9. It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

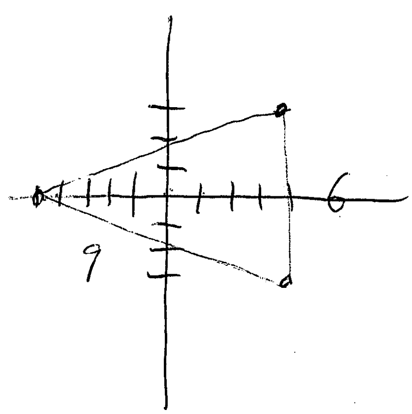
(A) 36 **(B)** 40 **(C)** 42 **(D)** 48 **(E)** 52

9. **Answer (B):** Let x be Clea's rate of walking and r be the rate of the moving escalator. Because the distance is constant, $24(x+r) = 60x$. Solving for r yields $r = \frac{3}{2}x$. Let t be the time required for Clea to make the escalator trip while just standing on it. Then $rt = 60x$, so $\frac{3}{2}xt = 60x$. Therefore $t = 40$ seconds.

10. What is the area of the polygon whose vertices are the points of intersection of the curves $x^2 + y^2 = 25$ and $(x-4)^2 + 9y^2 = 81$?

(A) 24 **(B)** 27 **(C)** 36 **(D)** 37.5 **(E)** 42

10. **Answer (B):** Solve the first equation for y^2 and substitute into the second equation to get $x^2+x-20 = 0$, so $x = 4$ or $x = -5$. This leads to the intersection points $(-5,0)$, $(4,3)$, and $(4,-3)$. The vertical side of the triangle with these three vertices has length $3-(-3) = 6$, and the horizontal height to that side has length $4-(-5) = 9$, so its area is $\frac{1}{2} \cdot 6 \cdot 9 = 27$.



11. In the equation below, A and B are consecutive positive integers, and A, B, and $A+B$ represent number bases:

$$132_A + 43_B = 69_{A+B}.$$

What is $A + B$?

(A) 9 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 17

11. **Answer (C):** First assume $B = A - 1$. By the definition of number bases,

$$A^2 + 3A + 2 + 4(A-1) + 3 = 6(A + A - 1) + 9.$$

Simplifying yields $A^2 - 5A - 2 = 0$, which has no integer solutions.

Next assume $B = A + 1$. In this case

$$A^2 + 3A + 2 + 4(A+1) + 3 = 6(A + A + 1) + 9,$$

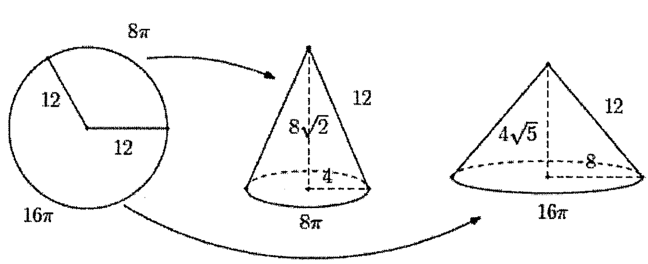
which simplifies to $A^2 - 5A - 6 = (A-6)(A+1) = 0$. The only positive solution is $A = 6$. Letting $A = 6$ and $B = 7$ in the original equation produces $132_6 + 43_7 = 69_{13}$, or $56 + 31 = 87$, which is true. The required sum is $A + B = 13$.

15. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{\sqrt{10}}{10}$ **(D)** $\frac{\sqrt{5}}{6}$ **(E)** $\frac{\sqrt{10}}{5}$

15. **Answer (C):** Each sector forms a cone with slant height 12. The circumference of the base of the smaller cone is $\frac{120}{360} \cdot 2 \cdot 12 \cdot \pi = 8\pi$. Hence the radius of the base of the smaller cone is 4 and its height is $\sqrt{12^2 - 4^2} = 8\sqrt{2}$. Similarly, the circumference of the base of the larger cone is 16π. Hence the radius of the base of the larger cone is 8 and its height is $4\sqrt{5}$. The ratio of the volume of the smaller cone to the volume of larger cone is

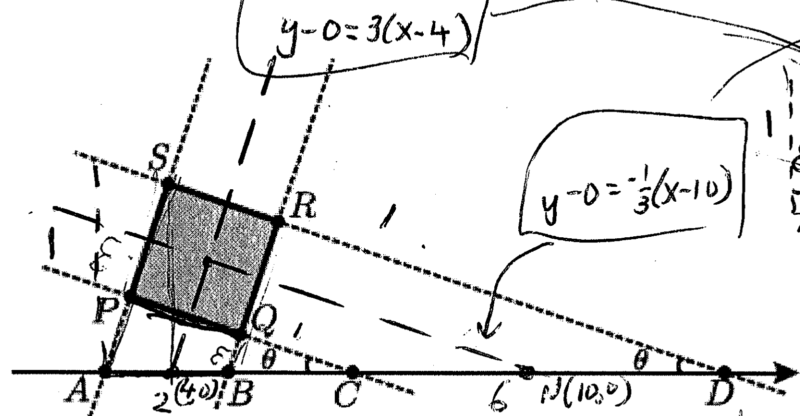
$$\frac{\frac{1}{3}\pi \cdot 4^2 \cdot 8\sqrt{2}}{\frac{1}{3}\pi \cdot 8^2 \cdot 4\sqrt{5}} = \frac{\sqrt{10}}{10}.$$

