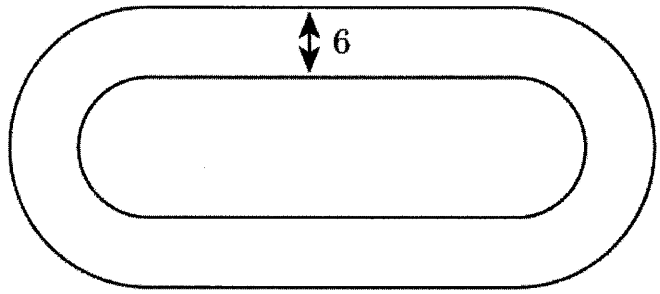
8. Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has width 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?



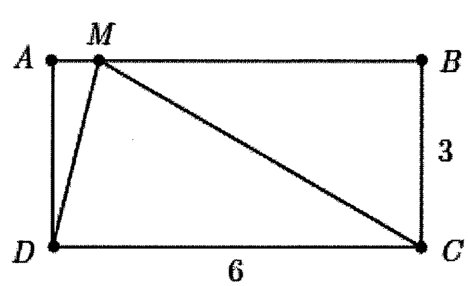
(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) π (D) $\frac{4\pi}{3}$ (E) $\frac{5\pi}{3}$

8. **Answer (A):** The only parts of the track that are longer walking on the outside edge rather than the inside edge are the two semicircular portions. If the radius of the inner semicircle is r, then the difference in the lengths of the two paths is $2\pi(r+6) - 2\pi r = 12\pi$. Let x be her speed in meters per second. Then $36x = 12\pi$, and $x = \frac{\pi}{3}$.

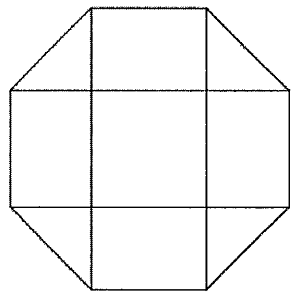
10. Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

(A) 15 (B) 30 (C) 45 (D) 60 (E) 75

10. **Answer (E):** Sides $\overline{AB}$ and $\overline{CD}$ are parallel, so $\angle CDM = \angle AMD$. Because $\angle AMD = \angle CMD$, it follows that ΔCMD is isosceles and $CD = CM = 6$. Therefore ΔMCB is a $30-60-90°$ right triangle with $\angle BMC = 30°$. Finally, $2 \cdot \angle AMD + 30° = \angle AMD + \angle CMD + 30° = 180°$, so $\angle AMD = 75°$.

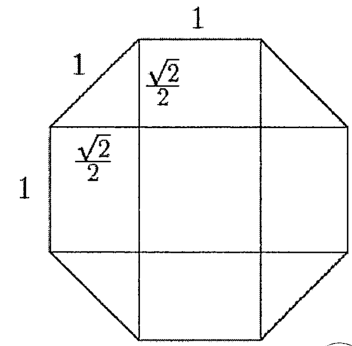


12. A dart board is a regular octagon divided into regions as shown. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?



(A) $\frac{\sqrt{2}-1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{2-\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{4}$ (E) $2-\sqrt{2}$

12. **Answer (A):** Assume the octagon's edge is 1. Then the corner triangles have hypotenuse 1 and thus legs $\frac{\sqrt{2}}{2}$ and area $\frac{1}{4}$ each; the four rectangles are 1 by $\frac{\sqrt{2}}{2}$ and have area $\frac{\sqrt{2}}{2}$ each, and the center square has area 1. The total area is $4\cdot\frac{1}{4}+4\cdot\frac{\sqrt{2}}{2}+1=2+2\sqrt{2}$. The probability that the dart hits the center square is $\frac{1}{2+2\sqrt{2}}=\frac{\sqrt{2}-1}{2}$.



13. Brian writes down four integers $w>x>y>z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6,$ and 9. What is the sum of the possible values for w?

(A) 16 (B) 31 (C) 48 (D) 62 (E) 93

13. **Answer (B):** The largest pairwise difference is 9, so $w-z=9$. Let n be either x or y. Because n is between w and z,

$$9=w-z=(w-n)+(n-z).$$

Therefore the positive differences $w-n$ and $n-z$ must sum to 9. The given pairwise differences that sum to 9 are $3+6$ and $4+5$. The remaining pairwise difference must be $x-y=1$.

The second largest pairwise difference is 6, so either $w-y=6$ or $x-z=6$. In the first case the set of four numbers may be expressed as $\{w, w-5, w-6, w-9\}$. Hence $4w-20=44$, so $w=16$. In the second case $w-x=3$, and the four numbers may be expressed as $\{w, w-3, w-4, w-9\}$. Therefore $4w-16=44$, so $w=15$. The sum of the possible values for w is $16+15=31$.

Note: The possible sets of four numbers are $\{16, 11, 10, 7\}$ and $\{15, 12, 11, 6\}$.

15. How many positive two-digit integers are factors of $2^{24}-1$?

(A) 4 (B) 8 (C) 10 (D) 12 (E) 14

15. **Answer (D):** Factoring results in the following product of primes:

$$2^{24}-1=(2^{12}-1)(2^{12}+1)=(2^6-1)(2^6+1)(2^4+1)(2^8-2^4+1)$$
$$=63\cdot 65\cdot 17\cdot 241=3\cdot 3\cdot 5\cdot 7\cdot 13\cdot 17\cdot 241.$$

The two-digit integers that can be formed from these prime factors are:

$$17,\quad 3\cdot 17=51,\quad 5\cdot 17=85,$$
$$13,\quad 3\cdot 13=39,\quad 5\cdot 13=65,\quad 7\cdot 13=91,$$
$$3\cdot 7=21,\quad 5\cdot 7=35,\quad 3\cdot 3\cdot 7=63,$$
$$3\cdot 5=15,\quad \text{and}\quad 3\cdot 3\cdot 5=45.$$

Thus there are 12 positive two-digit factors.

16. Rhombus $ABCD$ has side length 2 and $\angle B=120°$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

(A) $\frac{\sqrt{3}}{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{2\sqrt{3}}{3}$ (D) $1+\frac{\sqrt{3}}{3}$ (E) 2

16. **Answer (C):** Let E and H be the midpoints of $\overline{AB}$ and $\overline{BC}$, respectively. The line drawn perpendicular to $\overline{AB}$ through E divides the rhombus into two regions: points that are closer to vertex A than B, and points that are closer to vertex B than A. Let F be the intersection of this line with diagonal $\overline{AC}$. Similarly, let point G be the intersection of the diagonal $\overline{AC}$ with the perpendicular to $\overline{BC}$ drawn from H. Then the desired region R is the pentagon $BEFGH$.

Note that $\triangle AFE$ is a $30-60-90°$ triangle with $AE=1$. Hence the area of $\triangle AFE$ is $\frac{1}{2}\cdot 1\cdot \frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{6}$. Both $\triangle BFE$ and $\triangle BGH$ are congruent to $\triangle AFE$, so they have the same areas. Also $\angle FBG=120°-\angle FBE-\angle GBH=60°$, so $\triangle FBG$ is an equilateral triangle. In fact, the altitude from B to $\overline{FG}$ divides $\triangle FBG$ into two triangles, each congruent to $\triangle AFE$. Hence the area of $BEFGH$ is $4\cdot\frac{\sqrt{3}}{6}=\frac{2\sqrt{3}}{3}$.