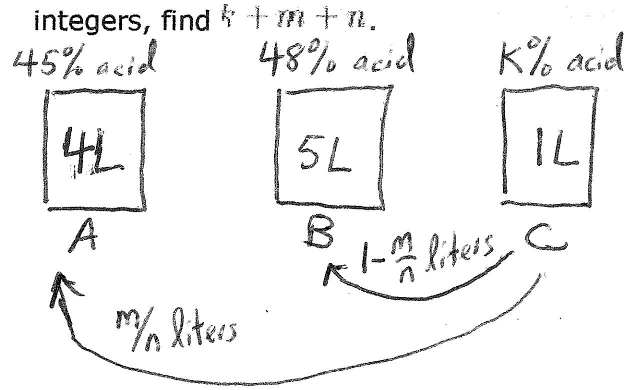
$\frac{4\cancel{AC}}{t\cancel{BC}} = \frac{\cancel{AC}t}{9\cancel{BC}} \qquad \frac{4}{t} = \frac{t}{9}$

$t^2 = 36 \quad t = 6$

$\boxed{\text{sunrise at 6am}}$

3. 2011 AIME 1

Jar A contains four liters of a solution that is 45% acid. Jar B contains five liters of a solution that is 48% acid. Jar C contains one liter of a solution that is $k\%$ acid. From jar C, $\frac{m}{n}$ liters of the solution is added to jar A, and the remainder of the solution in jar C is added to jar B. At the end both jar A and jar B contain solutions that are 50% acid. Given that m and n are relatively prime positive integers, find $k+m+n$.



$A \rightarrow \frac{45}{100}(4) = \frac{9}{5}L$ of acid

$B \rightarrow \frac{48}{100}(5) = \frac{12}{5}L$ of acid

$C \rightarrow \frac{K}{100}(1) = \frac{K}{100}L$ of acid

$$2\left(\frac{9}{5} + \frac{K}{100}\left(\frac{m}{n}\right)\right) = 4 + \frac{m}{n}$$

$$2\left[\frac{12}{5} + \left(1-\frac{m}{n}\right)\left(\frac{K}{100}\right)\right] = 6 - \frac{m}{n}$$

$$\frac{21}{5} + \cancel{\frac{Km}{100n}} + \frac{K}{100} - \cancel{\frac{Km}{100n}} = \frac{10}{2} = 5$$

$$\frac{21}{5} + \frac{K}{100} = 5 \quad \boxed{K=80}$$

$$2\left(\frac{9}{5}\right) + \frac{80}{100}\left(\frac{m}{n}\right) = 4 + \frac{m}{n} \rightarrow \boxed{\frac{m}{n} = \frac{2}{3}}$$

$K+m+n = 80+2+3 = \boxed{85}$

After transfering solution:

$A \rightarrow \frac{9}{5} + \frac{K}{100}\left(\frac{m}{n}\right)$ liters of acid

$B \rightarrow \frac{12}{5} + \frac{K}{100}\left(1-\frac{m}{n}\right)$ liters of acid

solutions in each jar is 50% acid

*multply acid by 2 and equate to respective solution amount in each jar

4. 2009 AIME II Suppose that a, b, and c are positive real numbers such that $a^{\log_3 7} = 27$, $b^{\log_7 11} = 49$, and $c^{\log_{11} 25} = \sqrt{11}$. Find $a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}$.

$\boxed{\text{Recall: } b^{\log_b n} = n}$

$$a^{(\log_3 7)^2} = a^{(\log_3 7)(\log_3 7)} = 27^{\log_3 7} = 3^{3\log_3 7} = 3^{\log_3 7 \cdot 3} = 7^3 = 343$$

$$b^{(\log_7 11)^2} = b^{(\log_7 11)(\log_7 11)} = 49^{\log_7 11} = 7^{2\log_7 11} = 7^{\log_7 11 \cdot 2} = 11^2 = 121$$

$$c^{(\log_{11} 25)^2} = c^{(\log_{11} 25)(\log_{11} 25)} = \sqrt{11}^{\log_{11} 25} = 11^{\frac{1}{2}\log_{11} 25} = 11^{\log_{11} 25 \cdot \frac{1}{2}} = 25^{\frac{1}{2}} = 5$$

$$343 + 121 + 5 = \boxed{469}$$