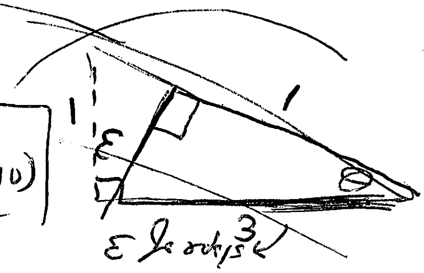


17. Square $PQRS$ lies in the first quadrant. Points $(3,0)$, $(5,0)$, $(7,0)$, and $(13,0)$ lie on lines SP, RQ, PQ, and SR, respectively. What is the sum of the coordinates of the center of the square $PQRS$?

(A) 6 (B) 6.2 (C) 6.4 (D) 6.6 (E) 6.8

17. **Answer (C):** Let $A = (3,0)$, $B = (5,0)$, $C = (7,0)$, $D = (13,0)$, and θ be the acute angle formed by the line PQ and the x-axis. Then $SR = PQ = AB\cos\theta = 2\cos\theta$, and $SP = QR = CD\sin\theta = 6\sin\theta$. Because $PQRS$ is a square, it follows that $2\cos\theta = 6\sin\theta$ and $\tan\theta = \frac{1}{3}$. Therefore lines SP and RQ have slope 3, and lines SR and PQ have slope $-\frac{1}{3}$. Let the points $M = (4,0)$ and $N = (10,0)$ be the respective midpoints of segments AB and CD. Let ℓ_1 be the line through M parallel to line SP. Let ℓ_2 be the line through N parallel to line SR. Lines ℓ_1 and ℓ_2 intersect at the center of the square $PQRS$. Line ℓ_1 satisfies the equation $y = 3(x-4)$, and line ℓ_2 satisfies the equation $y = -\frac{1}{3}(x-10)$. Thus the lines ℓ_1 and ℓ_2 intersect at the point $(4.6, 1.8)$, and the required sum of coordinates is 6.4.





$3x-12=-\frac{1}{3}x+\frac{10}{3}$

$\frac{10}{3}x=\frac{46}{3}$

$x=4.6$

Center of Square $(4.6, 1.8)$

Sum = 6.4

$\cos\theta = \frac{PQ}{2}$ $\sin\theta = \frac{SP}{6}$

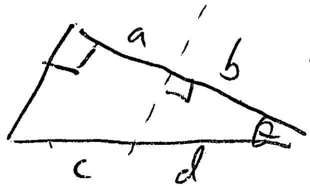
$PQ = 2\cos\theta$ $SP = 6\sin\theta$

$PQ = 2\cos\theta$

$\cos\theta = \frac{PQ}{2}$

$2\cos\theta = 6\sin\theta$

$\frac{1}{3} = \tan\theta$



$\cos\theta = \frac{b}{d} = \frac{a}{c}$

$\cos\theta = \frac{a+b}{c+d}$

$\frac{a}{b} = \frac{c}{d}$ $ad = bc$ $\frac{a}{c} = \frac{b}{d}$

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$B = A-1$

$B = A+1$ ~~$A = B-1$~~ $B = A-1$

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$$132_A + 43_B = 69_{A+B}.$$

$A-1$

$132_{10} =$

~~10~~ 1×10^2

3×10^1

2×10^0

What is $A + B$?

$2A-1 = A+B$

$A = B+1$

(A) 9 (B) 11 (C) 13 (D) 15 (E) 17

$132_A + 43_{A-1} = 69_{A+A-1}$

$1 \times A^2 + 3 \times A^1 + 2 \times A^0 + 4 \times (A-1) + 3(A-1)^0 = 6 \times (2A-1) + 9(1)$

$A^2 + 3A + 2 + 4A - 4 + 3 = 6(2A-1) + 9$

$A^2 + 7A + 1 = 12A - 6 + 9$

$A^2 - 5A - 2 = 0$

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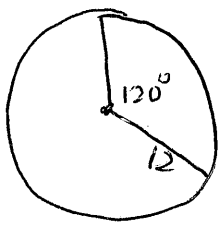
(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{\sqrt{10}}{10}$ (D) $\frac{\sqrt{5}}{6}$ (E) $\frac{\sqrt{10}}{5}$

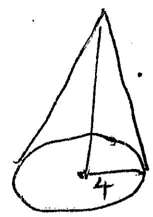


$C = 2\pi r$

12

$\frac{120}{360} \cdot 2\pi(r)$

$\frac{1}{3}(24\pi) = 8\pi$



$C = 8\pi$ $r = 4$

$C = 2\pi r$

$V = \frac{1}{3}\pi r^2 h$

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(A) 6 (B) 6.2 (C) 6.4 (D) 6.6 (E) 6.8